The birth of modern physics

Lord Kelvin said in the late 19th century that physics is a finished science:

"There is no problem that science cannot solve. Physics is a finished science, our theories work so well that they must be correct. Maybe there are two tiny clouds in the clear blue sky."

However, these clouds (light propagation and thermal radiation) shook physics to its foundations and led to the creation of two new theories:

- Relativity (special and general)

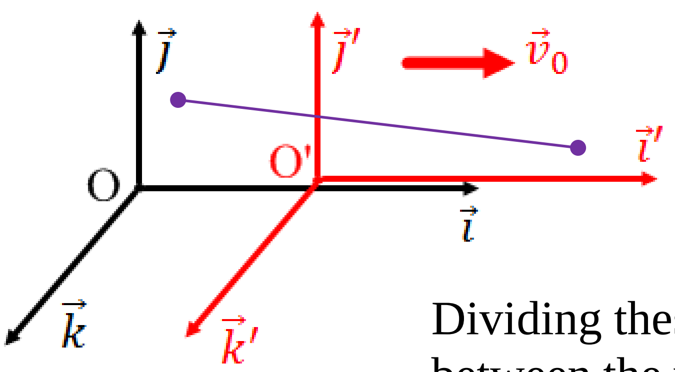
- Quantum physics

Thus, the beginning of the 20th century also marked the beginning of modern physics.

Galileo's principle of relativity

In any two reference frames moving at **constant** velocity relative to each other, **mechanical phenomena** occur in the same way.
E.g., apart from the vibration, we cannot feel whether the train is moving if it is moving at a constant speed. The dropped coin also falls vertically with uniform acceleration. Therefore, none of these reference systems is distinguished, there is no absolutely stationary reference system.

<u>Relationship between systems moving relative to each other:</u>
Let system *K*' move relative to *K* in the positive *x* direction with **constant** velocity v_0.



After time Δt, distance between origins: $\overline{OO'} = v_0 \Delta t$
Measured coordinate differences in *K*' are:

$$\Delta x' = \Delta x - v_0 \Delta t$$
$$\Delta y' = \Delta y$$
$$\Delta z' = \Delta z \qquad \text{Also:} \qquad \Delta t' = \Delta t \text{ (clocks in sync)}$$

Dividing these by Δt (or $\Delta t'$) we get the relationship between the velocities (purple line is the trajectory of a moving body):

$$v_x' = v_x - v_0$$
$$v_y' = v_y$$
$$v_z' = v_z$$

The speed of light

Writing Maxwell's equations in a *K* reference frame, we obtain the homogeneous wave equation:

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} - \varepsilon\mu \frac{\partial^2 E_i}{\partial t^2} = 0$$

Comparing with the general wave equation, we get for the propagation velocity:

$$v = \frac{1}{\sqrt{\varepsilon\mu}}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0$$

Question: With respect to what should this speed be measured in case of electromagnetic wave (light)? With respect to the medium in which light travels?

But what about light propagating in a vacuum?!

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \approx 3 \cdot 10^8 \ \frac{\mathrm{m}}{\mathrm{s}}$$

Proposal: There is a hypothetical all-pervading medium ("**ether**") that provides the "elastic" medium for the propagation of electromagnetic waves, in which the wave can propagate.

This would mean that an absolute resting frame of reference could be selected using electromagnetic phenomena.
This distinguished system would be fixed to the ether, and the electromagnetic wave (light) would propagate at a speed *c* relative to it.

Propagation of light examined in a moving system

Flash of light starting from origin reaches spherical surface of radius $c\Delta t$ in time Δt viewed in K (frame at rest with respect to ether): $\Delta x^2 + \Delta y^2 + \Delta z^2 = c^2\Delta t^2$

or $$c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = 0$$

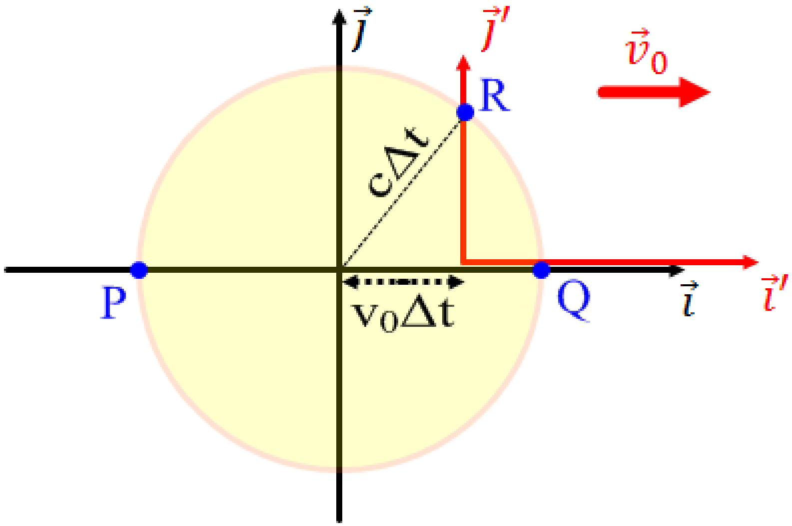
So the points P, Q, and R are all at a distance $c\Delta t$ from the origin measured in K.

The same distances in the K' system:

$$l_P = c\Delta t + v_0\Delta t = (c + v_0)\Delta t$$

$$l_Q = c\Delta t - v_0\Delta t = (c - v_0)\Delta t$$

$$l_R = \sqrt{c^2\Delta t^2 - {v_0}^2\Delta t^2} = \sqrt{c^2 - {v_0}^2}\,\Delta t$$



