The birth of modern physics

Lord Kelvin said in the late 19th century that physics is a finished science:

"There is no problem that science cannot solve. Physics is a finished science, our theories work so well that they must be correct. Maybe there are two tiny clouds in the clear blue sky."

However, these clouds (light propagation and thermal radiation) shook physics to its foundations and led to the creation of two new theories:

- Relativity (special and general)

- Quantum physics

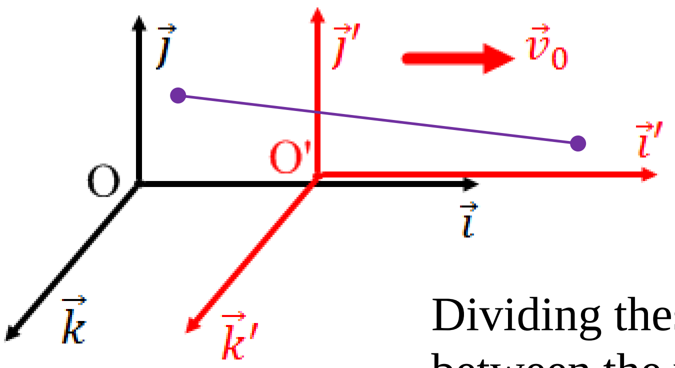
Thus, the beginning of the 20th century also marked the beginning of modern physics.

Galileo's principle of relativity

In any two reference frames moving at **constant** velocity relative to each other, **mechanical phenomena** occur in the same way.
E.g., apart from the vibration, we cannot feel whether the train is moving if it is moving at a constant speed. The dropped coin also falls vertically with uniform acceleration.
Therefore, none of these reference systems is distinguished, there is no absolutely stationary reference system.

Relationship between systems moving relative to each other:

Let system *K*' move relative to *K* in the positive *x* direction with **constant** velocity v_0.



After time Δt, distance between origins: $\overline{OO'} = v_0 \Delta t$

Measured coordinate differences in *K*' are:

$$\Delta x' = \Delta x - v_0 \Delta t$$
$$\Delta y' = \Delta y$$
$$\Delta z' = \Delta z$$

Also: $\Delta t' = \Delta t$ (clocks in sync)

Dividing these by Δt (or $\Delta t'$) we get the relationship between the velocities (purple line is the trajectory of a moving body):

$$v_x' = v_x - v_0$$
$$v_y' = v_y$$
$$v_z' = v_z$$

The speed of light

Writing Maxwell's equations in a *K* reference frame, we obtain the homogeneous wave equation:

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} - \varepsilon\mu \frac{\partial^2 E_i}{\partial t^2} = 0$$

Comparing with the general wave equation, we get for the propagation velocity:

$$v = \frac{1}{\sqrt{\varepsilon\mu}}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0$$

<u>Question:</u> With respect to what should this speed be measured in case of electromagnetic wave (light)? With respect to the medium in which light travels?

But what about light propagating in a vacuum?!

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \approx 3 \cdot 10^8 \ \frac{\mathrm{m}}{\mathrm{s}}$$

<u>Proposal:</u> There is a hypothetical all-pervading medium ("**ether**") that provides the "elastic" medium for the propagation of electromagnetic waves, in which the wave can propagate.

This would mean that an absolute resting frame of reference could be selected using electromagnetic phenomena.
This distinguished system would be fixed to the ether, and the electromagnetic wave (light) would propagate at a speed *c* relative to it.

Propagation of light examined in a moving system

Flash of light starting from origin reaches spherical surface of radius $c\Delta t$ in time Δt viewed in K (frame at rest with respect to ether): $\Delta x^2 + \Delta y^2 + \Delta z^2 = c^2\Delta t^2$

or $$c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = 0$$

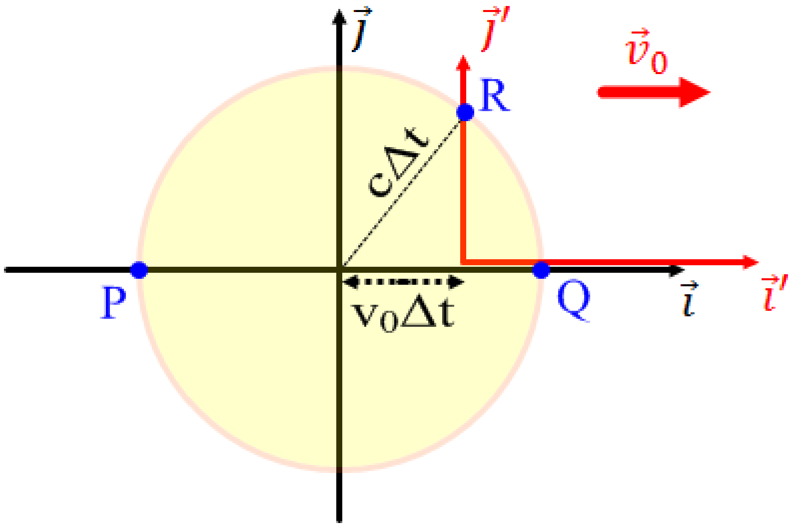
So the points P, Q, and R are all at a distance $c\Delta t$ from the origin measured in K.

The same distances in the K' system:

$$l_P = c\Delta t + v_0\Delta t = (c + v_0)\Delta t$$

$$l_Q = c\Delta t - v_0\Delta t = (c - v_0)\Delta t$$

$$l_R = \sqrt{c^2\Delta t^2 - v_0{}^2\Delta t^2} = \sqrt{c^2 - v_0{}^2}\Delta t$$



So the speed of light in different directions in the moving system is different:
(directions marked with points P, Q, R)

$$c_P' = \frac{l_P}{\Delta t} = c + v_0 \qquad c_Q' = \frac{l_Q}{\Delta t} = c - v_0 \qquad c_R' = \frac{l_R}{\Delta t} = \sqrt{c^2 - v_0{}^2}$$

This, in principle, allows us to determine the speed of our movement relative to the ether.

The Michelson experiment

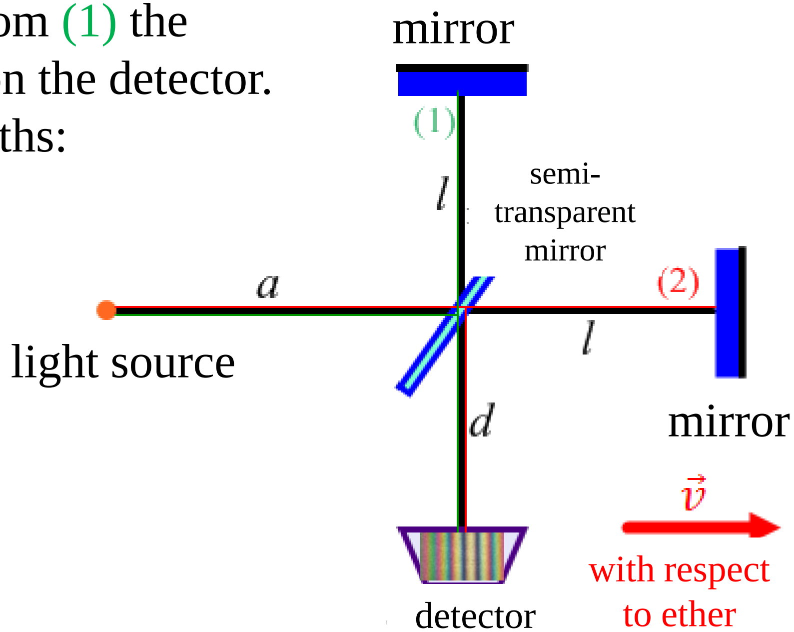
The purpose of the experiment was to determine the speed of the Earth relative to the ether. The rays passing through (2) and reflecting from (1) the semi-transparent mirror create interference fringes on the detector. Determining the time difference between the two paths:

$$t_1 = \frac{2l}{\sqrt{c^2 - v^2}} = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx \frac{2l}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{l(c+v) + l(c-v)}{(c-v)(c+v)} =$$

$$= \frac{2lc}{c^2 - v^2} = \frac{2l/c}{1 - \frac{v^2}{c^2}} \approx \frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right)$$

$$\Delta t = t_2 - t_1 \approx \frac{2l}{c}\frac{1}{2}\frac{v^2}{c^2} = \frac{lv^2}{c^3}$$



If we rotate the device by 90 degrees, the roles of the arms are reversed:

$$\Delta t^* = t_2 - t_1 = -\frac{lv^2}{c^3}$$

So the position of the fringes shifts because of this.

No shift was detected!

Nowadays, a velocity of 3 m/s relative to the ether could be detected, but the result is negative.

<u>Thus:</u> there is **no ether**, light propagates in all reference frames, in all directions, with *c*.

<u>The principle of special relativity:</u> Frames moving at constant speed relative to each other are equivalent from the point of view of the laws of physics. Equations are of similar form.

Lorentz transformation

The negative result of Michelson's experiment suggests that we cannot distinguish between systems moving at a constant velocity relative to each other using electromagnetic phenomena. Since the phase of the light wave in both *K* and *K*' is a spherical surface expanding at speed *c*:

$$c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = 0$$
$$c^2 \Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2 = 0$$

Galileo's method of adding velocities cannot be valid for the motion of light; it is an approximation only valid for slowly moving (relative to light) reference frames.
<u>Einstein:</u> Let's give up the $\Delta t = \Delta t'$ constraint. Just as we cannot speak of absolute space, we cannot speak of absolute time either.

Let's find the transformation rule that connects the coordinate and time differences Δx, Δy, Δz, Δt and the coordinate and time differences $\Delta x'$, $\Delta y'$, $\Delta z'$, $\Delta t'$!

<u>Conditions:</u>
1. Events that happen at same time and position in one system should also be at same place and time in the other system. If $\Delta t = \Delta x = \Delta y = \Delta z = 0$, then $\Delta t' = \Delta x' = \Delta y' = \Delta z' = 0$.
2. The quantities of *K* are transformed into *K*' by a function of similar form as the quantities of *K*' into *K* (neither is distinguished). The transformation is linear.
3. In the limiting case $v \ll c$ we get back the Galilean transformation.
4. The speed of light should be the same in all reference frames.

Lorentz transformation formulas

Let the system *K*' move relative to *K* in the positive *x* direction with a constant velocity *v*. The transformation can be written in the following general form:

$$\Delta x' = \xi_x \Delta x + \xi_t \Delta t \qquad \Delta t' = \tau_x \Delta x + \tau_t \Delta t \qquad \Delta y' = \kappa \Delta y \qquad \Delta z' = \kappa \Delta z$$

where $\xi_x, \xi_t, \tau_x, \tau_t,$ κ are factors independent of the coordinates, only dependent on the velocity *v*.

Velocity of *O*' ($\Delta x' = 0$) in *K* is *v*, so: $v = \frac{\Delta x}{\Delta t} = -\frac{\xi_t}{\xi_x} \rightarrow \xi_t = -v\xi_x = -v\xi$

Using this for $\Delta x'$: $\Delta x' = \xi(\Delta x - v\Delta t)$

Since the wave propagates at the same speed *c* in both systems:

$$c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = c^2\Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2$$

By writing the transformation formulas and collecting the like terms, we obtain four equations for ξ, τ_x, τ_t, and κ. From these:

$$\xi = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \tau_t \qquad \kappa = 1 \qquad \tau_x = -\frac{\xi v}{c^2}$$

So the transformation formulas are:

$$\Delta x' = \frac{\Delta x - v\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}; \quad \Delta y' = \Delta y; \quad \Delta z' = \Delta z; \quad \Delta t' = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

If we want to express quantities without commas, we must write –*v* everywhere instead of *v*.

Relativity of simultaneity

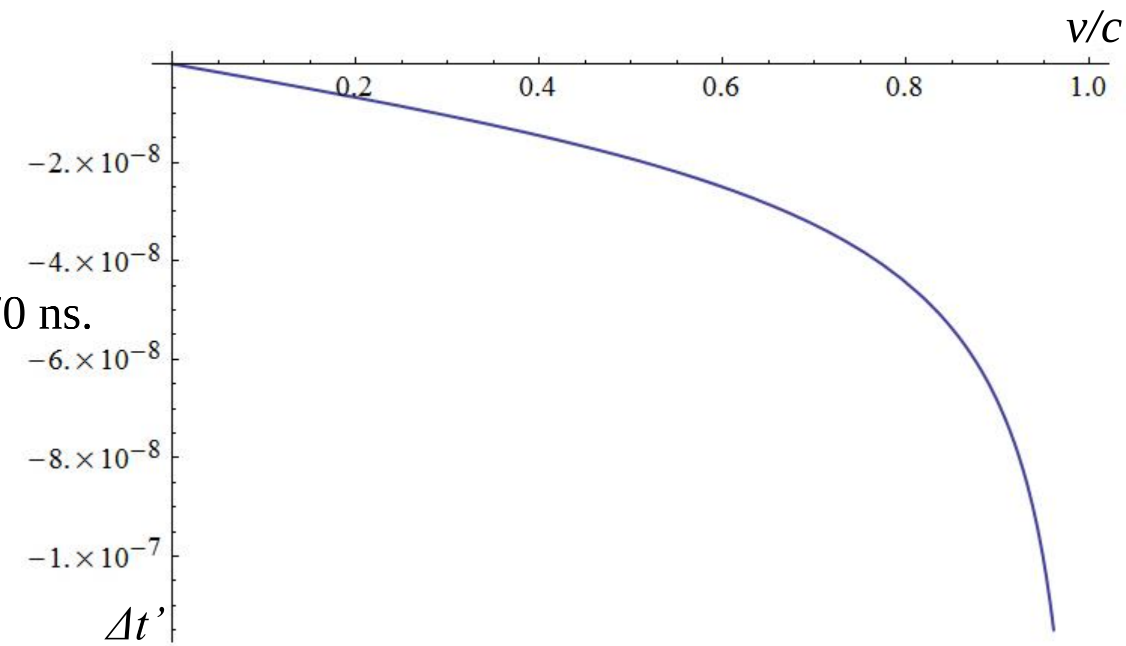
Let two events (observed in the system *K*) occur at different locations ($\Delta x \neq 0$) but at the same time ($\Delta t = 0$).

Then the time difference between the two events observed from the *K'* system:

$$\Delta t' = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{\frac{v\Delta x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \neq 0$$

So for an observer moving with velocity *v* relative to *K* (resting in *K'*), the two events will not happen at the same time!

For example:
If Δx = 10 m and v = 0.9c
then the time difference is about 70 ns.



Time dilation

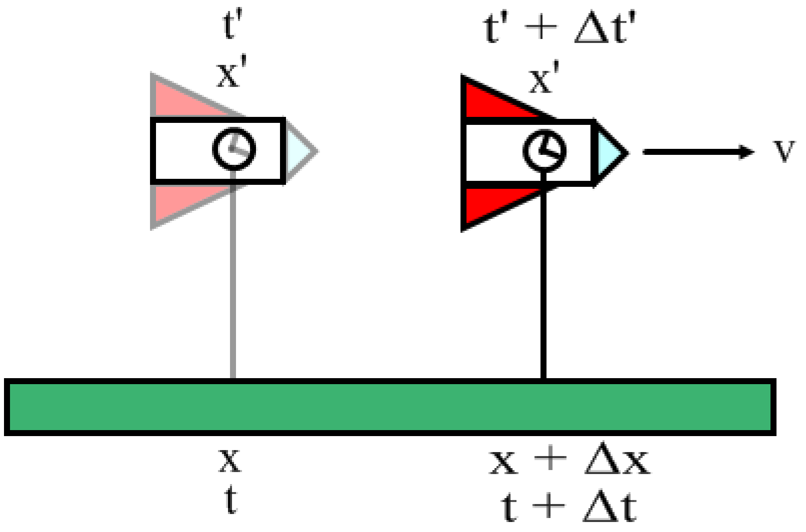
If a clock with a constant speed v passes a point and then after a time Δt (measured in K) passes another point at a distance Δx, then this time is measured differently by the moving clock.

