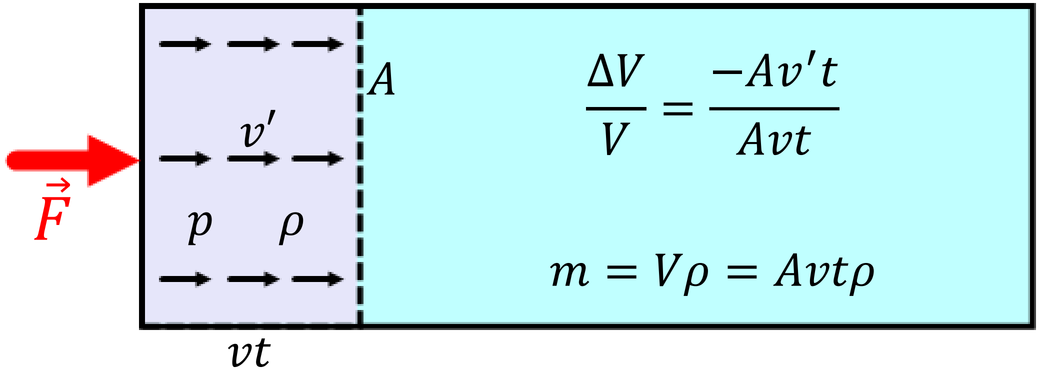
Speed of sound in air

Force acting on air column causes adiabatic compression.
After *t* time, particles in *Avt* volume move at *v'* velocity.

A: cross sectional area
p: pressure
ρ: density
v: speed of sound



$$\frac{\Delta V}{V} = \frac{-Av't}{Avt}$$

$$m = V\rho = Avt\rho$$

From Poisson equation:

$$pV^\kappa = (p+\Delta p)(V+\Delta V)^\kappa$$

$$p = (p+\Delta p)\left(1+\frac{\Delta V}{V}\right)^\kappa$$

$$p = (p+\Delta p)\left(1+\kappa\frac{\Delta V}{V}\right)$$

$$p = p + \Delta p + p\kappa\frac{\Delta V}{V} + 1 + \overset{\ll 1}{\Delta p\kappa\frac{\Delta V}{V}}$$

$$\Delta p = -p\kappa\frac{\Delta V}{V} \quad \rightarrow \quad F = \Delta p A$$

Bulk modulus:

$$B = p\kappa = -V\frac{dp}{dV}$$

From momentum-force law:

$$Ft = |\Delta\vec{p}| = Avt\rho v'$$

$$Ft = -p\kappa\frac{\Delta V}{V}At = Avt\rho v'$$

$$p\kappa\frac{Av't}{Avt}At = Avt\rho v'$$

$$p\kappa = v^2\rho \quad \rightarrow \quad v = \sqrt{\frac{p\kappa}{\rho}} = \sqrt{\frac{B}{\rho}}$$