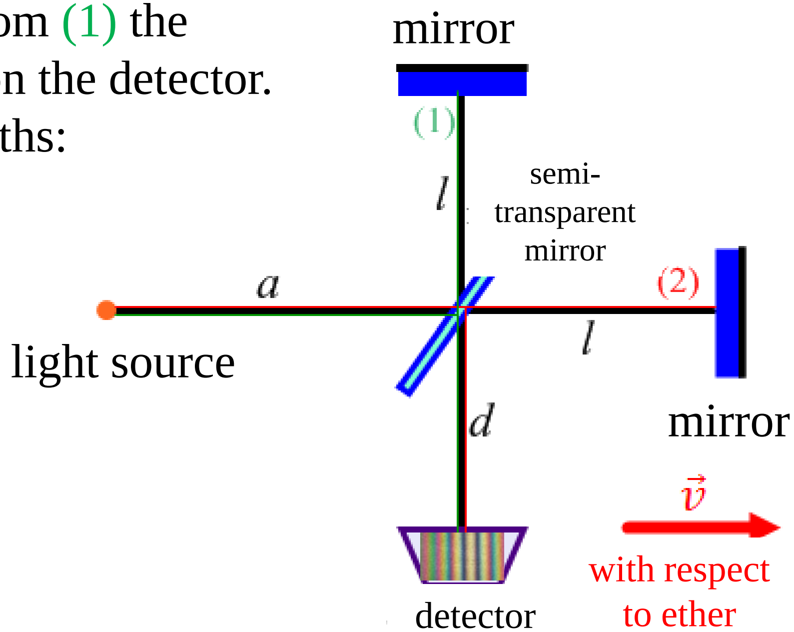
So the speed of light in different directions in the moving system is different:
(directions marked with points P, Q, R)

$$c'_P = \frac{l_P}{\Delta t} = c + v_0 \qquad c'_Q = \frac{l_Q}{\Delta t} = c - v_0 \qquad c'_R = \frac{l_R}{\Delta t} = \sqrt{c^2 - {v_0}^2}$$

This, in principle, allows us to determine the speed of our movement relative to the ether.

The Michelson experiment

The purpose of the experiment was to determine the speed of the Earth relative to the ether. The rays passing through (2) and reflecting from (1) the semi-transparent mirror create interference fringes on the detector.
Determining the time difference between the two paths:



$$t_1 = \frac{2l}{\sqrt{c^2 - v^2}} = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx \frac{2l}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$t_2 = \frac{l}{c - v} + \frac{l}{c + v} = \frac{l(c+v) + l(c-v)}{(c-v)(c+v)} =$$

$$= \frac{2lc}{c^2 - v^2} = \frac{2l/c}{1 - \frac{v^2}{c^2}} \approx \frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right)$$

$$\Delta t = t_2 - t_1 \approx \frac{2l}{c}\frac{1}{2}\frac{v^2}{c^2} = \frac{lv^2}{c^3}$$

If we rotate the device by 90 degrees, the roles of the arms are reversed:

$$\Delta t^* = t_2 - t_1 = -\frac{lv^2}{c^3}$$

So the position of the fringes shifts because of this.

No shift was detected!

Nowadays, a velocity of 3 m/s relative to the ether could be detected, but the result is negative.
Thus: there is **no ether**, light propagates in all reference frames, in all directions, with c.
The principle of special relativity: Frames moving at constant speed relative to each other are equivalent from the point of view of the laws of physics. Equations are of similar form.

Lorentz transformation

The negative result of Michelson's experiment suggests that we cannot distinguish between systems moving at a constant velocity relative to each other using electromagnetic phenomena. Since the phase of the light wave in both *K* and *K*' is a spherical surface expanding at speed *c*:

$$c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = 0$$
$$c^2\Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2 = 0$$

Galileo's method of adding velocities cannot be valid for the motion of light; it is an approximation only valid for slowly moving (relative to light) reference frames.
Einstein: **Let's give up the $\Delta t = \Delta t'$ constraint.** Just as we cannot speak of absolute space, we cannot speak of absolute time either.

Let's find the transformation rule that connects the coordinate and time differences Δx, Δy, Δz, Δt and the coordinate and time differences $\Delta x'$, $\Delta y'$, $\Delta z'$, $\Delta t'$!

Conditions:
1. Events that happen at same time and position in one system should also be at same place and time in the other system. If $\Delta t = \Delta x = \Delta y = \Delta z = 0$, then $\Delta t' = \Delta x' = \Delta y' = \Delta z' = 0$.
2. The quantities of *K* are transformed into *K*' by a function of similar form as the quantities of *K*' into *K* (neither is distinguished). The transformation is linear.
3. In the limiting case $v << c$ we get back the Galilean transformation.
4. The speed of light should be the same in all reference frames.

Lorentz transformation formulas

Let the system *K*' move relative to *K* in the positive *x* direction with a constant velocity *v*. The transformation can be written in the following general form:

$$\Delta x' = \xi_x \Delta x + \xi_t \Delta t \qquad \Delta t' = \tau_x \Delta x + \tau_t \Delta t \qquad \Delta y' = \kappa \Delta y \qquad \Delta z' = \kappa \Delta z$$

where $\xi_x, \xi_t, \tau_x, \tau_t$, and κ are factors independent of the coordinates, and only dependent on the velocity *v* of *K*' relative to *K*.

Velocity of *O*' ($\Delta x' = 0$) in *K* is *v*, so:

$$v = \frac{\Delta x}{\Delta t} = -\frac{\xi_t}{\xi_x} \rightarrow \xi_t = -v\xi_x = -v\xi$$

Using this for $\Delta x'$:

$$\Delta x' = \xi(\Delta x - v\Delta t)$$

Since the wave propagates at the same speed *c* in both systems:

$$c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = c^2\Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2$$

By writing the transformation formulas and collecting the like terms, we obtain four equations for ξ, τ_x, τ_t, and κ. From these:

$$\xi = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \tau_t \qquad \kappa = 1 \qquad \tau_x = -\frac{\xi v}{c^2}$$

So the transformation formulas are:

$$\Delta x' = \frac{\Delta x - v\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \Delta y' = \Delta y; \quad \Delta z' = \Delta z; \quad \Delta t' = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If we want to express quantities without commas, we must write –*v* everywhere instead of *v*.