Let's fix the *K*' system to the moving clock, so $\Delta x' = 0$.
Expressing the time measured in *K* in terms of the time of the moving clock:

$$\Delta t = \frac{\Delta t' + v\Delta x'/c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

So the moving clock shows a shorter time (proper time) for the time required to cover the distance than the stationary clock.

<u>Experimental evidence:</u>
Due to cosmic rays, μ-mesons are produced at an altitude of about 100 km, which have a half-life of 2.2 μs. Even with the speed of light 100 km takes 0.333 ms to cover.
The detection of mesons at sea level therefore proves time dilation.



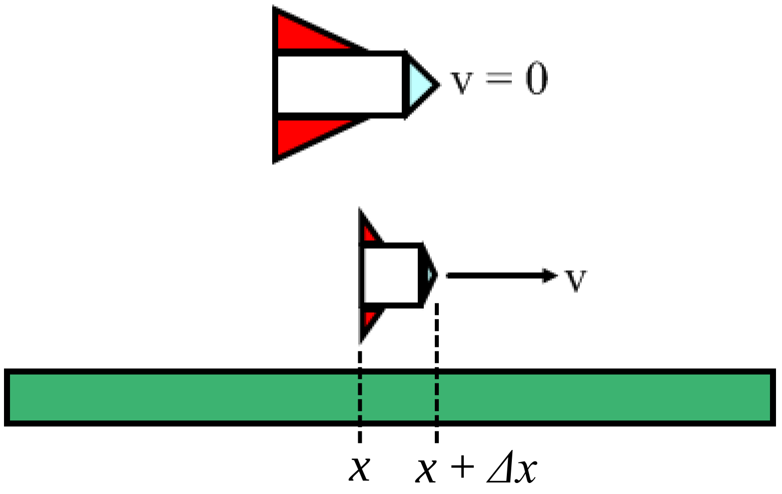
Length contraction

In the system *K*' moving with velocity *v*, a rigid rod rests in the *x* direction. The start and end of the rod are at a distance $\Delta x'$ from each other (length of the rod: L_0).

If we want to measure the length of the rod in *K*, we need to determine the distance Δx, such that the beginning of the rod passes through $x + \Delta x$ and the end passes through *x* in the same time, so $\Delta t = 0$.

$$\Delta x' = \frac{\Delta x - v\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta x}{\sqrt{1-\frac{v^2}{c^2}}}$$

So the apparent length of the rod ($\Delta x = L$) is less than the rest length ($\Delta x' = L_0$)



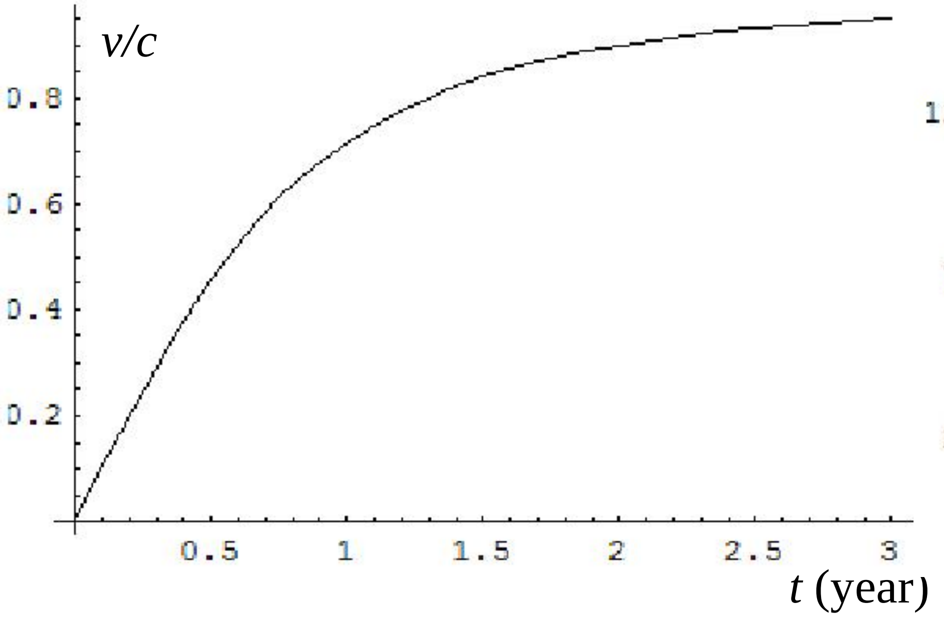
Interstellar travel

Due to time dilation, the crew will measure significantly less time than they would on Earth if the spacecraft approaches the speed of light.

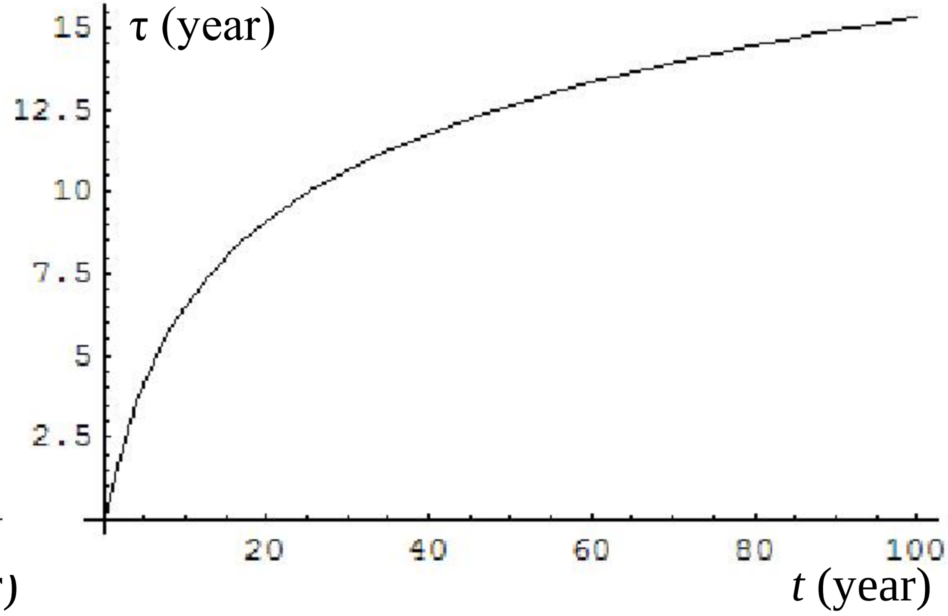
For example: Halfway through the journey, the ship accelerates at a constant acceleration of 1 g = 10 m/s^2, then slows down at the same rate, stopping at the destination, and then returning to Earth in the same way:

Bussard jet-powered starship

Spaceship speed as a function of Earth time



The round trip time (τ) measured on the spacecraft as a function of the time measured on Earth (t)



Equivalence principle

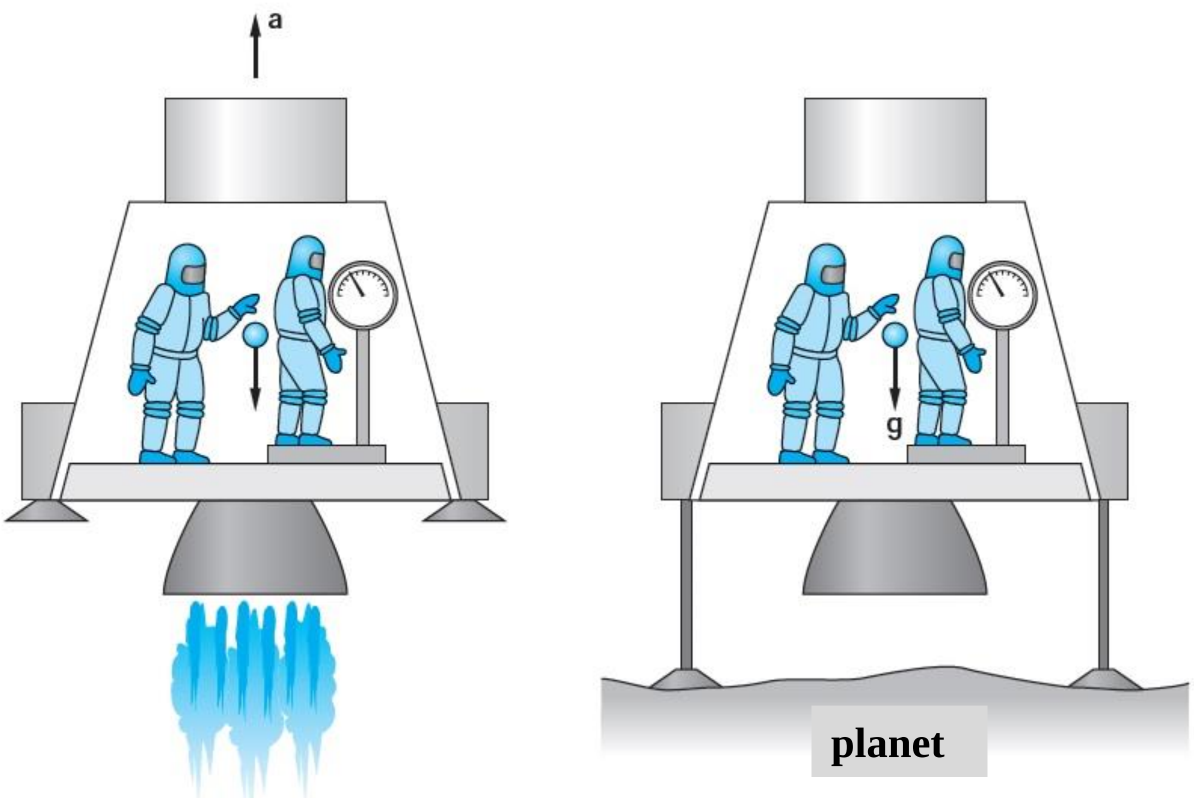
A homogeneous gravitational field is equivalent in all respects to a uniformly accelerated frame of reference.

There is no experiment that could help distinguish!
Gravity is also an apparent force (e.g. centrifugal) – it can be transformed away by choosing an appropriate system.

The principle of relativity:
for all systems!
inertial and
non-inertial (accelerating)

Acceleration is also relative

Principle of general relativity 1916

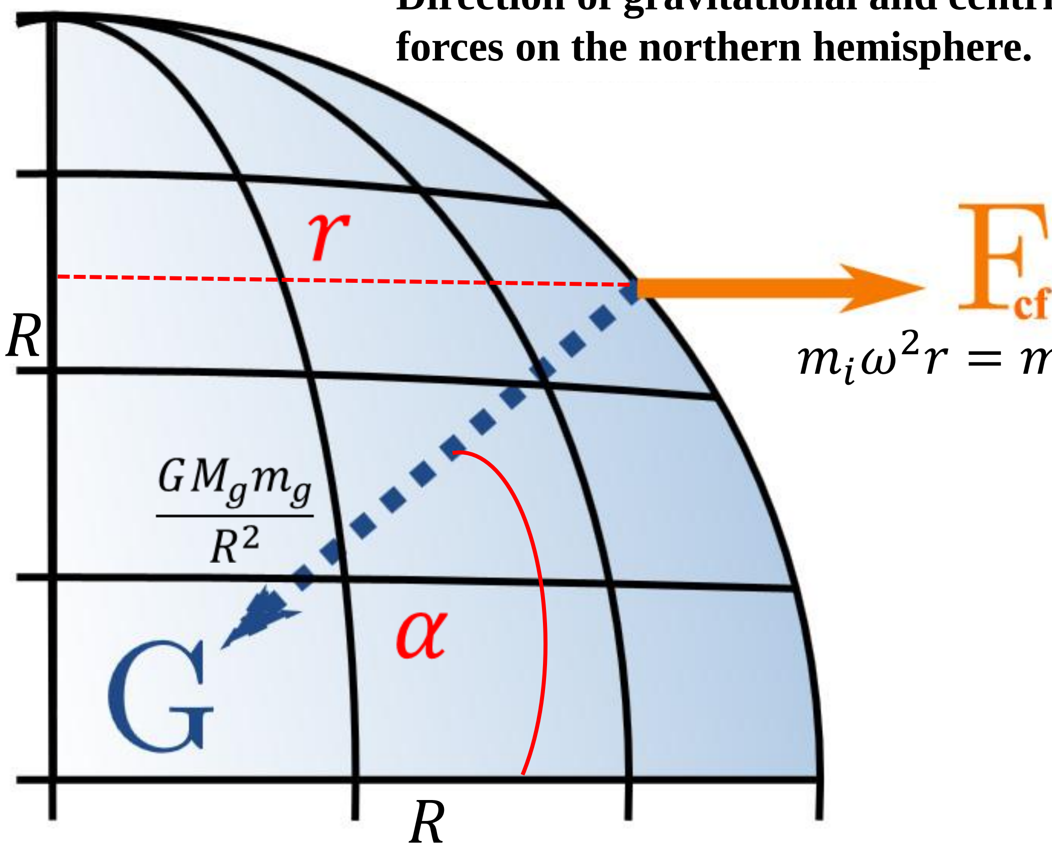


Gravitational mass and inertial mass

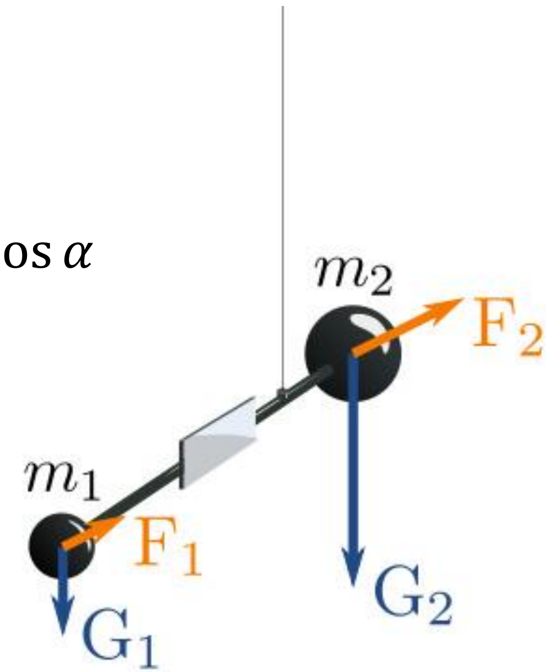
The equivalence principle means that gravitational mass and inertial mass are equal.

Proof: Using an Eötvös pendulum ($< 10^{-10}$ % deviance)

Direction of gravitational and centrifugal forces on the northern hemisphere.