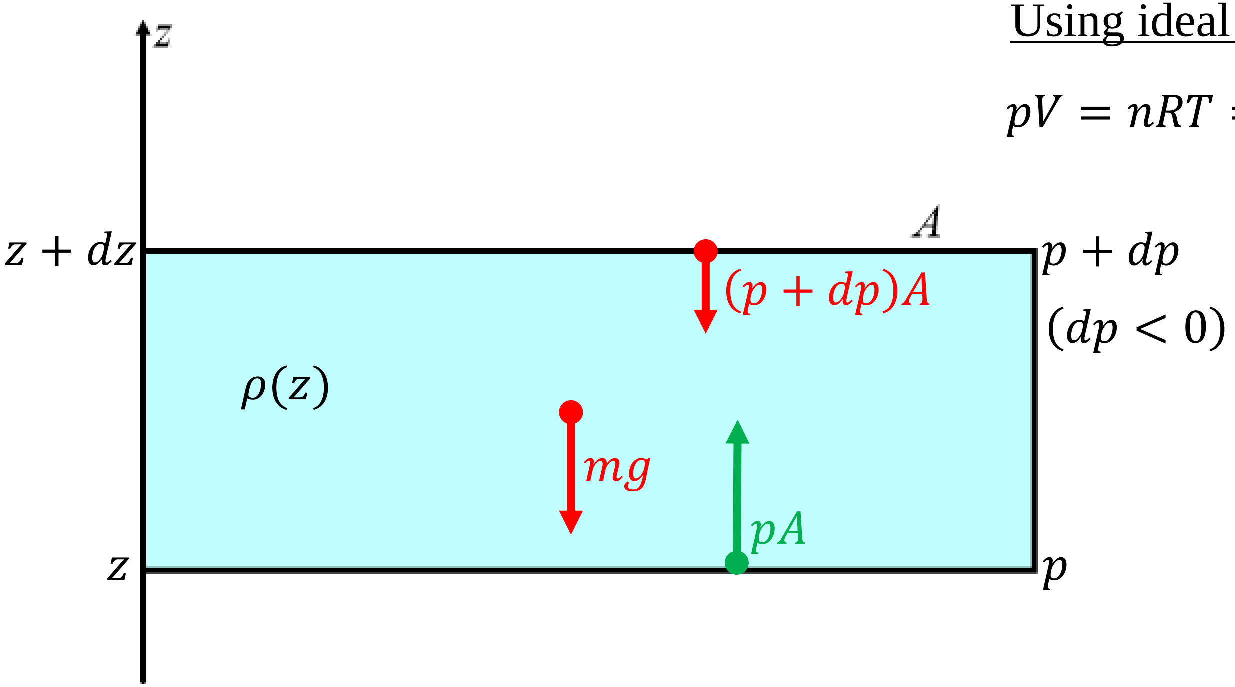
Using ideal gas law (eq. of state) to express density:

$$pV = nRT = \frac{m}{M}RT \quad \rightarrow \quad \rho = \frac{m}{V} = \frac{pM}{RT}$$

$$v = \sqrt{\frac{p\kappa}{\rho}} = \sqrt{\frac{p\kappa RT}{pM}} = \sqrt{\frac{\kappa RT}{M}}$$

Barometric formula

Weight of air layer is balanced by the pressure gradient.



Using ideal gas law to express density:

$$pV = nRT = \frac{m}{M}RT \quad \rightarrow \quad \rho = \frac{m}{V} = \frac{pM}{RT}$$

Forces must cancel out: $(p + dp)A + mg = pA$

$$dpA + \rho A dz g = 0$$

$$\frac{dp}{dz} = -\rho g = -\frac{pMg}{RT}$$

$$\frac{dp}{p} = -\frac{Mg}{RT}dz \quad \rightarrow *$$

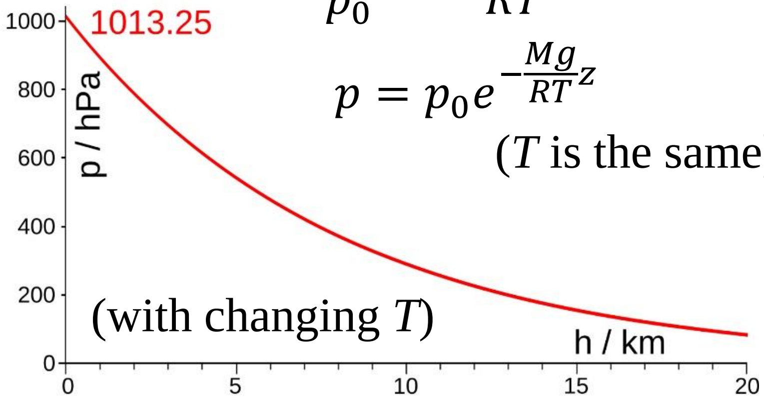
$$* \rightarrow \quad \int_{p_0}^{p} \frac{dp'}{p'} = -\frac{Mg}{RT}\int_0^z dz'$$