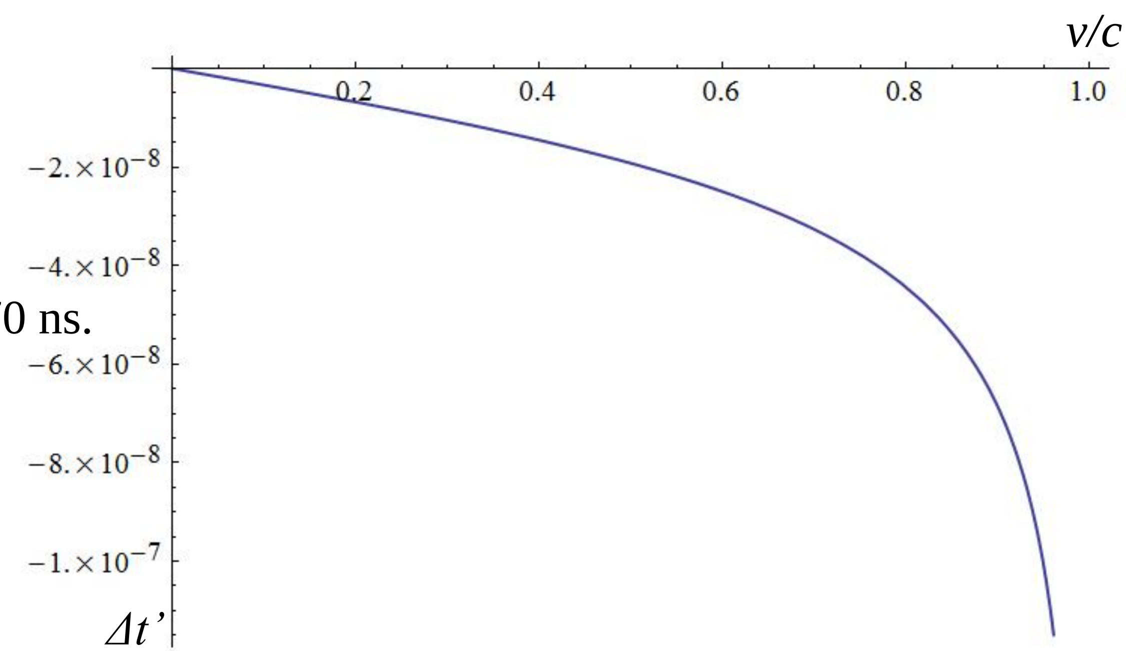
Relativity of simultaneity

Let two events (observed in the system K) occur at different locations ($\Delta x \neq 0$) but at the same time ($\Delta t = 0$).

Then the time difference between the two events observed from the K' system:

$$\Delta t' = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{\frac{v\Delta x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \neq 0$$

So for an observer moving with velocity v relative to K (resting in K'), the two events will not happen at the same time!



For example:
If Δx = 10 m and v = 0.9c
then the time difference is about 70 ns.

Time dilation

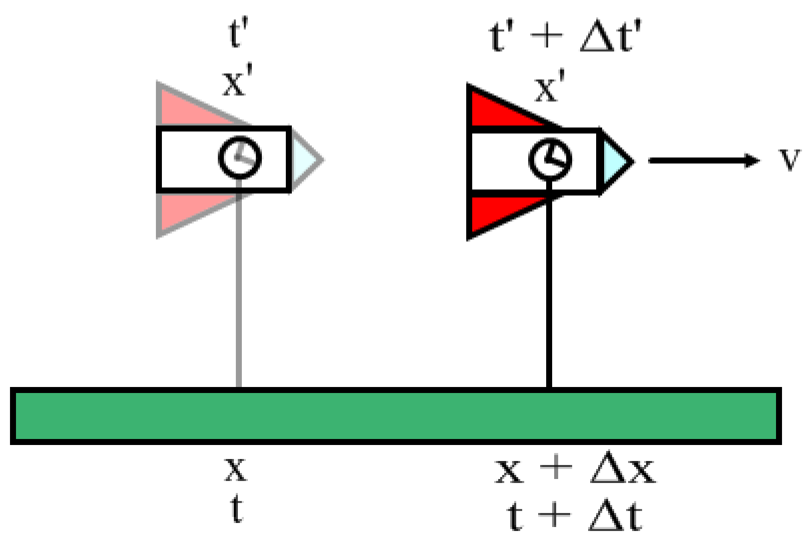
If a clock with a constant speed *v* passes a point and then after a time *Δt* (measured in *K*) passes another point at a distance *Δx*, then this time is measured differently by the moving clock.

Let's fix the *K*' system to the moving clock, so *Δx*' = 0.
Expressing the time measured in *K* in terms of the time of the moving clock:

$$\Delta t = \frac{\Delta t' + v\Delta x'/c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

So the moving clock shows a shorter time (proper time) for the time required to cover the distance than the stationary clock.

<u>Experimental evidence:</u>
Due to cosmic rays, μ-mesons are produced at an altitude of about 100 km, which have a half-life of 2.2 μs. Even with the speed of light 100 km takes 0.333 ms to cover.
The detection of mesons at sea level therefore proves time dilation.



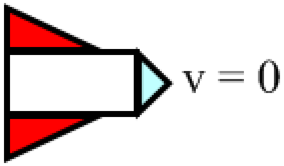
Length contraction

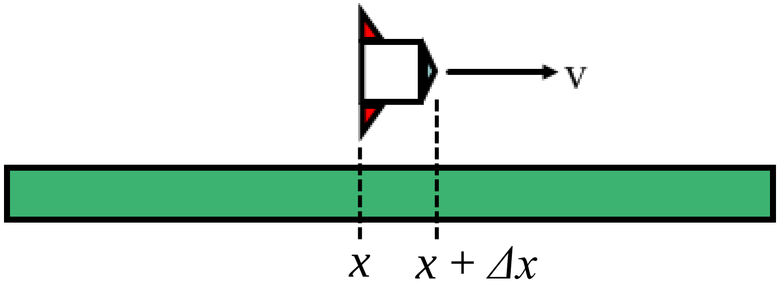
In the system K' moving with velocity v, a rigid rod rests in the x direction. The start and end of the rod are at a distance $\Delta x'$ from each other (length of the rod: L_0).

If we want to measure the length of the rod in K, we need to determine the distance Δx, such that the beginning of the rod passes through $x + \Delta x$ and the end passes through x in the same time, so $\Delta t = 0$.

$$\Delta x' = \frac{\Delta x - v\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta x}{\sqrt{1-\frac{v^2}{c^2}}}$$

So the apparent length of the rod ($\Delta x = L$) is less than the rest length ($\Delta x' = L_0$)