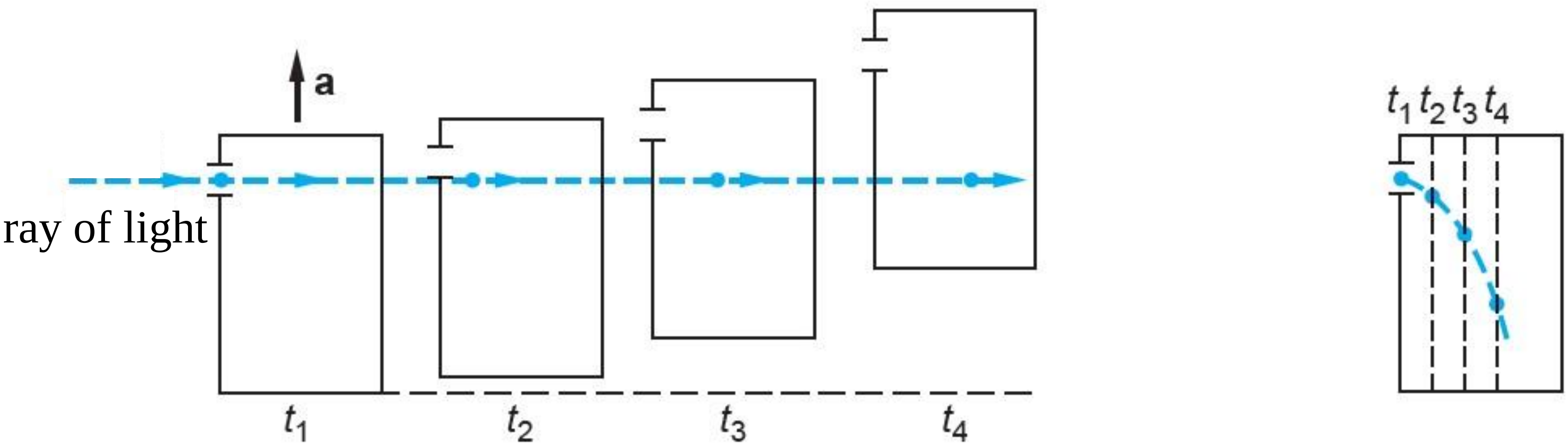
$$m_i \omega^2 r = m_i \omega^2 R \cos\alpha$$



Equilibrium if: $\frac{F_1}{F_2} = \frac{G_1}{G_2}$

Light deflection in gravitational field

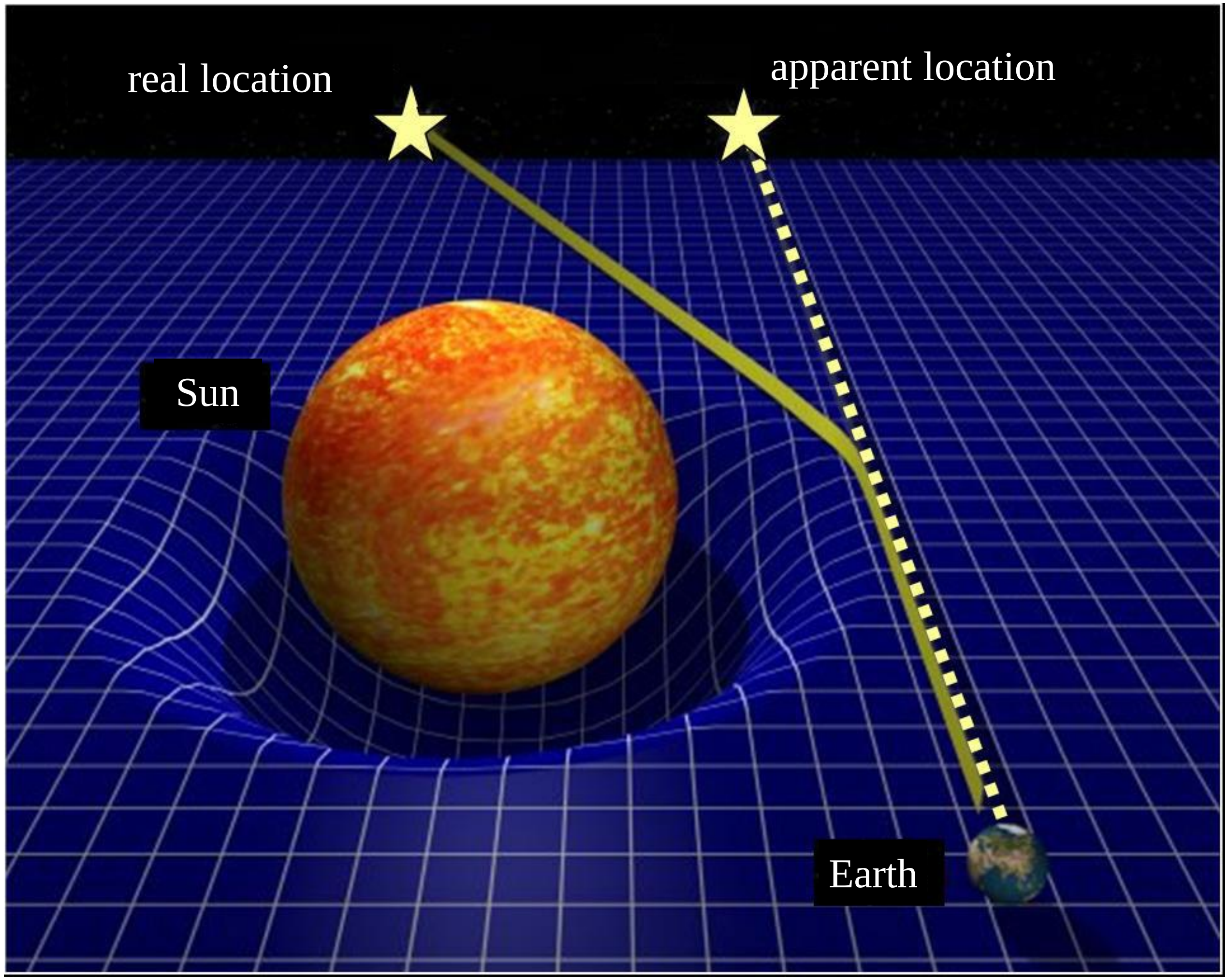
Using the equivalence principle, we examine the path of light in an accelerating system. Trajectory observed in accelerating system will match the trajectory observed in gravitational field.



Example:
We shine a powerful laser horizontally on the Earth's surface. How much does the path deviate from a straight line after traveling 1km due to Earth's gravity?

Experimental proof of light deflection

Eddington confirmed the deflection angles predicted by Einstein by observing stars visible near the Sun during a solar eclipse.



It can be explained based on Snell's law.

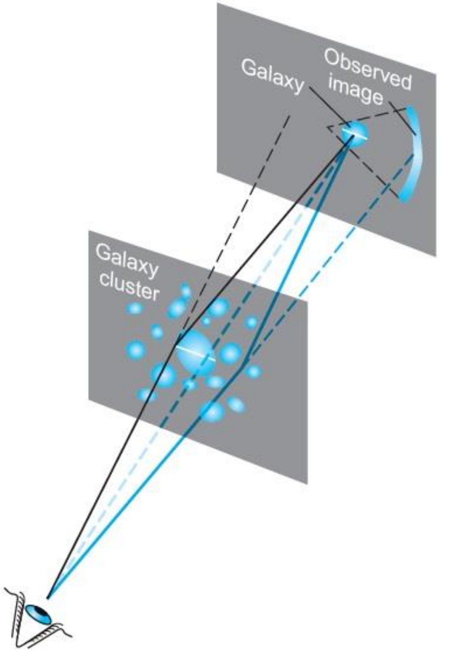
Light slows down in the gravitational field of the Sun to $\gamma(r)c$ speed.

$$\gamma(r) = \sqrt{1 - \frac{2GM}{c^2 r}}$$

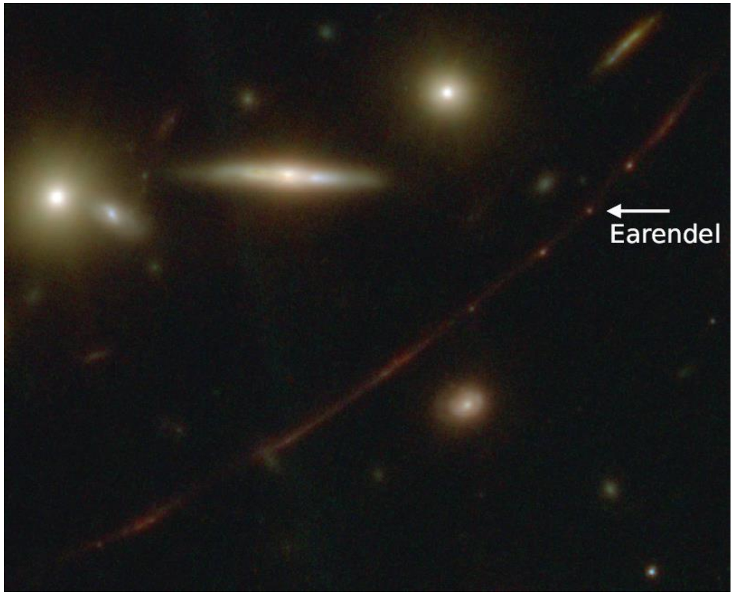
This leads to a rotation of the wavefront.

Gravitational lensing

The massive foreground galaxy cluster acts like a lens, distorting and magnifying the distant galaxies behind it.

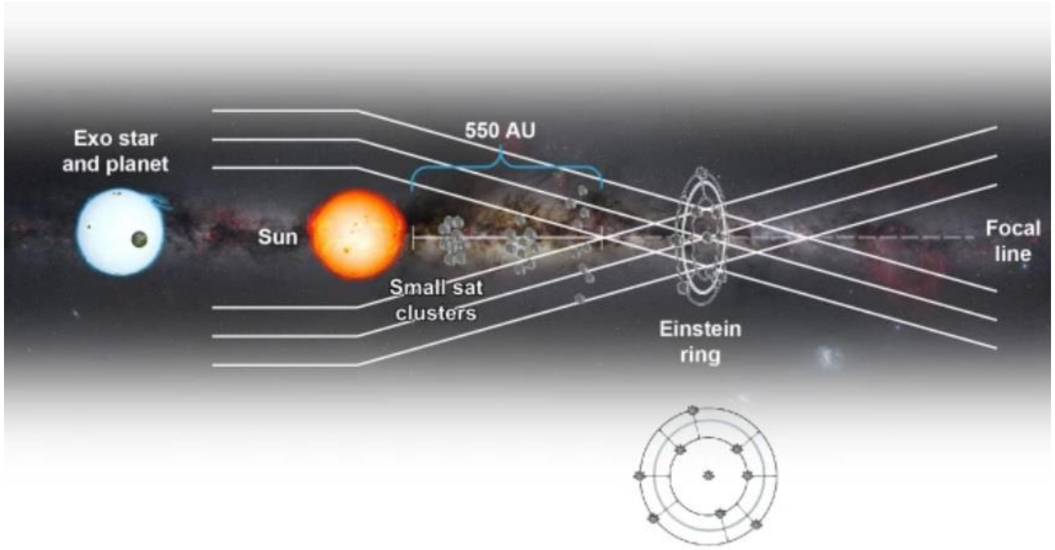


observer

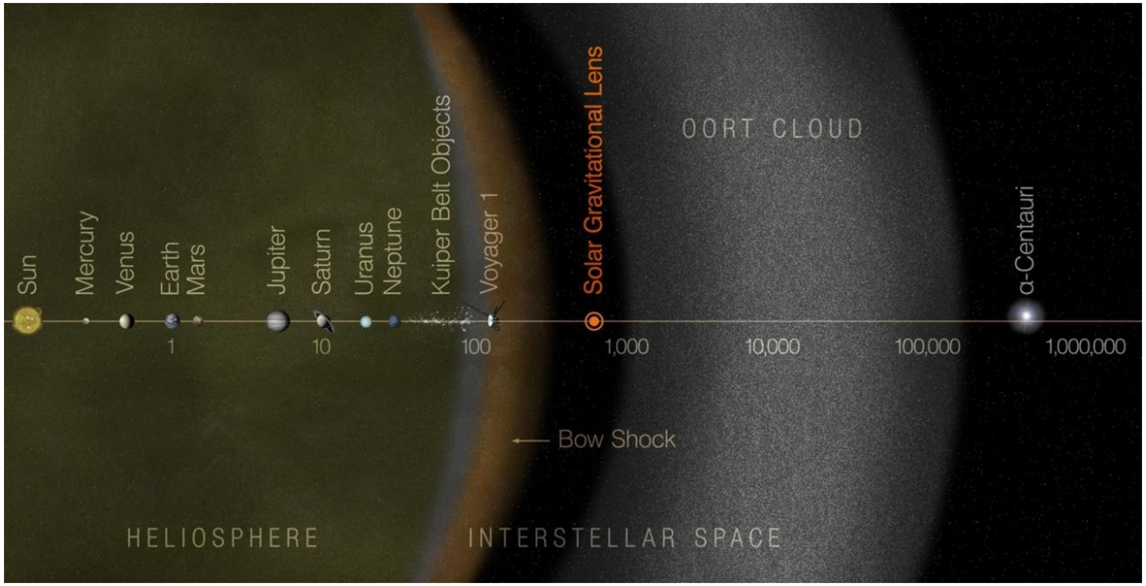


Gravitational telescope

The Sun's gravity can be used as a kind of lens by a swarm of satellites placed at a suitable distance.
100 light years ~10km/pixel



Einstein ring observed by the Hubble Space Telescope



Effect of mass on spacetime

Special relativity: invariant arc element in differential form

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

Minkowski spacetime (flat spacetime)

$d\tau$: proper time (time for observer resting in reference frame)

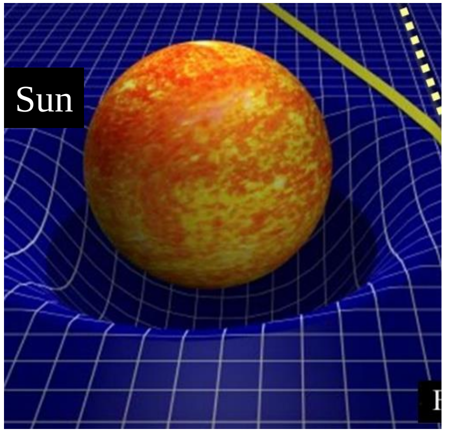
Converted to spherical polar coordinates:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

In the case of spherically symmetric mass distribution, the invariant arc element:

$$ds^2 = c^2 d\tau^2 = \gamma(r)^2 c^2 dt^2 - \frac{dr^2}{\gamma(r)^2} - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