$$[\ln p']_{p_0}^{p} = -\frac{Mg}{RT}z$$

$$\ln\frac{p}{p_0} = -\frac{Mg}{RT}z$$

$$p = p_0 e^{-\frac{Mg}{RT}z}$$

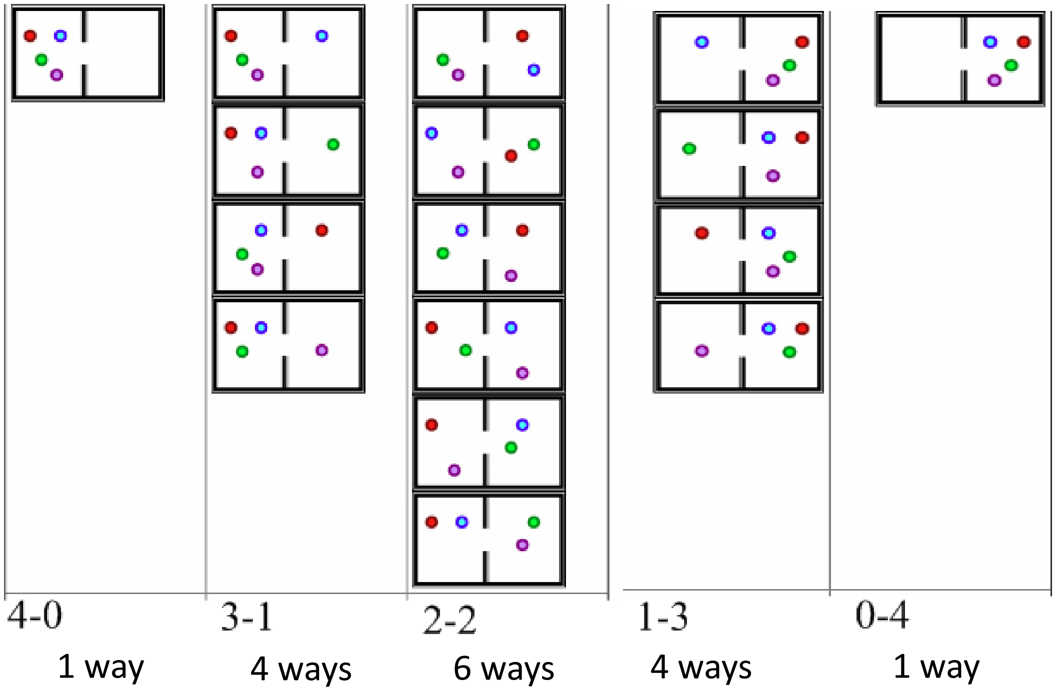
(T is the same)



(with changing T)

Microstates, state sum, entropy

A macroscopically measurable and distinguishable state of the system can be created by multiple microscopic arrangements (a state that also takes into account microscopic differences that are not perceptible to us, e.g., swapping two identical atoms in a container).



Example with 4 identical particles:

microstate:
which particle is on which side

macrostate:
how many particles on one side

state sum: Z
number of microstates
belonging to a macrostate
(how many ways can it be realized)

Even for 4 atoms it can be seen that the most probably arrangement is when atoms are distributed evenly. This is much more dominant for huge number of particles.

Entropy (disorder): $S = k\ln Z$ $k = 1{,}38\cdot 10^{-23}$ J/K (Boltzmann constant)

If the system receives δQ heat at temperature T, its entropy change is:

$$\boxed{dS = \frac{dQ}{T}}$$

Second law of thermodynamics

There are processes that are **irreversible**.
Examples:
- kinetic energy converted into heat is not converted back
- a broken glass does not bounce back intact
- gas molecules opened to an empty part will not all return to the original chamber

These processes are not prohibited by the laws of physics, but their probability is negligible. Therefore, it can be stated statistically that processes can only proceed in a direction that **increases disorder**. The measure of disorder is the **entropy**.
Entropy can be reduced locally, but only at the cost of increasing it elsewhere.
The entropy of the universe is therefore constantly increasing.