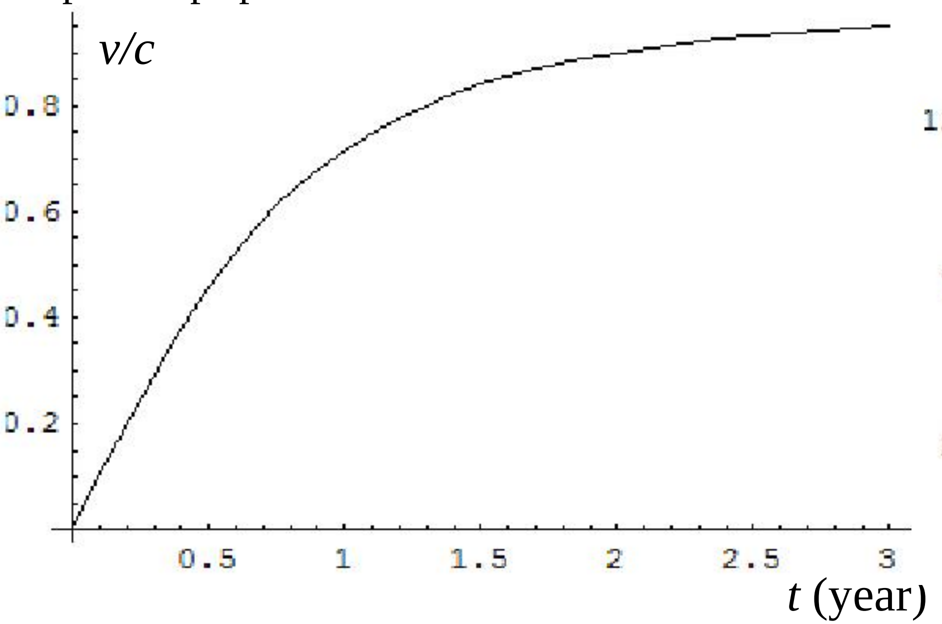
Interstellar travel

Due to time dilation, the crew will measure significantly less time than they would on Earth if the spacecraft approaches the speed of light.

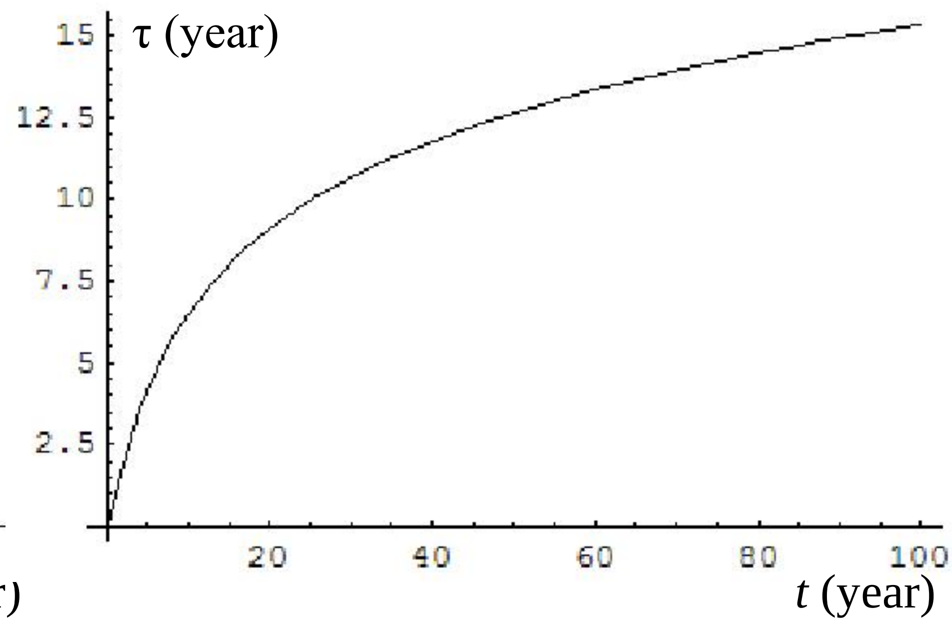
<u>For example:</u> Halfway through the journey, the ship accelerates at a constant acceleration of 1 g = 10 m/s^2, then slows down at the same rate, stopping at the destination, and then returning to Earth in the same way:

Bussard jet-powered starship

Spaceship speed as a function of Earth time



The round trip time (τ) measured on the spacecraft as a function of the time measured on Earth (t)



Equivalence principle

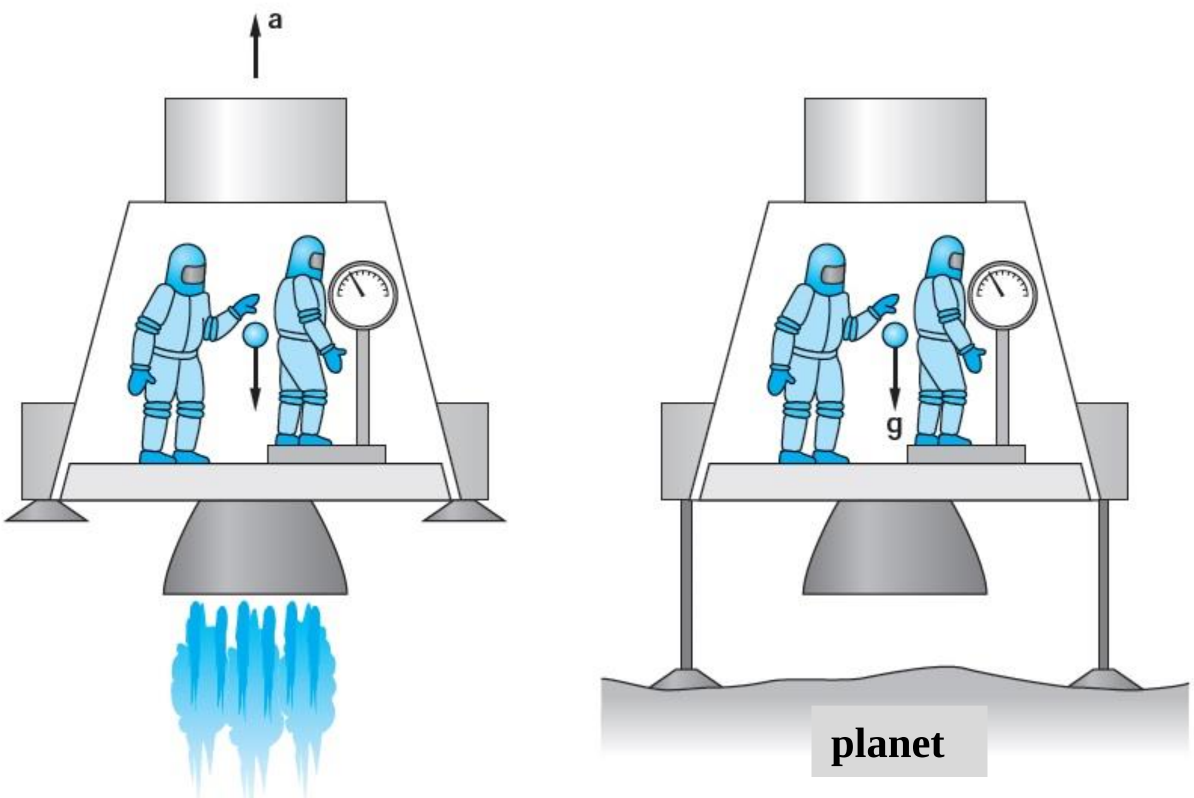
A homogeneous gravitational field is equivalent in all respects to a uniformly accelerated frame of reference.