Schwarzschild spacetime (curved spacetime)



$$\gamma(r) = \sqrt{1 - \frac{2GM}{c^2 r}} = \sqrt{1 - \frac{r_S}{r}}$$

r_S: Schwarzschild radius

Gravitational time dilation and redshift

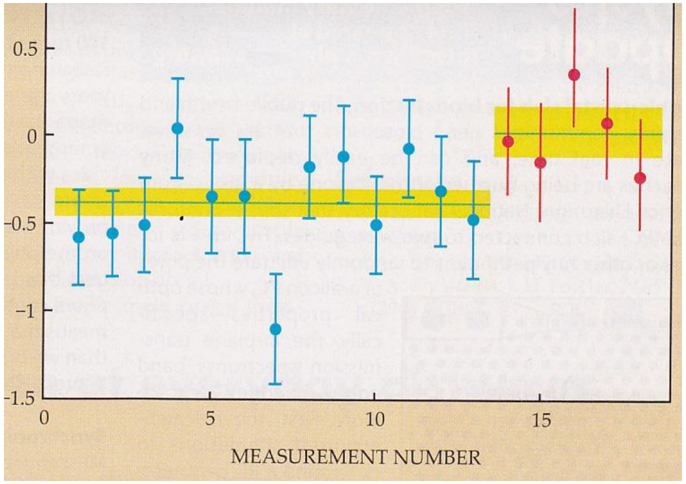
Comparing observers at rest located at infinity and at distance r from spherically symmetric object of mass M: $dr = 0, d\theta = 0, d\varphi = 0$

$$ds^2 = c^2 d\tau^2 = \gamma(r)^2 c^2 dt^2$$

$$\boxed{d\tau = \gamma(r)\, dt}$$

$$\gamma(r) = \sqrt{1 - \frac{2GM}{c^2 r}} = \sqrt{1 - \frac{r_S}{r}} < 1$$

The closer it is, the less time passes. Stronger gravity - slower clocks

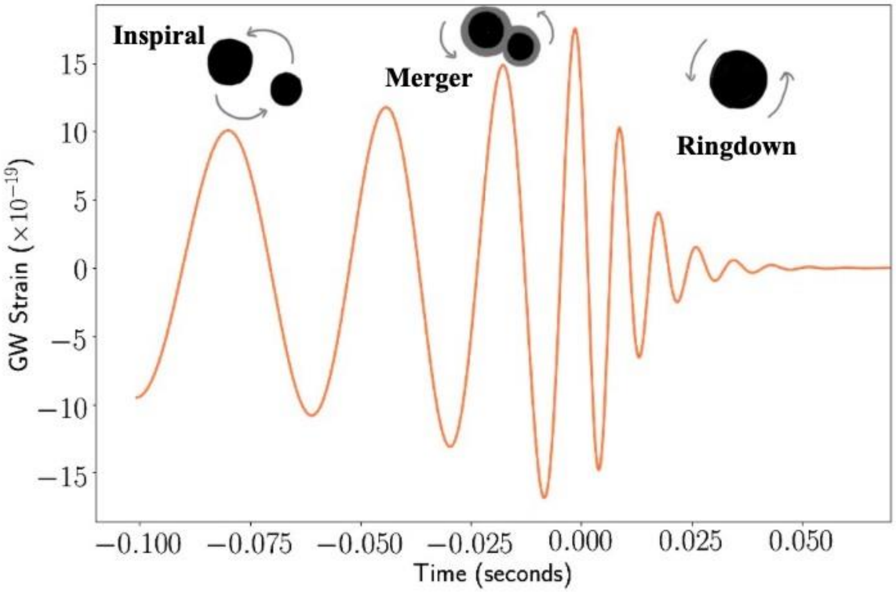


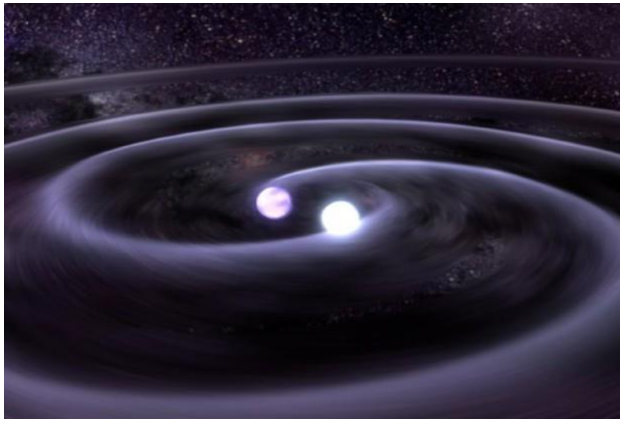
Due to the slowing down of time, the frequency of light waves will be lower due to gravity:
redshift if light travels upwards
blueshift if light travels downwards
(detected for gamma photons for 22 meters)

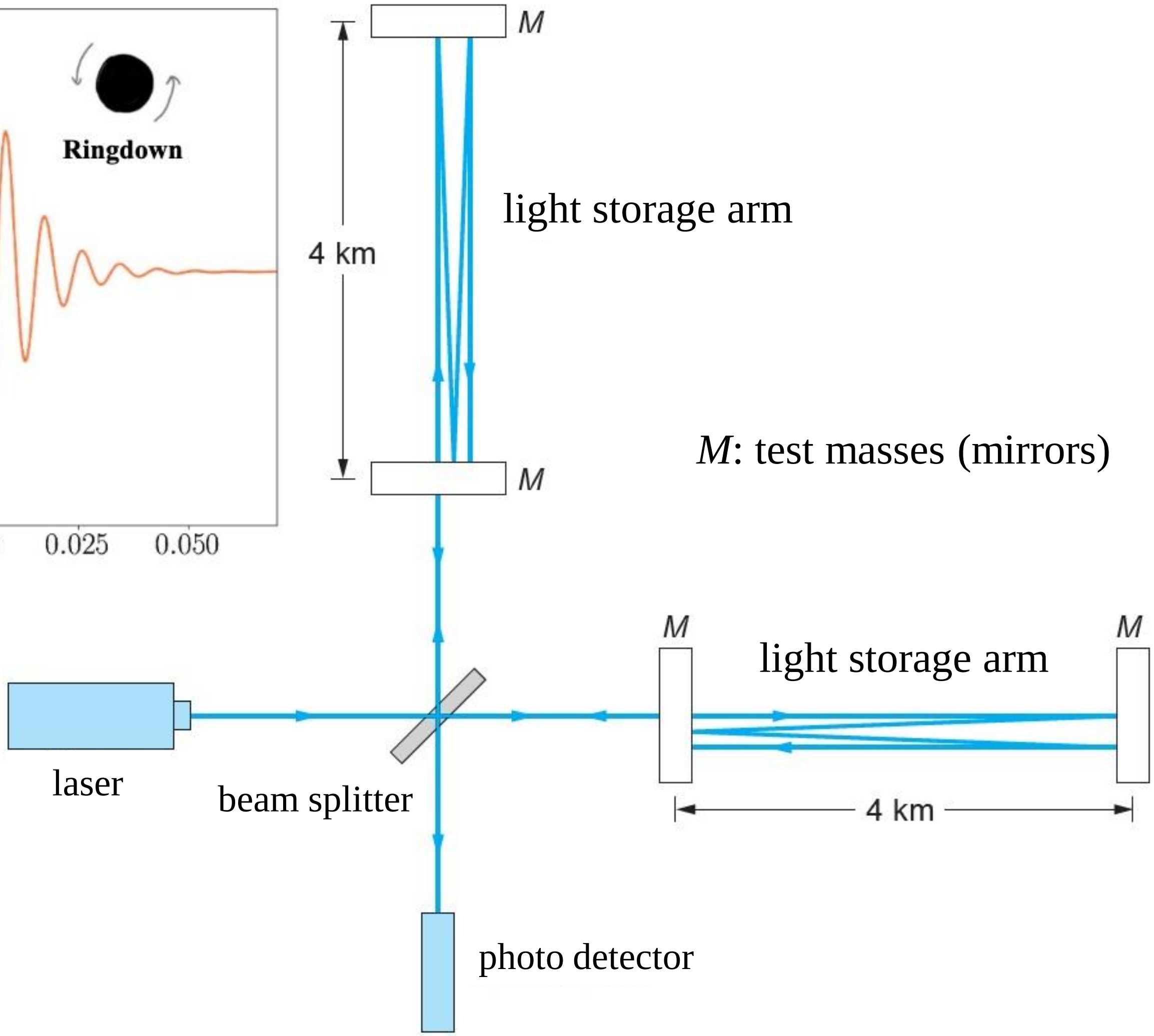
Atomic clocks have detected a **33cm** height difference on Earth!

Gravitational waves

When massive celestial bodies are accelerated, changes in spacetime propagate at the speed of light, e.g. when neutron stars or black holes merge.



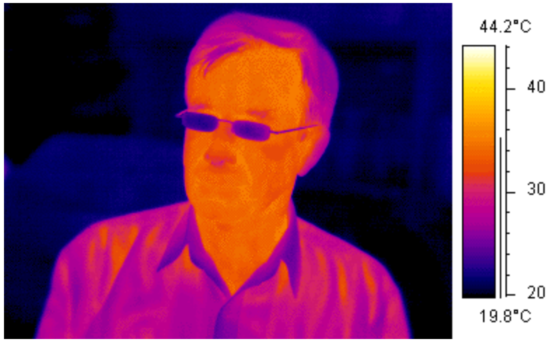




Thermal radiation

In the case of heated objects, a faint reddish light can be observed when the temperature reaches about 525 ºC, and then upon further heating they glow yellow or white, i.e. they emit light (electromagnetic waves in the visible range).

- So with increasing temperature the spectrum **shifts** to shorter wavelengths and the emitted **power** rapidly**increases**.



Although we can only see the radiation of very hot bodies with our own eyes, we can also measure the radiation of bodies at lower temperatures with the help of instruments. Every object whose temperature is not absolute zero radiates.

Thermal radiation is also called blackbody radiation.

Ideal black body: one that completely absorbs the radiation incident on it, and the radiation emitted depends only on the temperature. This can be achieved with a hollow body made of any material and a small hole in it, because it is true for a hole that all

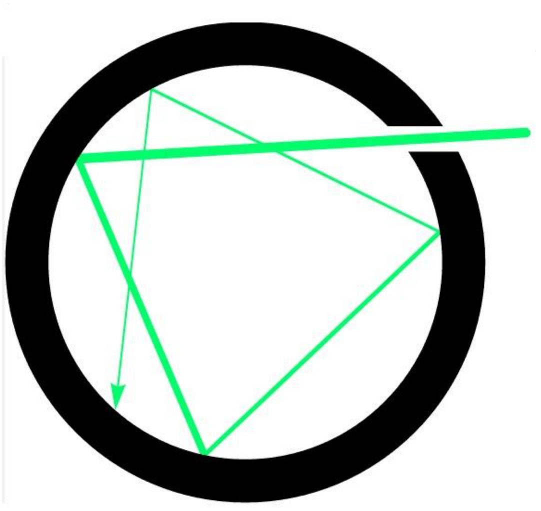
• the radiation incident on it enters the cavity through the hole.

• the light reflected from the inner wall of the cavity most likely remains inside and is absorbed inside.

• thermodynamic equilibrium is established between the electromagnetic radiation and the material.

• the spectrum of the radiation then depends only on the temperature of the material.

Generation of thermal radiation

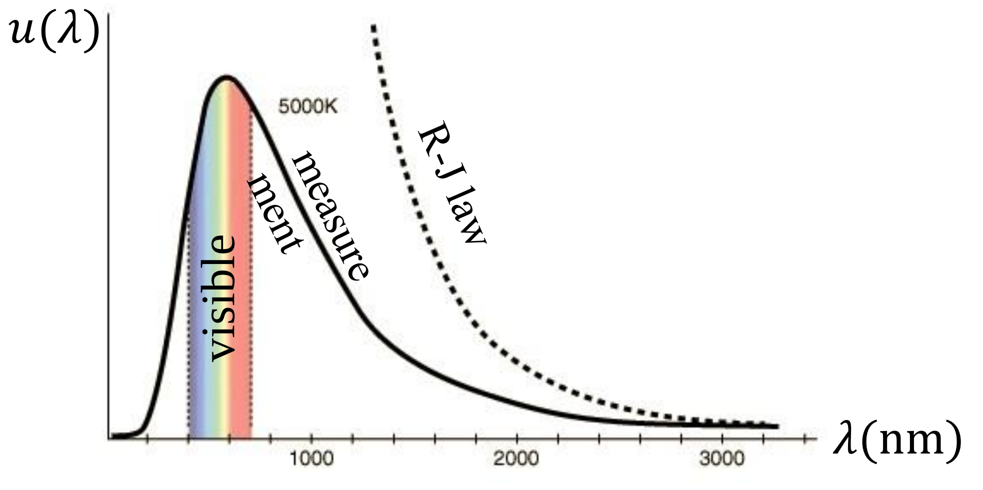


A large number of oscillators in the wall of the material perform irregular vibrations at all kinds of frequencies.

Emission: vibrating charges emit radiation, which includes all kinds of frequencies and therefore wavelengths.

Absorption: radiation incident on a material causes oscillators with the appropriate natural frequency to resonate, so they absorb energy from the radiation.

Rayleigh-Jeans law: taking into account this interaction between radiation and matter, the classical spectral energy density derived using Maxwell's equations tends to infinity for short wavelengths (ultraviolet catastrophe).



Oscillators with quantized energy

Planck (1900): the energy of an oscillator with frequency *f* cannot be a continuous arbitrary value, but an integer number time the ε energy quantum.

$\varepsilon = hf$ where *h* is the Planck's constant: $h = 6{,}626\cdot 10^{-34}$ Js

This assumption meant the beginning of **quantum physics**.
He obtained the following for the emission as a function of λ (spectral energy density):

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/(\lambda kT)-1}}$$ where *k* is Boltzmann's constant: $k = 1{,}38\cdot 10^{-23}$ J/K

Wien's displacement law: differentiating $u(\lambda, T)$ with respect to the wavelength the position of the peak:

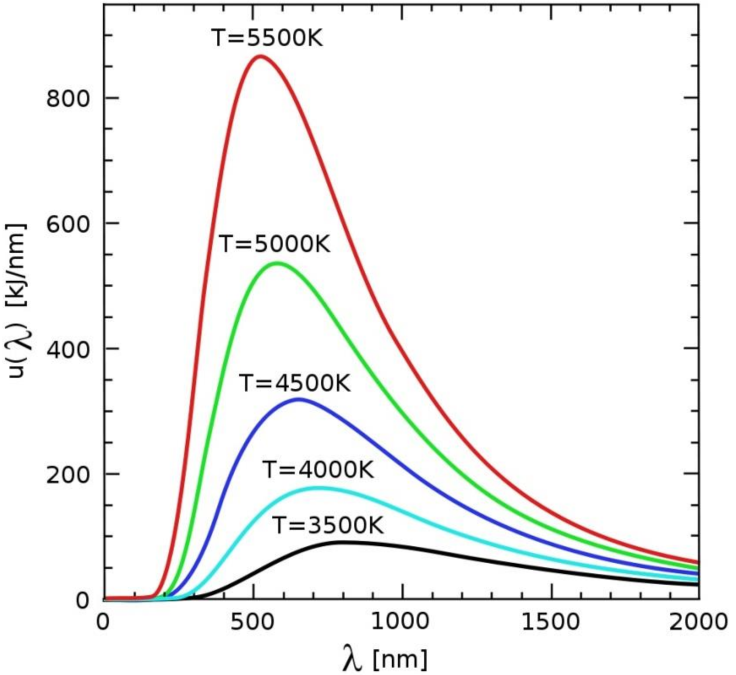
$$\lambda_{max}\cdot T = \text{constant}$$

The Wien constant is $2{,}9\cdot 10^{-3}$ Km.

Stefan-Boltzmann law: integrating the $u(\lambda, T)$ with respect to the wavelength (area under curve) we get the total radiated power. An ideal blackbody with *T* temperature and *A* surface area:

$$P = \sigma\cdot T^4\cdot A$$

where $\sigma = 5{,}67\cdot 10^{-8}$ W/(m^2·K^4) is the Stefan-Boltzmann constant.