Second law of thermodynamics: The entropy of a closed system can never decrease.

$$\boxed{\Delta S \geq 0}$$

In the case of reversible processes, disorder remains unchanged.
e.g. elastic collision, adiabatic change of state

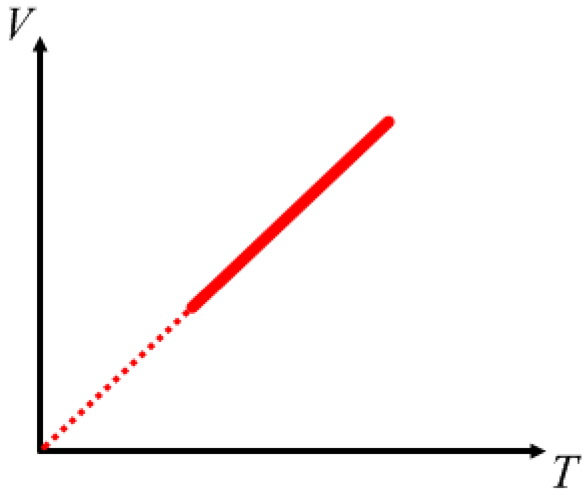
Real materials

Comparison of real substances and ideal gases

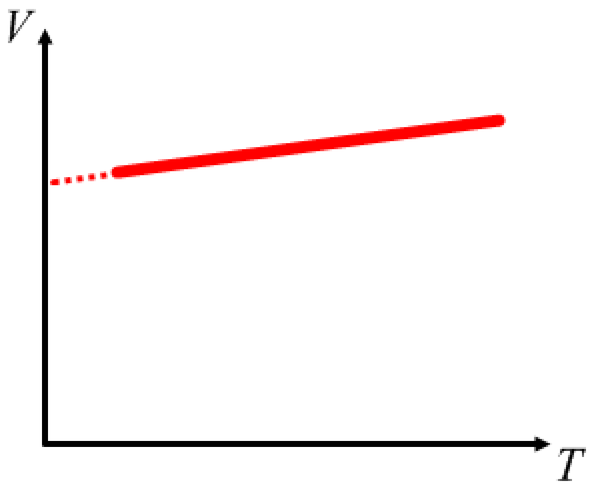
For ideal gases at constant pressure: $pV = nRT$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

As we approach absolute zero, the volume tends to zero for ideal gases.
Of course, this does not happen because at low temperatures the ideal gas approximation is not valid.



ideal gas



condensed body

Van der Waals equation of state for real gases

The pressure and volume of real gases do not approach zero with decreasing temperature.
The volume occupied by the particles is not negligible. Volume of 1 mole of particles: *b*
There is an attractive force between particles that come close to each other, the effect of which reduces the pressure.
The Van der Waals equation of state for 1 mole of gas ($v = V/n$):

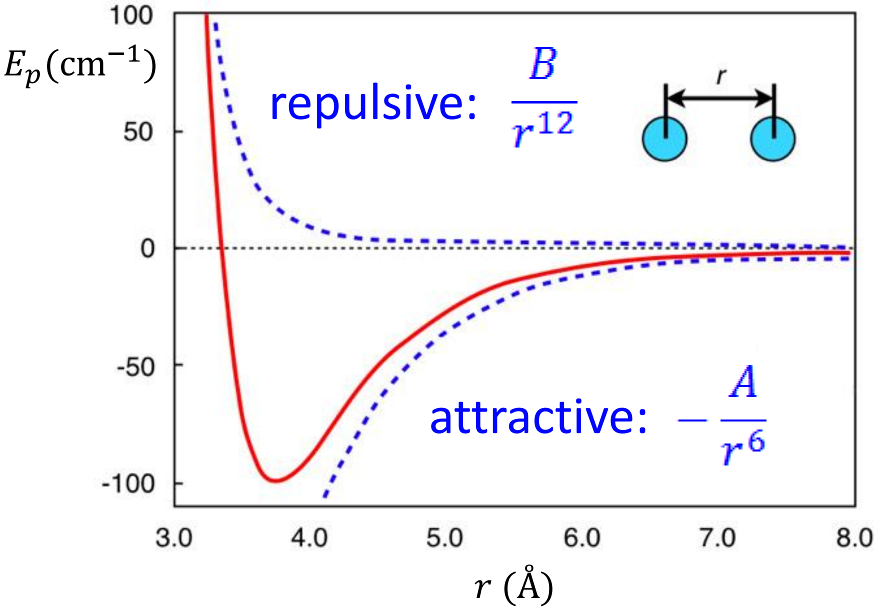
$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