There is no experiment that could help distinguish!
Gravity is also an apparent force (e.g. centrifugal) – it can be transformed away by choosing an appropriate system.

The principle of relativity:
for all systems!
inertial and
non-inertial (accelerating)

Acceleration is also relative

Principle of general relativity
1916

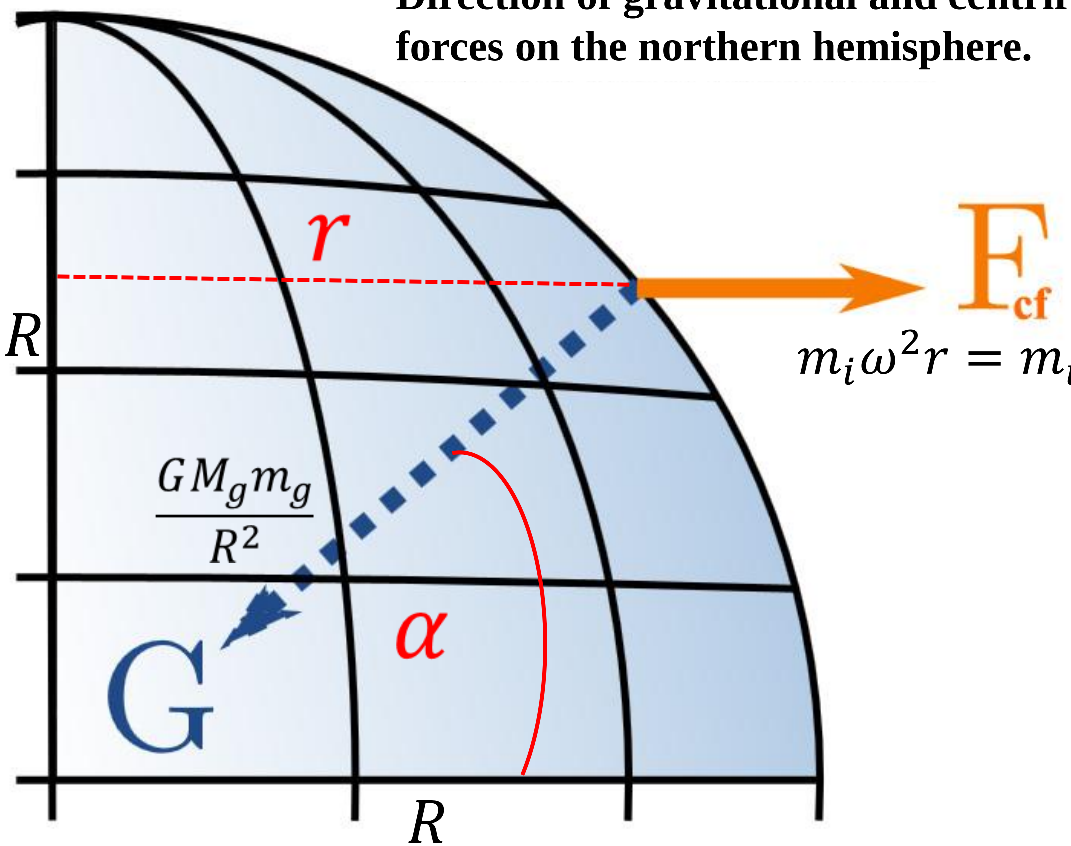


Gravitational mass and inertial mass

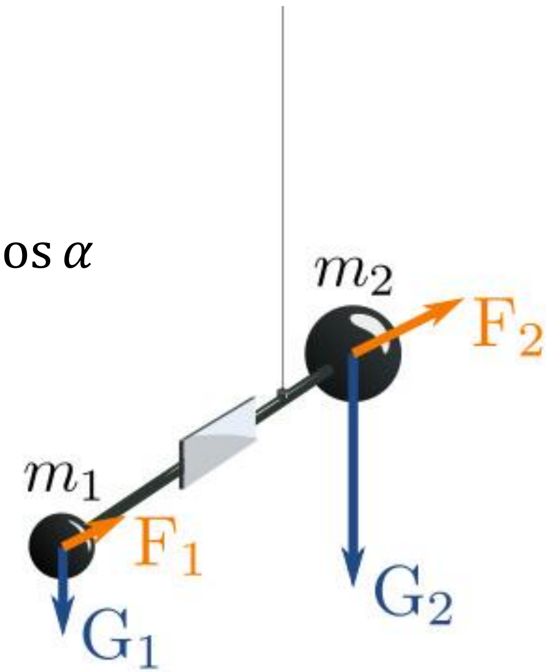
The equivalence principle means that gravitational mass and inertial mass are equal.

Proof: Using an Eötvös pendulum ($< 10^{-10}$ % deviance)

Direction of gravitational and centrifugal forces on the northern hemisphere.



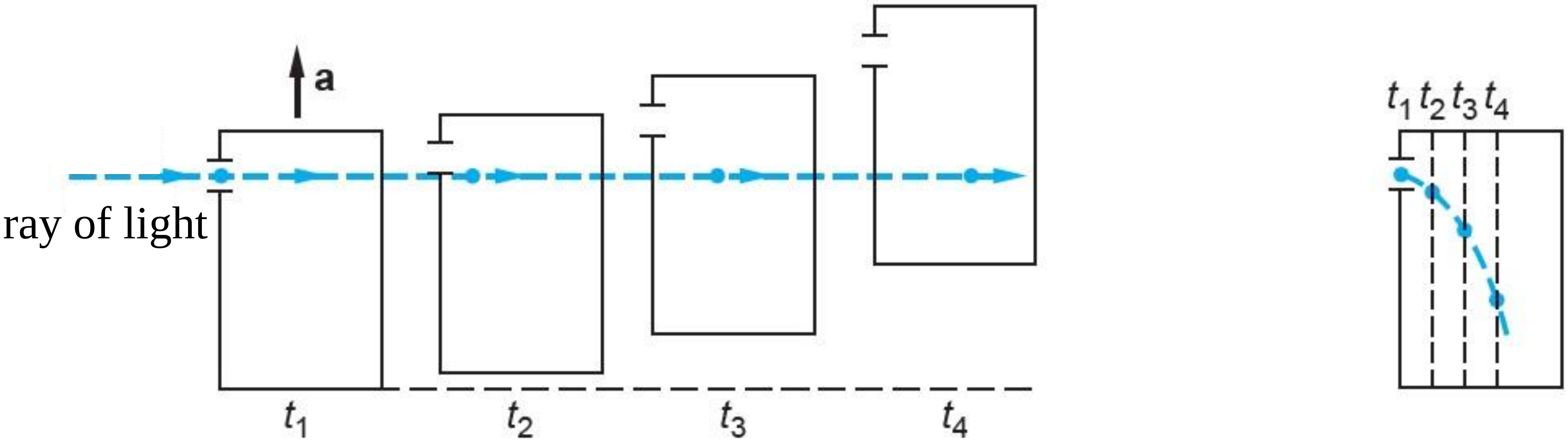
$$m_i\omega^2 r = m_i\omega^2 R\cos\alpha$$