Pressure caused by light

Intensity: energy reaching a unit area perpendicular to the direction of light per unit time

$$I = \frac{E}{tA} = \frac{Nhf}{tA} = \frac{Nhc}{tA\lambda}$$

Change in momentum for one photon:

- when absorbed $\left|\overrightarrow{\Delta p_f}\right| = h/\lambda$
- when reflected $\left|\overrightarrow{\Delta p_f}\right| = 2h/\lambda$

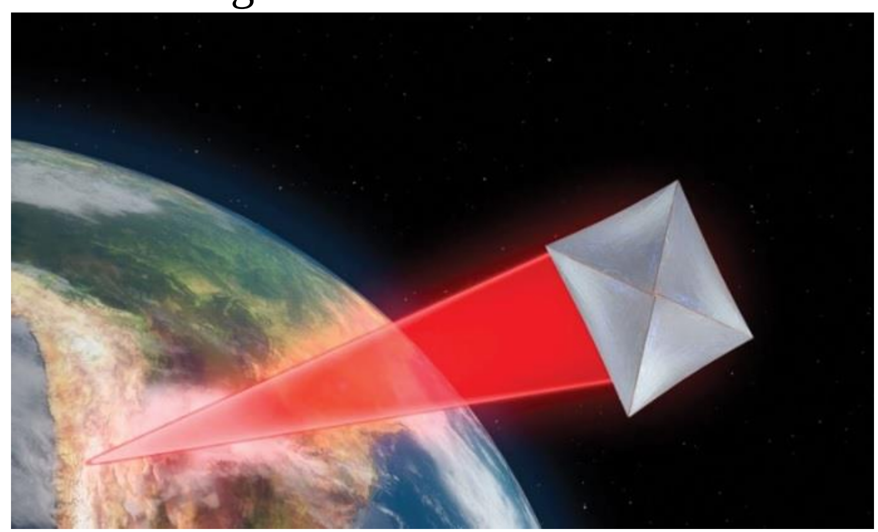
The pressure exerted by light on an absorbing surface:

$$p = \frac{F}{A} = \frac{\left|\overrightarrow{\Delta p}\right|/t}{A} = \frac{N\left|\overrightarrow{\Delta p_f}\right|}{tA} = \frac{Nh}{tA\lambda} = \frac{I}{c}$$

for reflecting surface: $p = \frac{2I}{c}$

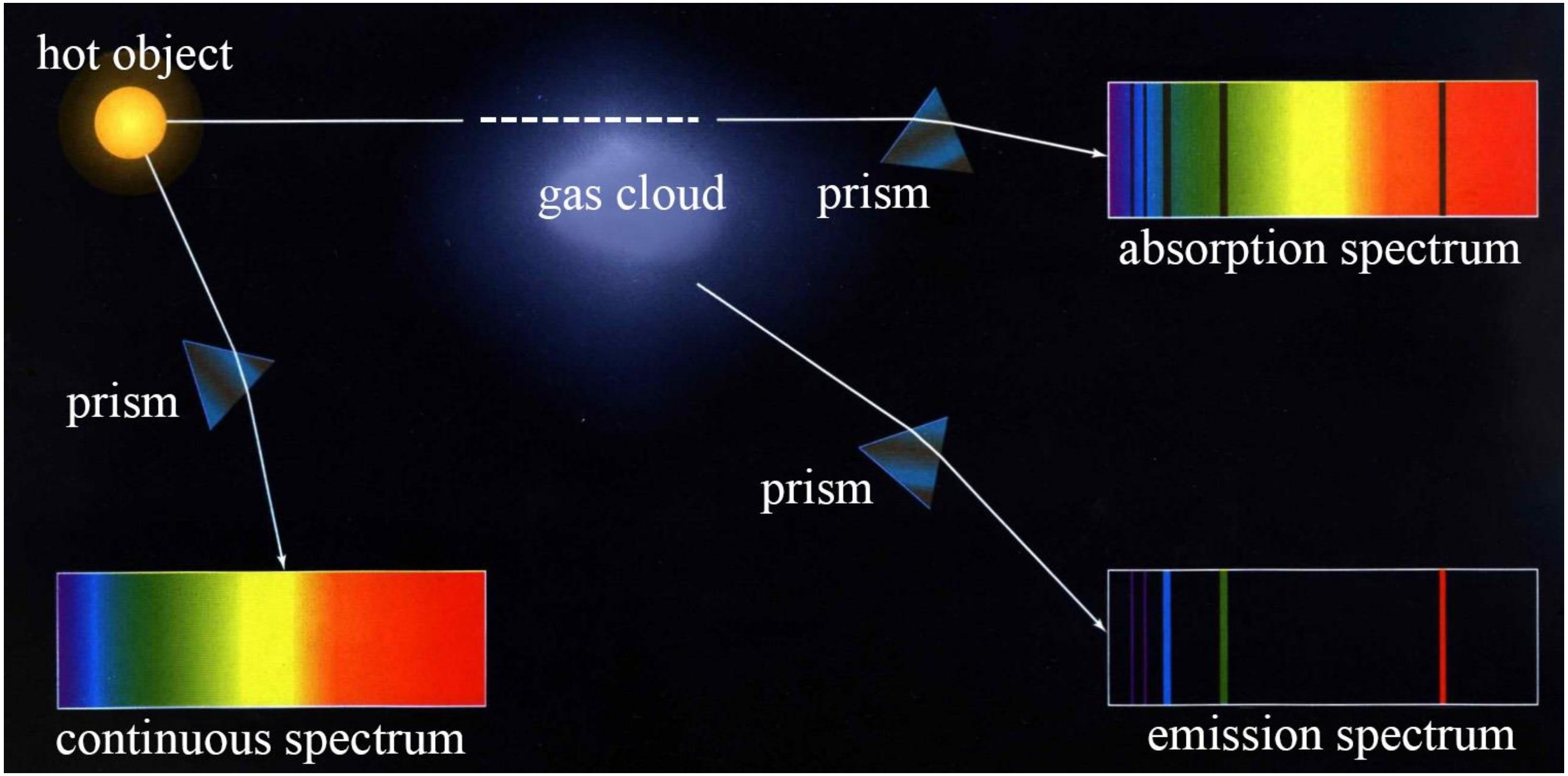
one side absorbs light, other side reflects: spun up by pressure difference

Breakthrough Starshot



Emission and absorption spectra of gases

In contrast to the continuous spectrum thermal radiation of a solid body, atomic gases or vapors only emit and absorb radiation at certain frequencies.



The lines of the spectrum can be used as a kind of fingerprint and with their help distant celestial bodies can be identified.

Explanation of the spectra of gases – Bohr’s postulates

From the emitted and absorbed photons with well-defined frequencies, it can be concluded that only certain energy transitions are possible in atoms.

<u>Bohr’s postulates:</u>

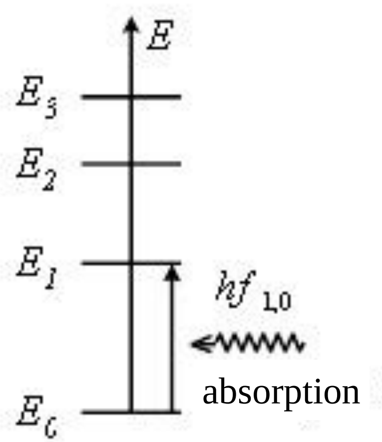
• In atoms, electrons reside only in discrete energy levels E_1, E_2, …, E_i and do not radiate in these stationary orbits.

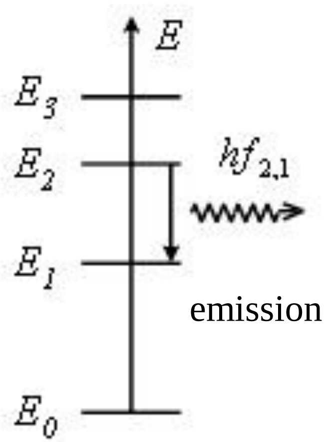
• Atoms only emit radiation when an electron moves from a higher energy orbit to a lower one.

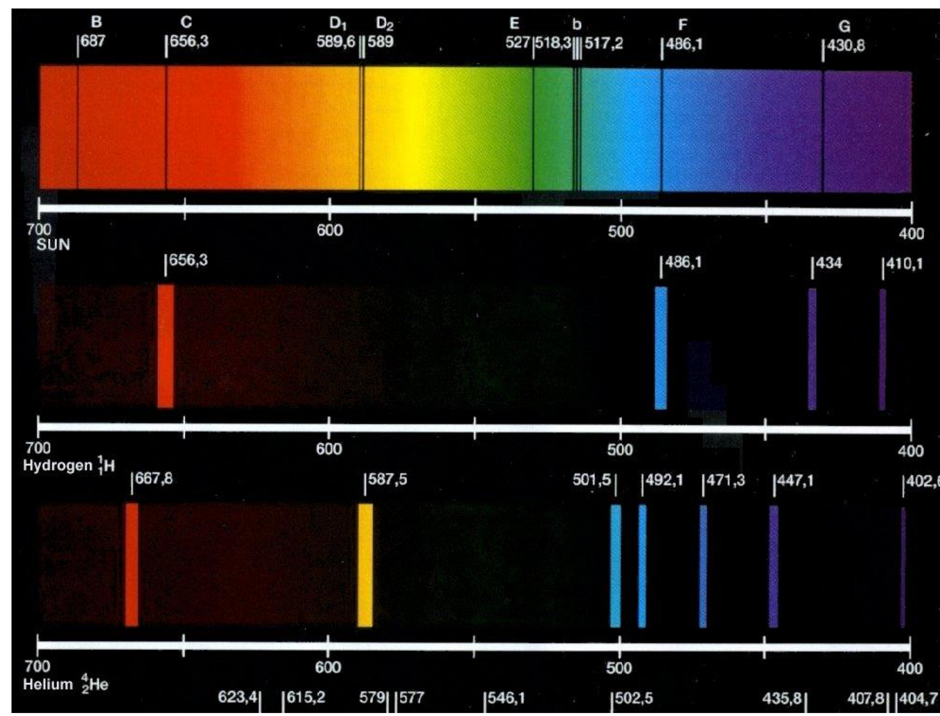
The inverse of emission is absorption.

Bohr's frequency condition:

$$E_i - E_j = hf_{ij}$$

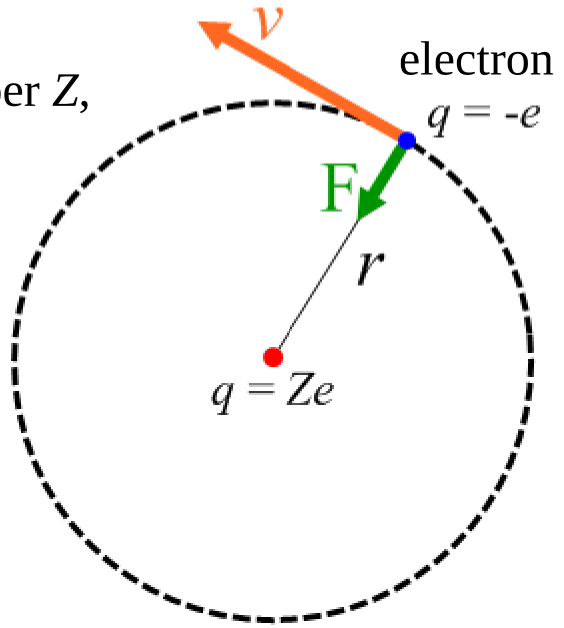






Bohr model of the hydrogen* atom

The model must provide the discrete E_n energies of the electron.
The orbital angular momentum of the electron: $L = mvr$
Like energy, this is also quantized: $L = nh/(2\pi) = n\hbar$
*Not only good for hydrogen, but also for ions with atomic number Z, which contain only one electron (hydrogen-like):



$$\frac{kZe^2}{r^2} = m\frac{v^2}{r} \rightarrow \quad kZe^2 = mvr \cdot v = n\hbar \cdot v$$

$$v = \frac{kZe^2}{n\hbar}$$

The total (mechanical) energy of the electron:

$$E = E_{kin} + E_{pot} = \frac{1}{2}mv^2 - \frac{kZe^2}{r} = \frac{1}{2}mv^2 - mv^2 = -\frac{1}{2}mv^2$$

$$E_n = -\frac{1}{2}m{v_n}^2 = -\frac{mk^2Z^2e^4}{2\hbar^2}\frac{1}{n^2} = -E^*Z^2\frac{1}{n^2}$$

$$E^* = \frac{m_0e^4k^2}{2\hbar^2} = 2{,}176 \cdot 10^{-18}\ \mathrm{J} = 13{,}6\ \mathrm{eV}$$

($m = m_0$: rest mass of electron)

(ionization energy of hydrogen)

Energy levels of the hydrogen atom

From the previously derived formula, we obtain the energy levels of hydrogen for $Z = 1$:

$$E_n = -E^* \frac{1}{n^2} \qquad E^* = \frac{m_0 e^4 k^2}{2\hbar^2} = 2{,}176 \cdot 10^{-18}\ \mathrm{J} = 13{,}6\ \mathrm{eV}$$

For emission and absorption frequencies:

$$f_{nm} = \frac{E_n - E_m}{h} = \frac{E^*}{h}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

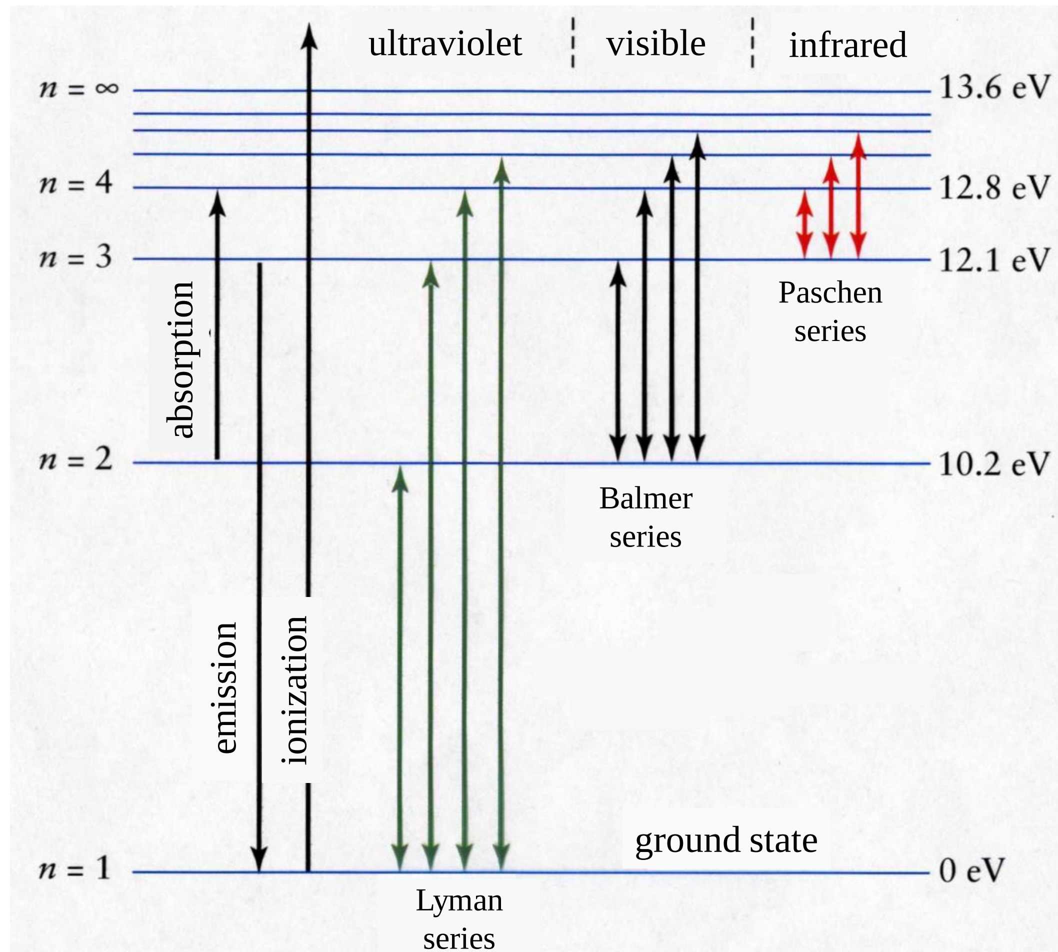
$$f_{nm} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

R: Rydberg constant

Lyman series: $$f_{n1} = R\left(1 - \frac{1}{n^2}\right)$$

Balmer series: $$f_{n2} = R\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

Paschen series: $$f_{n3} = R\left(\frac{1}{9} - \frac{1}{n^2}\right)$$

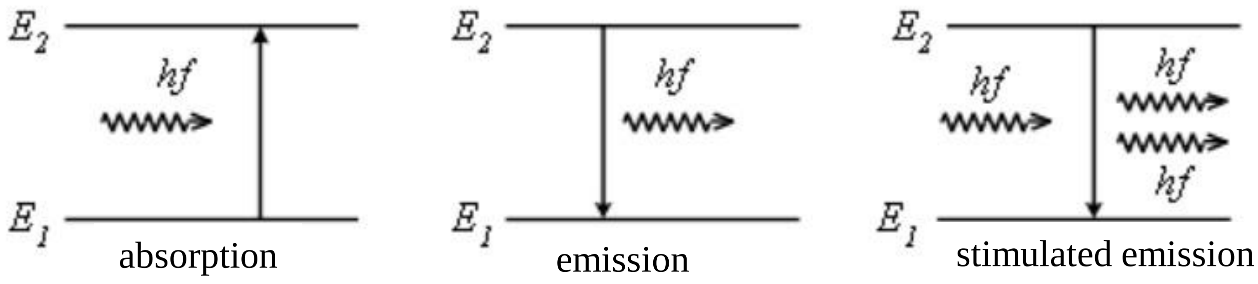


Excited state of atoms - Stimulated emission

By absorbing a photon of the appropriate energy, the atom enters an excited state, and the given electron jumps to a higher energy orbit. This is the process of **absorption**.
The lifetime of the excited state is about 10^{-8} s, but there also exist metastable states with approximately 10^{-3} s lifetime!