In real gases, potential energy also contributes to internal energy.
As the volume increases, the particles move further apart, and the potential energy increases.
So internal energy depends not only on temperature, but also on volume.

Potential energy of
Van der Waals interaction:

$$E_p = -\frac{A}{r^6} + \frac{B}{r^{12}}$$

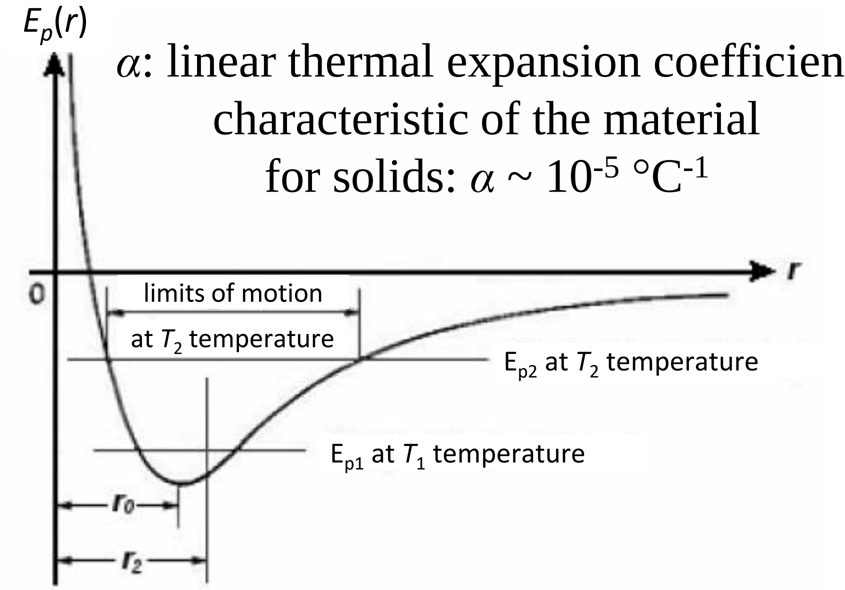
Lennard-Jones empirical formula.
A affects a, and B affects b.



Thermal expansion

Linear thermal expansion for small temperature changes: $\Delta h = \alpha h_1 \Delta T$ h_1: original length

So the new length: $\boxed{h_2 = h_1(1 + \alpha\Delta T)}$



α: linear thermal expansion coefficient
characteristic of the material
for solids: $\alpha \sim 10^{-5}$ °C^{-1}

The cause of thermal expansion is the asymmetry of the potential:
the average of the possible distances shifts towards larger values ($r_2 > r_0$).

Volumetric thermal expansion:
cube with side h_1
Initial volume $V_1 = h_1^3$, the new $V_2 = h_2^3$

Using the law of linear thermal expansion:

$$V_2 = h_2{}^3 = h_1{}^3(1+\alpha\Delta T)^3 = h_1{}^3(1 + 3\alpha\Delta T + 3\alpha^2\Delta T^2 + \alpha^3\Delta T^3) = h_1{}^3(1+3\alpha\Delta T)$$

$\alpha\Delta T$ is small, so higher order terms can be omitted

$$\boxed{V_2 = V_1(1+\beta\Delta T)}$$

$\beta = 3\alpha$ β: volumetric thermal expansion coefficient

It is valid for any isotropic solids and liquids. Does not apply for water, because its volume is minimal at 4 °C (it expands even when cooled!)