Equilibrium if: $\frac{F_1}{F_2} = \frac{G_1}{G_2}$

Light deflection in gravitational field

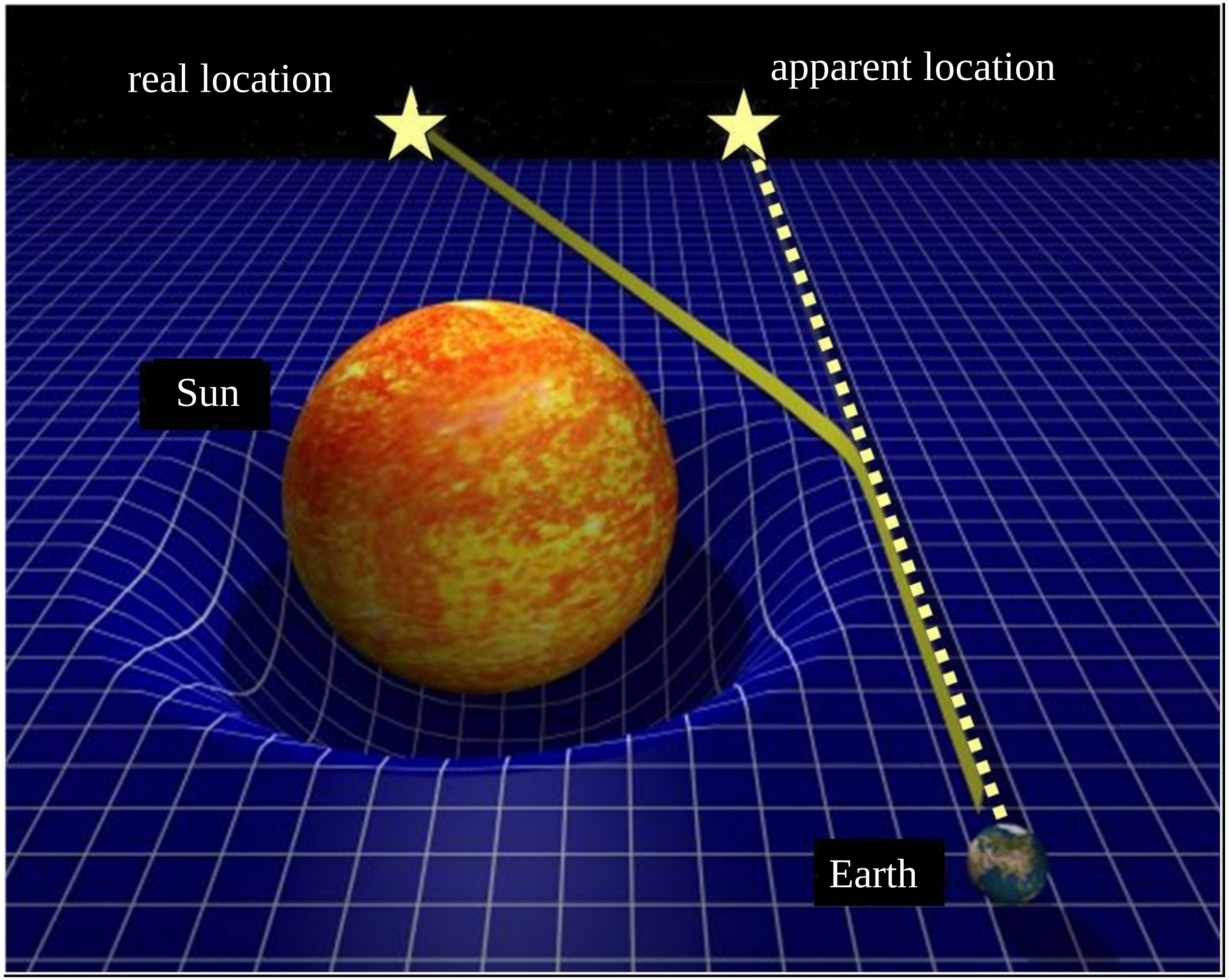
Using the equivalence principle, we examine the path of light in an accelerating system. Trajectory observed in accelerating system will match the trajectory observed in gravitational field.



Example:
We shine a powerful laser horizontally on the Earth's surface. How much does the path deviate from a straight line after traveling 1km due to Earth's gravity?

Experimental proof of light deflection

Eddington confirmed the deflection angles predicted by Einstein by observing stars visible near the Sun during a solar eclipse.



It can be explained based on Snell's law.