Then, through **spontaneous emission**, the electron jumps to a lower energy state with the emission of a photon of appropriate energy: $E_2 - E_1 = hf$

Einstein predicted a third type of process in 1916, **stimulated emission**.



Stimulated emission:
In the case of stimulated emission, de-excitation and emission do not occur spontaneously, but are triggered (**stimulated**) by a photon of the same energy passing by the excited atom. The emitted photon travels in the same direction as the inducing photon, and has the same frequency, phase, and plane of polarization, so the two photons are **coherent**.

Operation of laser

<u>LASER:</u> **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation

Stimulated emission makes it possible to amplify light.

<u>Operation:</u> By pumping energy, they achieve that there are more electrons in the excited state than in the low energy state (**population inversion**). Then there will be more stimulated emission than absorption, so the light is amplified.

<u>Properties:</u> monochromaticity (same frequency), low divergence, high coherence, high surface power density (can be enhanced with lenses), high spectral power density (since there is only one frequency).

<u>Applications of lasers:</u>

- machining, drilling, spot welding,
- surgical intervention, eye surgery
- gene surgery,
- barcode reading,
- CD player laser reading head,
- interference-based length and speed measurement,
- direction setting,
- holography

--- UP TO HERE ON EXAM BUT READ ALL! ---

X-rays

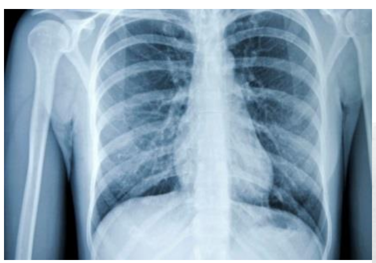
The wavelength of X-rays falls in the range between 10^{-11} and 10^{-8} m.
Unlike visible rays, these have great penetrating power.

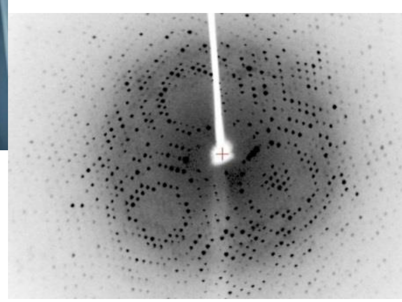
Production:

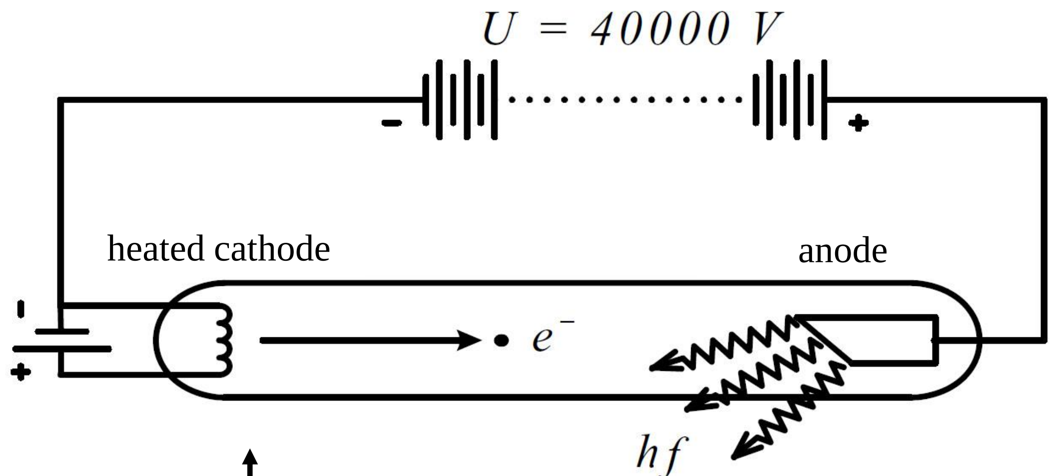
- electrons emerging from a heated cathode are accelerated by high voltage
- the electrons smash into the anode, which is a metal with high atomic number (e.g. tungsten)

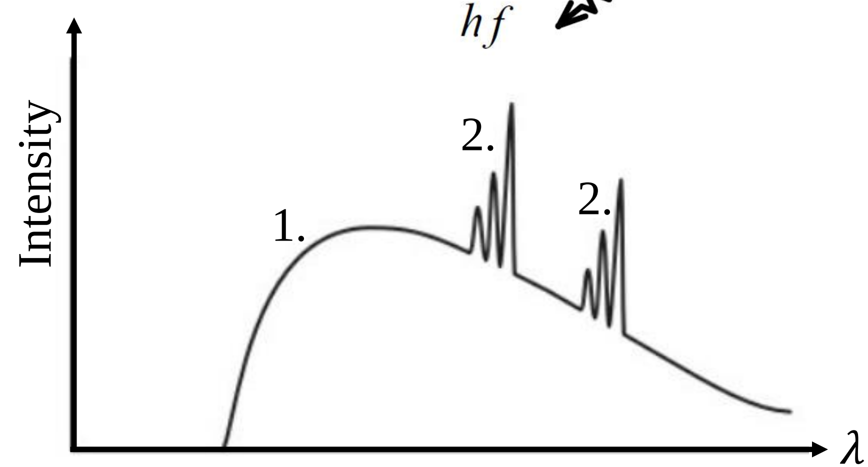
Main areas of use:

- medical imaging
- crystallography
- X-ray spectroscopy









Two components of the spectrum:

1. continuum (continuous spectrum)
2. characteristic radiation (peaks)

Continuum component - Bremsstrahlung (braking radiation)

Electron entering the Coulomb field of a heavy nucleus is deflected and slowed down.
<u>Bremsstrahlung:</u>
A charged particle (here e^-) undergoing acceleration loses energy and emits a photon.

$$E_{k1} - E_{k2} = E_f$$

$$\frac{1}{2}m{v_1}^2 - \frac{1}{2}m{v_2}^2 = hf$$

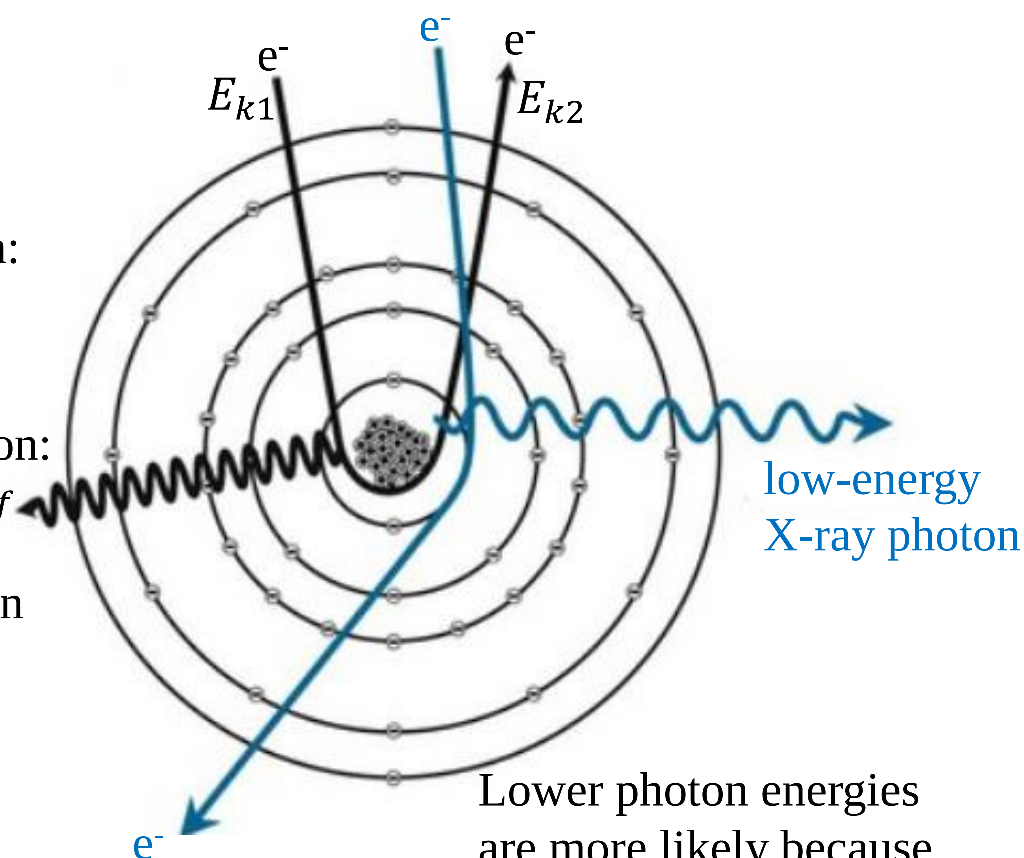
If the electron is completely slowed down:

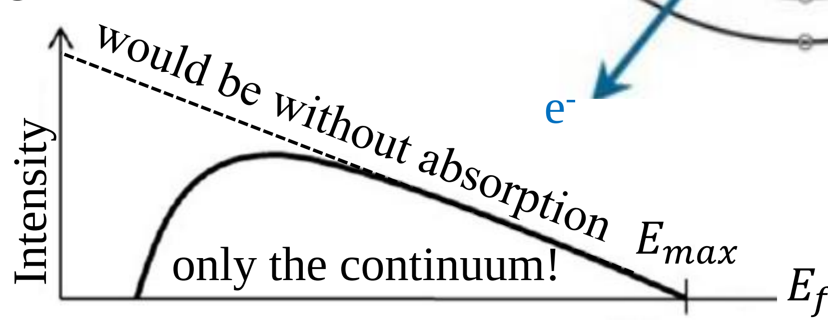
$$\frac{1}{2}m{v_1}^2 = hf_{max}$$

Since the energy of the accelerated electron:

$$E_k = eU$$

Thus, the maximum energy of the photon in eV is the accelerating voltage of the electron!

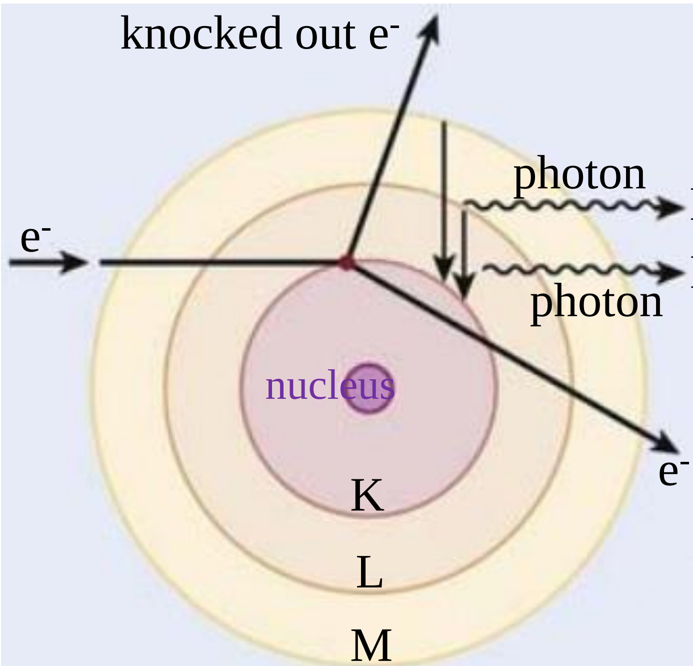


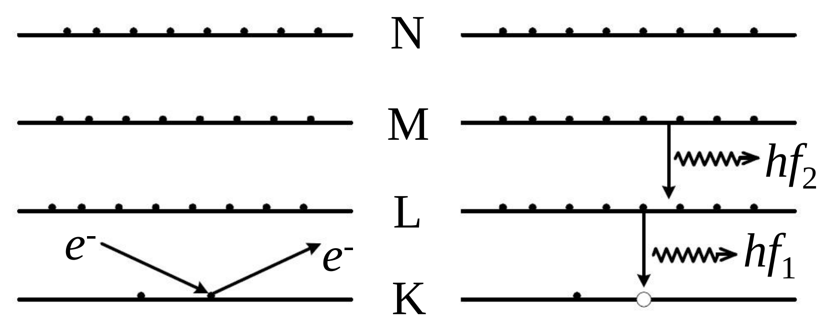


Lower photon energies are more likely because the electron can be deflected several times in the material. (straight line in the figure)

Characteristic radiation - component with lines

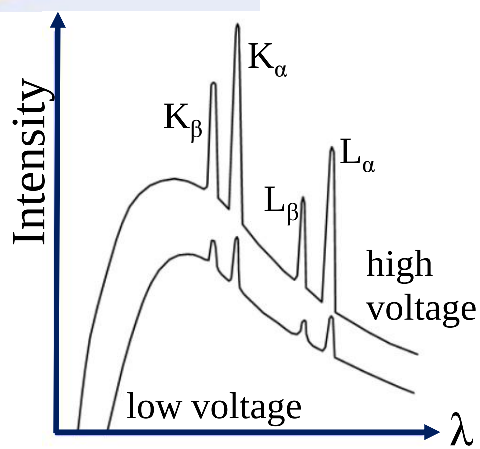
The accelerated electron knocks another electron out from one of the inner shells of the atom. This creates a vacancy, which causes further electron jumps.