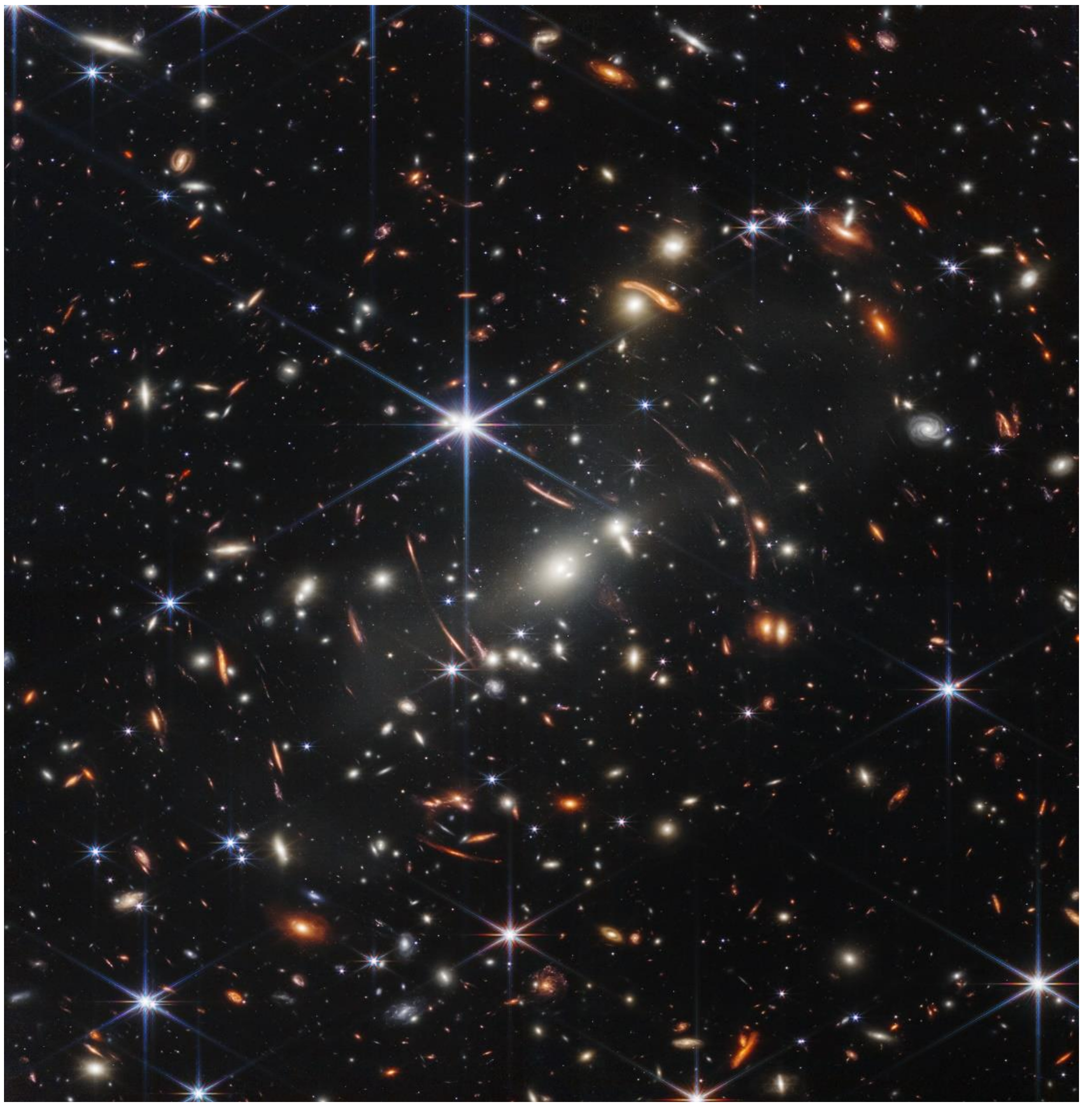
Light slows down in the gravitational field of the Sun to $\gamma(r)c$ speed.

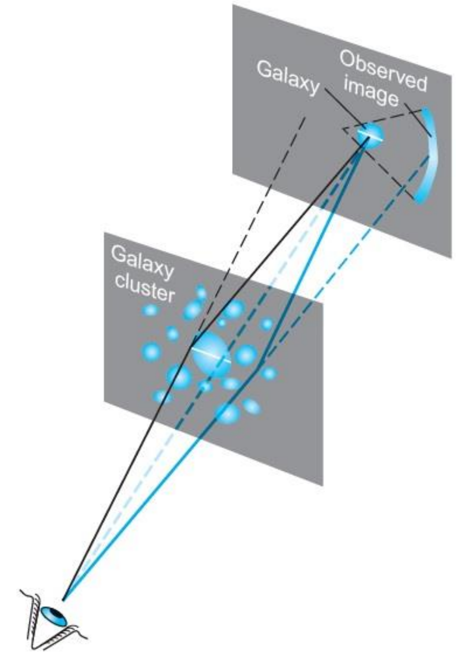
$$\gamma(r) = \sqrt{1 - \frac{2GM}{c^2 r}}$$

This leads to a rotation of the wavefront.

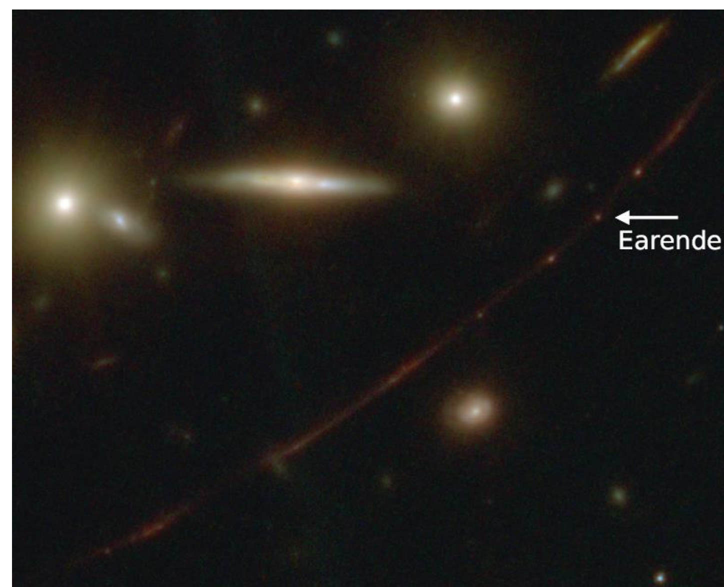
Gravitational lensing

The massive foreground galaxy cluster acts like a lens, distorting and magnifying the distant galaxies behind it.