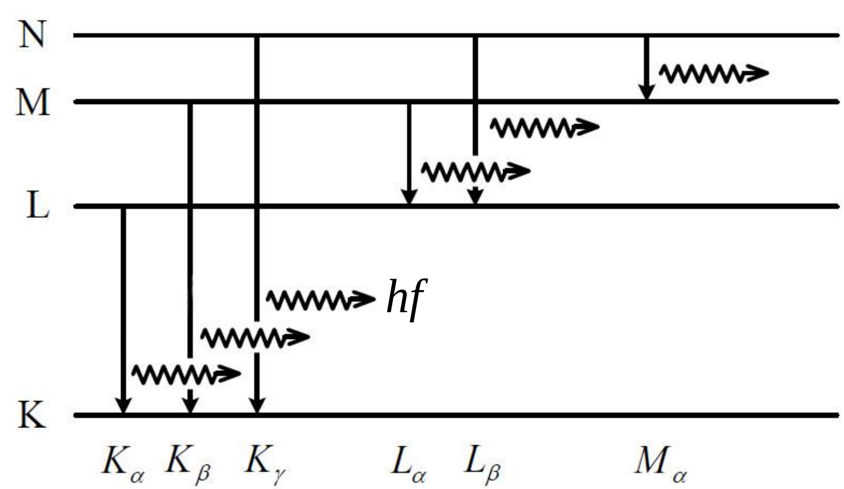




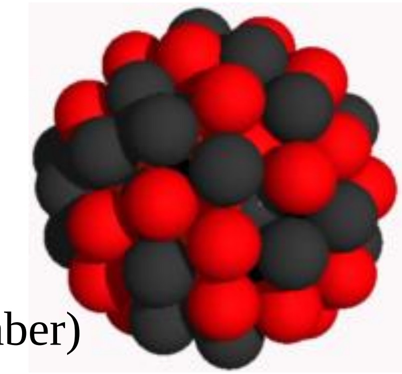
Discrete energy photons characteristic of transition:
- lines that can be arranged in series

$$E_2 - E_1 = hf$$





Structure of the atomic nucleus



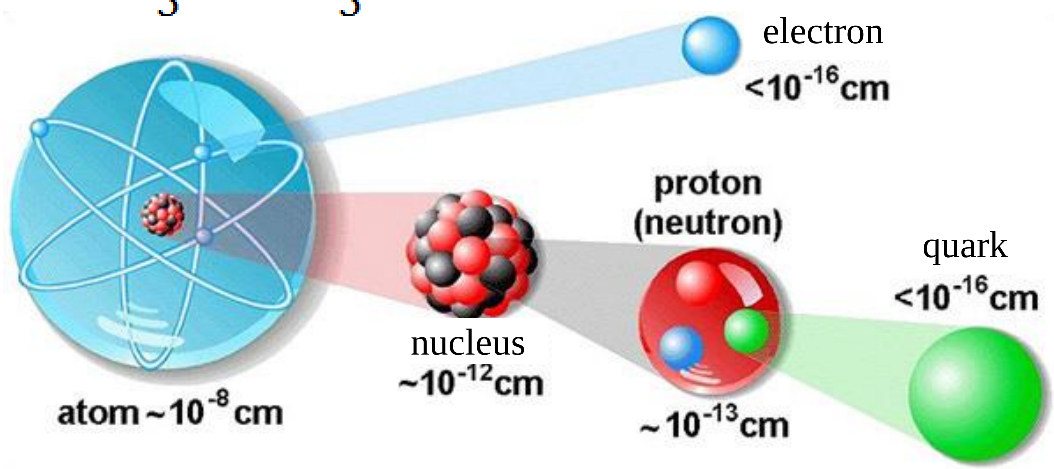
The nucleus contains positively charged protons and neutral neutrons.
Z: **atomic number** (number of protons, charge of nucleus in *e* units.)
The atomic number is also the number of electrons in a neutral atom.

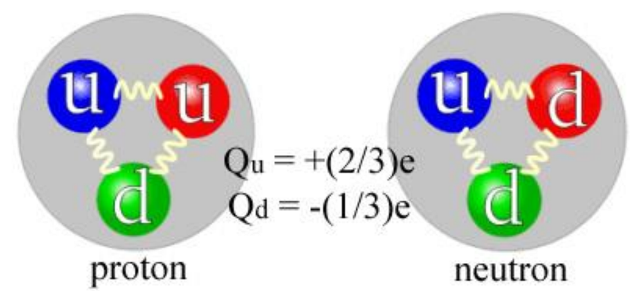
A: **mass number** (how many times the mass of the proton or neutron)
Mass number is also the number of nucleons: $A = N + Z$ (*N*: neutron number)

<u>isotopes:</u> for a given *Z*, *N* or *A* can be different, e.g. ${}^1_1H$, ${}^2_1H$, ${}^3_1H$ hydrogen (proton only), deuterium (proton + neutron), tritium (proton + 2 neutrons).

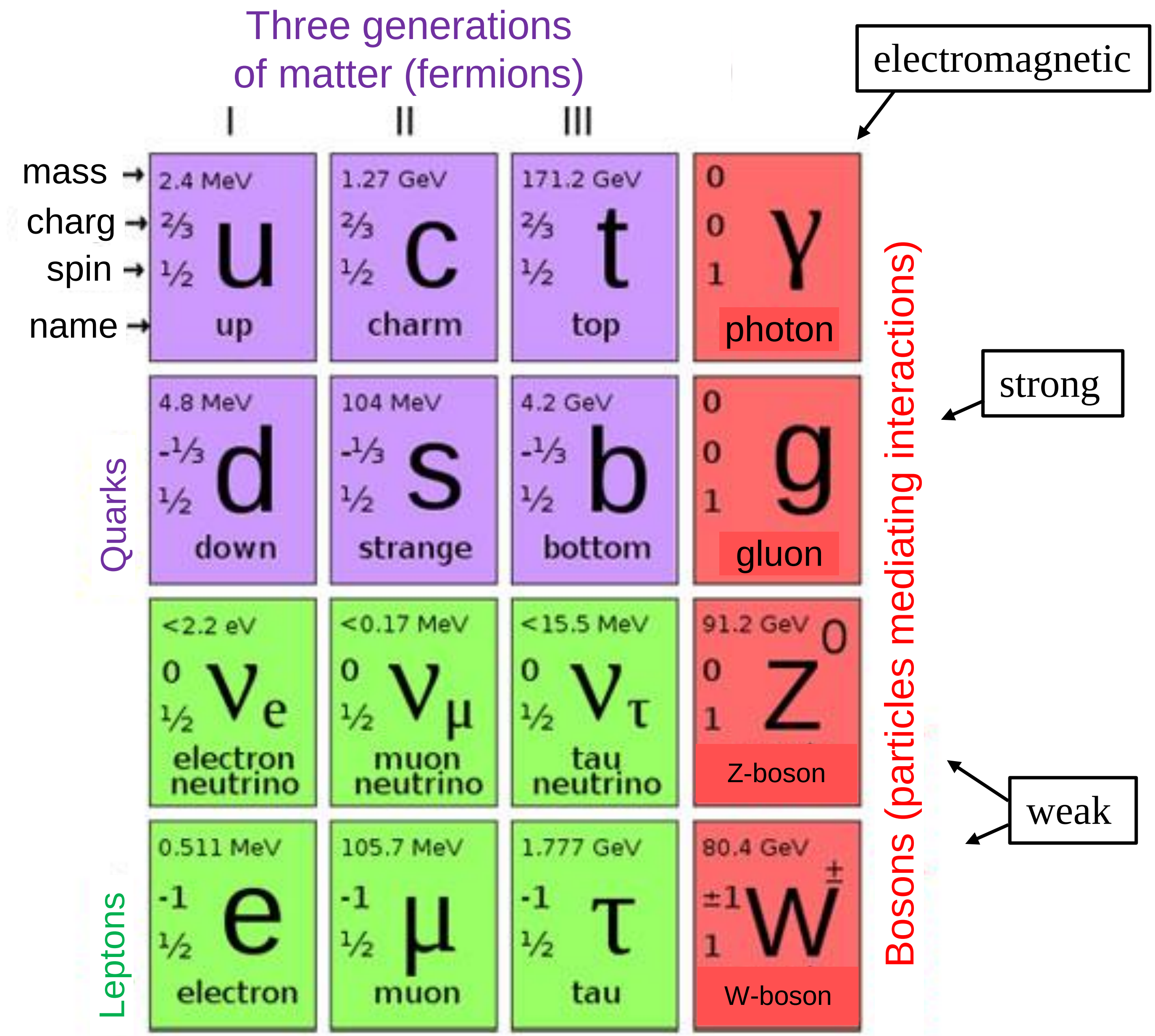
The density of the nucleus is independent of its size, therefore its volume is proportional to the mass number:

$$V = \frac{4R^3\pi}{3} = \frac{4{R_0}^3\pi}{3}A$$ so for nuclear radius: $R(A) = R_0 A^{1/3}$ $\quad R_0 = 1{,}4 - 1{,}5 \text{ fm}$





Basic building blocks and interactions



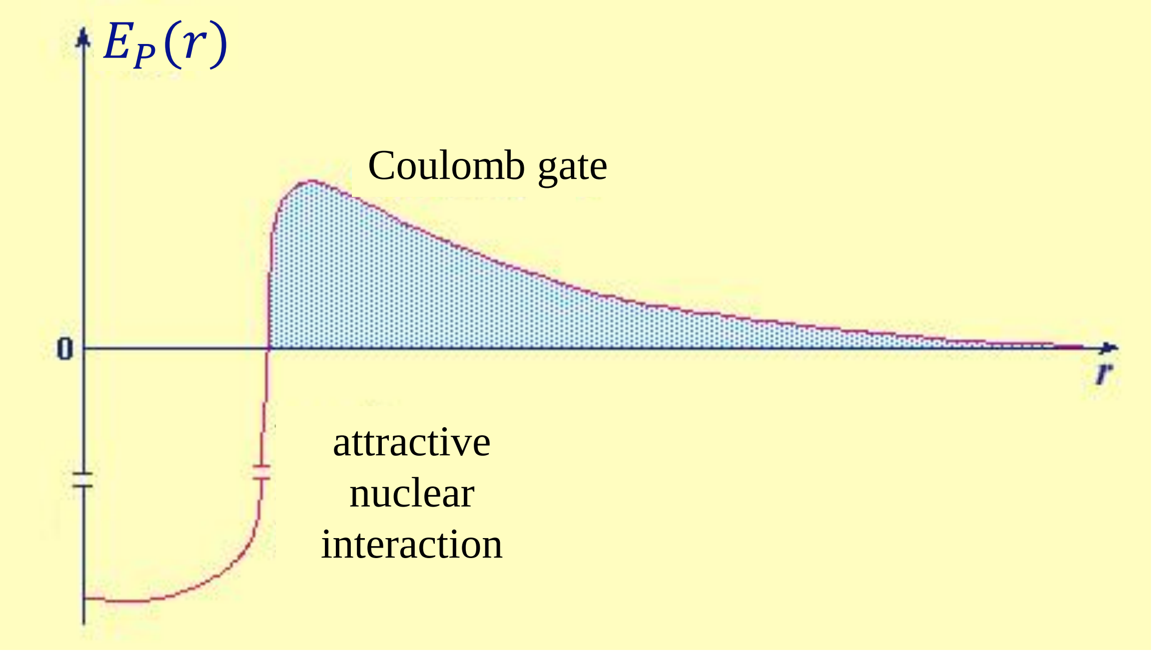
<u>Interactions</u>

1. electromagnetic
2. strong
3. weak
4. gravitational

Nuclear interaction

The nucleus contains Z number of protons, which repel each other due to their identical charge. However, in addition to the Coulomb interaction, a much stronger attractive force (nuclear or **strong interaction**) appears at very small distances (~ proton radius). This is charge independent, and is attractive between *p-p*, *p-n*, and *n-n*.

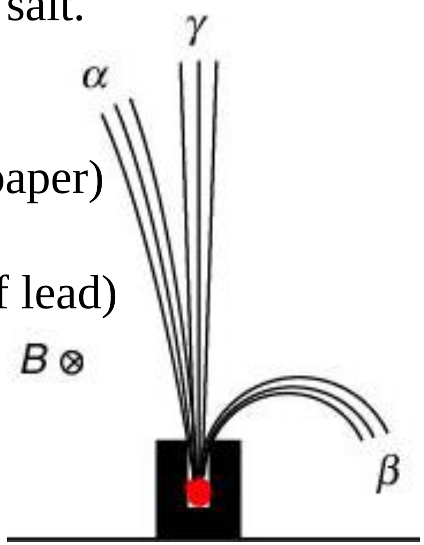
The nucleons are therefore in a bound state, their energy is negative ($E_M = E_k + E_p$)



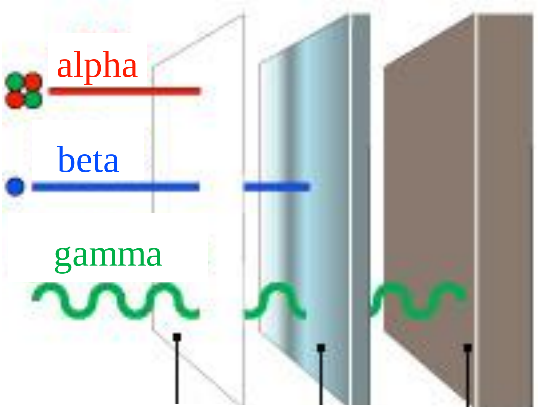
Quantum mechanics: protons and neutrons can only have discrete energies in the potential valley created by other nucleons, but the energies here are much higher than for electrons in the electron shell.

Radioactivity

<u>Becquerel (1896):</u> the photographic plate turns black when near uranium salt. Later, in a magnetic field, this radiation split into three: α, β, γ.



α: helium nuclei $^4_2He^{2+}$ (low penetrating power, absorbed by sheet of paper)
β: electrons (near the speed of light, absorbed by a few mm *Al* sheet)
γ: high-energy EM radiation (f > 10^{18} Hz, only absorbed by several cm of lead)



When radioactive radiation is emitted, element transmutation usually occurs (except γ).

The ejected particles are highly energetic because the nuclear forces are orders of magnitude stronger than the Coulomb force acting on electrons, so greater energies are released than during chemical reactions (electron transitions between energy levels).

Types of radioactive decay

α-decay: mass number decreases by 4, atomic number decreases by 2.

$$^{A}_{Z}X \rightarrow \, ^{A-4}_{Z-2}Y + \, ^{4}_{2}He$$

β-decay: two types (β^- and β^+) depending on whether electron (e^-) or **positron** (e^+) is created. The positron is the antiparticle of electron, with opposite charge, but everything else is same.

β^- $\quad ^{A}_{Z}X \rightarrow \, _{Z+1}^{\;\;A}Y + e^- + \bar{\nu}$ $\qquad$ for nucleons: $\qquad n \rightarrow p^+ + e^- + \bar{\nu}$

β^+ $\quad ^{A}_{Z}X \rightarrow \, _{Z-1}^{\;\;A}Y + e^+ + \nu$ $\qquad$ for nucleons: $\qquad p^+ + (e^- + e^+) \rightarrow n + e^+ + \nu$

The ν and $\bar{\nu}$ represent **neutrino** and antineutrino, respectively. These are uncharged, very low-mass particles that only react through the weak interaction. This makes them extremely difficult to detect. The positron leaves the nucleus and annihilates with an electron, producing two high-energy photons (matter + antimatter).
These also include electron capture, mostly from the innermost shell:

$^{A}_{Z}X + e^- \rightarrow \, _{Z-1}^{\;\;A}Y + \nu$ $\qquad$ for nucleons: $\qquad p^+ + e^- \rightarrow n + \nu$

γ-decay: does not involve element transmutation, only the transformation of the nucleus from an excited state to the ground state takes place. The energy difference is released in the form of a photon (the energy differences are large!).

$$^{A}_{Z}X^* \rightarrow \, ^{A}_{Z}X + \gamma$$

Law of radioactive decay

Radioactive decay is a random phenomenon. The nucleus of a radioactive isotope decays with the same probability per unit time, regardless of its age. The laws are statistical in nature, they only hold true for large numbers.
If λ is the probability that a nucleus will decay in the next second (**decay constant**), then for the change in the number *N* of the nuclei (*N* is large!) over time *dt*:

$$dN = -\lambda N dt$$

Rearranging the equation (separating variables): $\frac{dN}{N} = -\lambda dt \longrightarrow \int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$

After integration: $\ln N - \ln N_0 = -\lambda t$

For the **decay law**: $N = N_0 e^{-\lambda t}$ (exponential decline, $1/\lambda$ is the average lifetime.)