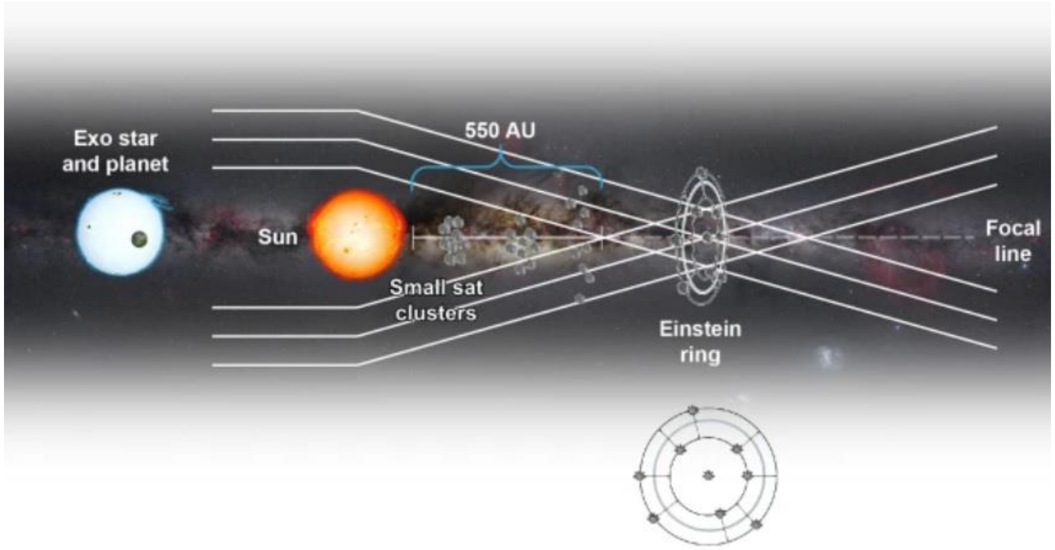


observer

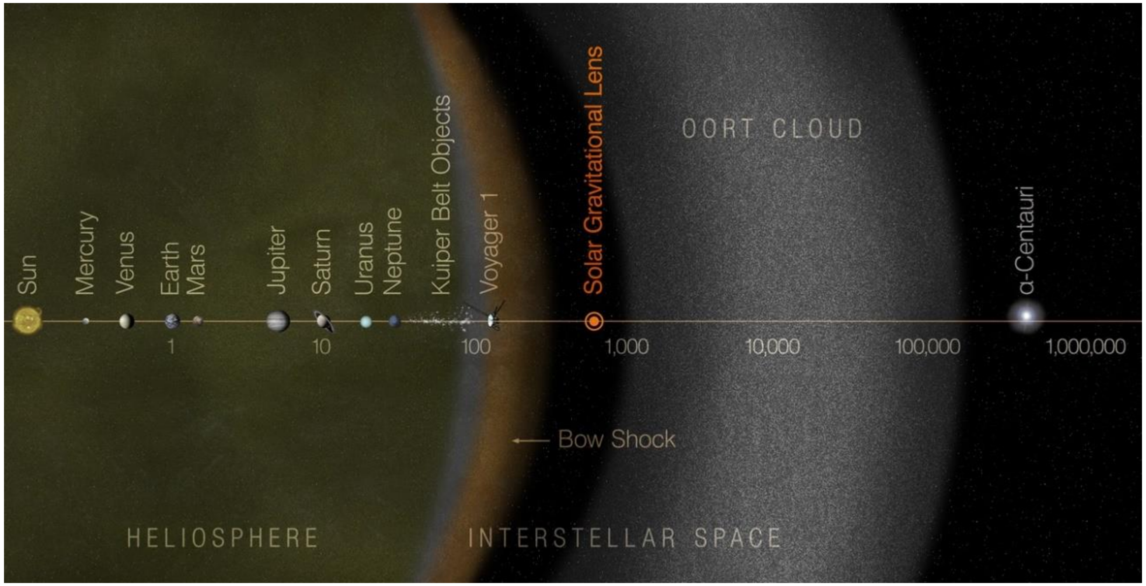


Gravitational telescope

The Sun's gravity can be used as a kind of lens by a swarm of satellites placed at a suitable distance.
100 light years ~10km/pixel



Einstein ring observed by the Hubble Space Telescope



Effect of mass on spacetime

Special relativity: invariant arc element in differential form

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

Minkowski spacetime (flat spacetime)

$d\tau$: proper time (time for observer resting in reference frame)

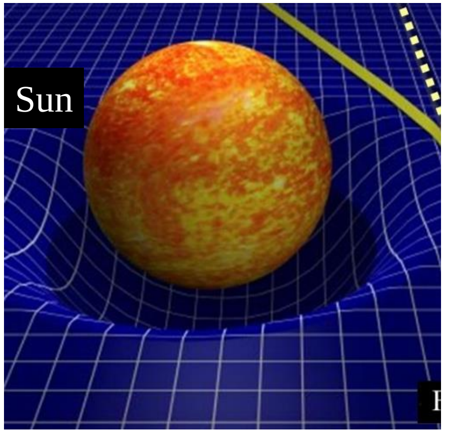
Converted to spherical polar coordinates:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

In the case of spherically symmetric mass distribution, the invariant arc element:

$$ds^2 = c^2 d\tau^2 = \gamma(r)^2 c^2 dt^2 - \frac{dr^2}{\gamma(r)^2} - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

Schwarzschild spacetime (curved spacetime)



$$\gamma(r) = \sqrt{1 - \frac{2GM}{c^2 r}} = \sqrt{1 - \frac{r_S}{r}}$$

r_S: Schwarzschild radius

Gravitational time dilation and redshift

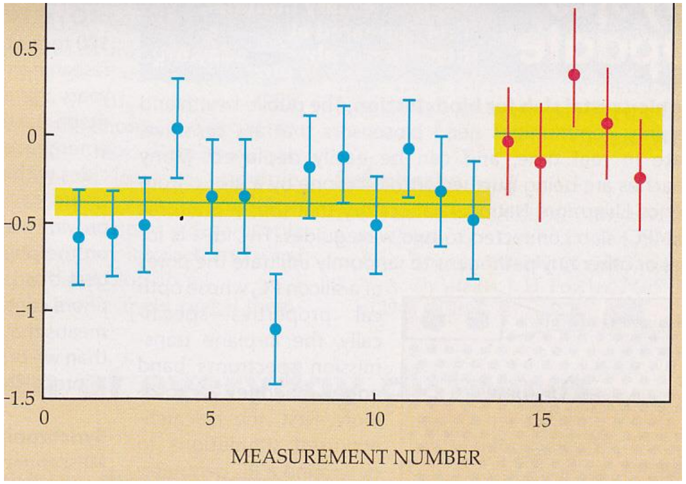
Comparing observers <u>at rest</u> located at infinity and at distance r from spherically symmetric object of mass M: $dr = 0, d\theta = 0, d\varphi = 0$

$$ds^2 = c^2 d\tau^2 = \gamma(r)^2 c^2 dt^2$$

$$\boxed{d\tau = \gamma(r)\, dt}$$

$$\gamma(r) = \sqrt{1 - \frac{2GM}{c^2 r}} = \sqrt{1 - \frac{r_S}{r}} < 1$$

The closer it is, the less time passes. Stronger gravity - slower clocks



Due to the slowing down of time, the frequency of light waves will be lower due to gravity:
redshift if light travels upwards
blueshift if light travels downwards
(detected for gamma photons for 22 meters)

Atomic clocks have detected a **33cm** height difference on Earth!

Gravitational waves

When massive celestial bodies are accelerated, changes in spacetime propagate at the speed of light, e.g. when neutron stars or black holes merge.