The **half-life** gives the time it takes for half of the original large number of radioactive nuclei to decay. Waiting for another half-life, the number of nuclei that have not yet decayed is halved again, and so on.

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$e^{\lambda T_{1/2}} = 2 \longrightarrow T_{1/2} = \frac{\ln 2}{\lambda}$$

Activity

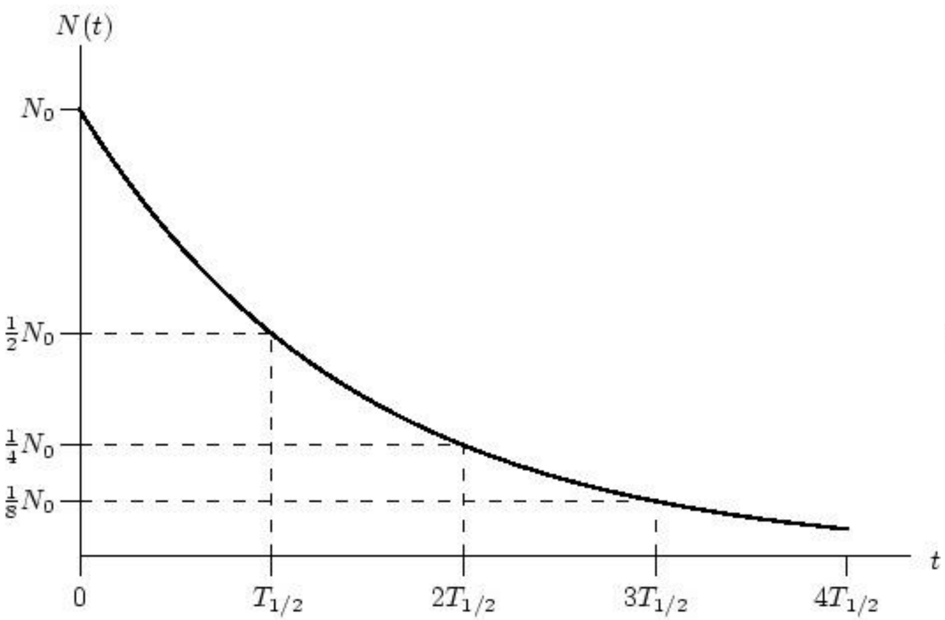
<u>Activity:</u> Number of decays occurring in the sample per unit time: $A = \left|\frac{dN}{dt}\right|$

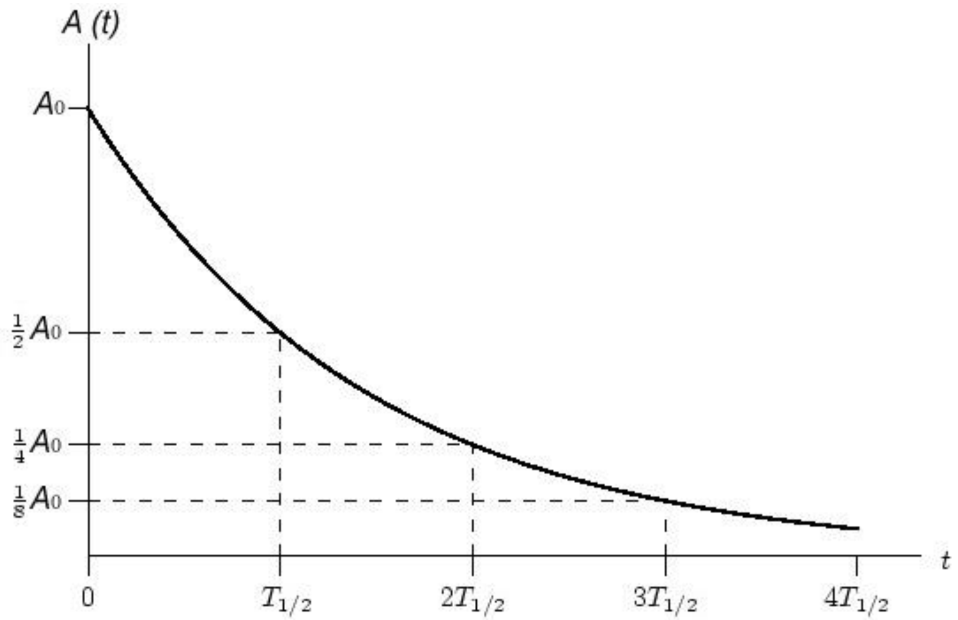
[A] = 1 Bq (becquerel) = 1 decay/second

$$A = \left|\frac{dN}{dt}\right| = N_0 \lambda e^{-\lambda t} = A_0 e^{-\lambda t}$$

So the activity decreases according to the same exponential function, and at any time:

$$A(t) = N(t)\lambda$$





Mass defect

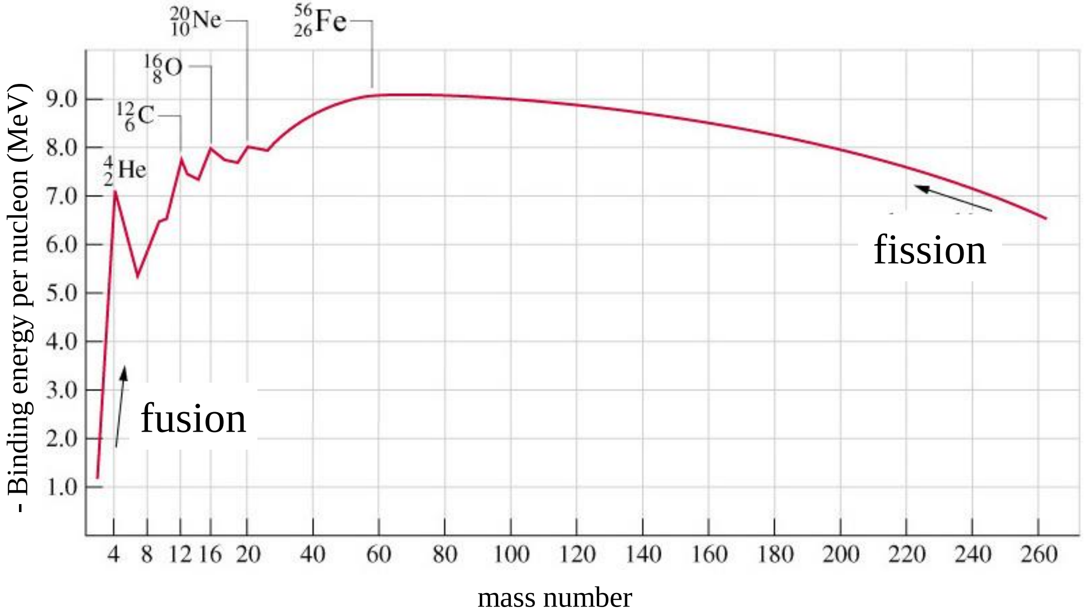
Let *M(A, Z)* denote the mass of the nucleus with mass number *A* and atomic number *Z*. Measured with mass spectrometer, we find that the mass of the nucleus is Δm smaller than the mass of its constituents (protons and neutrons):

$$\Delta m = M(A, Z) - Zm_p - (A - Z)m_n < 0$$

This **mass defect**, calculated based on Einstein's mass-energy equivalence, gives the **binding energy** (the ~ 0 energy of free components became negative because they entered a bound state). So the binding energy gives us how much energy we could invest to break the nucleus (or any bound system) back into its components.

$$E_K = \Delta mc^2 < 0$$

The binding energy per nucleon can be determined by measuring the masses: $\varepsilon = E_K/A$



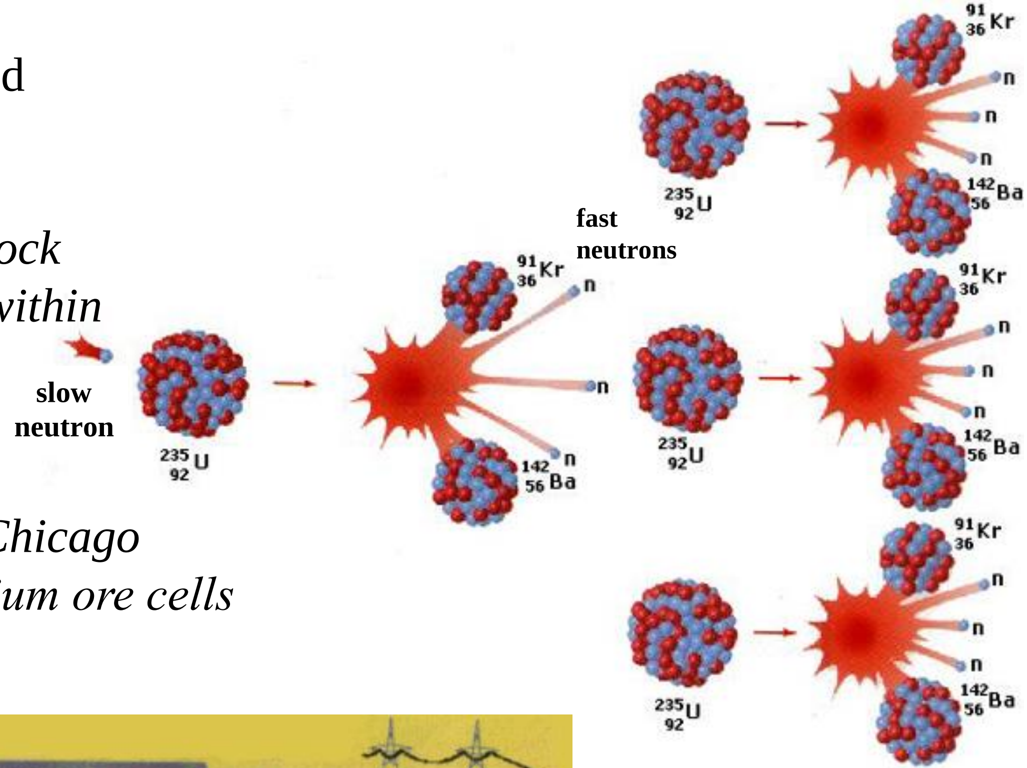
If ε decreases during a process, energy is released.

e.g. fusion of small nuclei or fission of large nuclei

ε is the smallest for iron.

Nuclear power plant

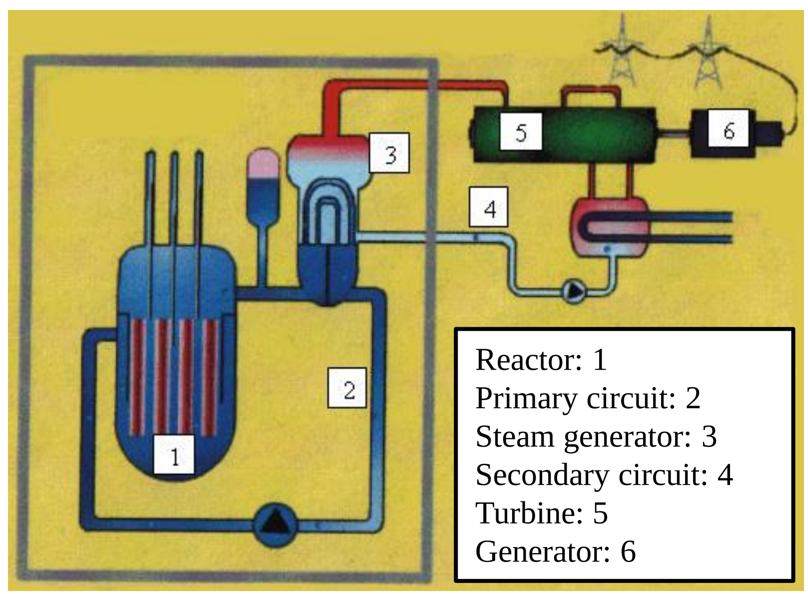
If fast neutrons slow down and get captured (on average ≥ 1) $\rightarrow$ **chain reaction**.
Uncontrolled: nuclear bomb
Critical mass: if the size of the uranium block is large enough, neutrons will slow down within it and get captured.



Controlled: **nuclear power plant**
First self-sustaining chain reaction 1942 Chicago
Fermi and Szilárd separated natural uranium ore cells with graphite bricks (moderator).

Multiplication factor ($k = n'/n$).
n': causes another fission
n: number of fissions

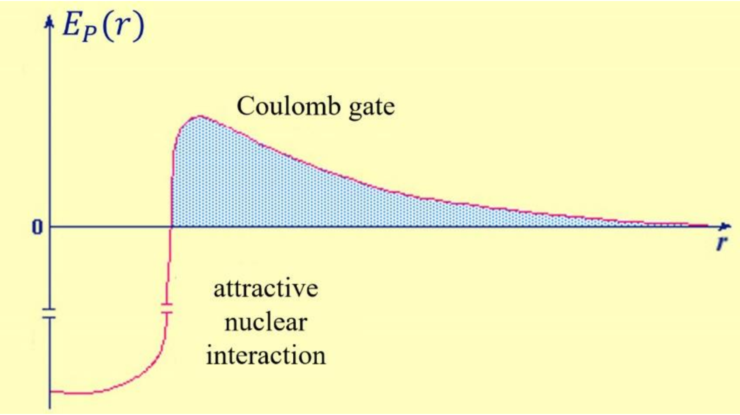
If k is kept below one, but close to it, then energy can be produced in a controlled manner.



moderator materials:
- graphite
- heavy water
- ordinary water

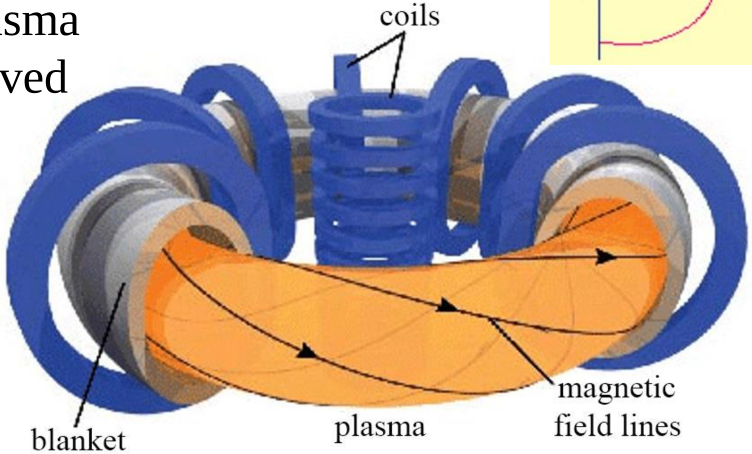
slowing down fast neutrons for fission.

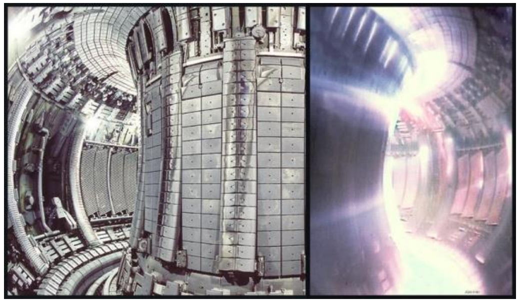
Fusion



Energy is also released when smaller nuclei fuse, e.g. in the Sun or in the hydrogen bomb hydrogen is converted into helium. Problem: Due to the Coulomb barrier, temperatures of tens of millions of degrees are required for fusion between nuclei to occur.
Bomb: fission bomb heats it up
Power plant: keeping hot plasma together has not yet been solved
<u>Two types:</u>

1. Tokamak (held together by a magnetic bottle)





2. fusion by lasers (hydrogen in a tiny drop is ignited by focused lasers)