

# **The special theory of relativity**

## **The preliminaries of special theory of relativity**

The Galilean theory of relativity states that it is impossible to find the absolute reference system with mechanical experiments. In other words: The laws of mechanics are the same in every inertial frame of reference.

By the end of the XIX century, the physicists still believe that there is an absolute frame of reference, and absolute time, but so far had been impossible to find it by mechanical experiments.

By this time however a lot of new and very sensitive experimental methods have been developed, using electromagnetic waves, and electrodynamics.

Physicists think that the universe is fill out with hypothetical aether or ether (it was assumed that the ether must be luminiferous, or very tenuous, in order to allow the planets to move freely through it), and this is the absolute frame of reference, and the light waves propagate in this hypothetical medium ether, just as the sound wave in the air.

The double role of ether is: it is the absolute frame of reference (the absolute rest), and the medium in which the light propagates.

If ether exists, it is possible to measure the speed of the Earth with respect to ether.

During the late nineteenth and early twentieth centuries, intensive efforts were made to find experimental evidence for the existence of the ether. The Michelson-Morley experiment was an effort to detect motion of the earth relative to ether. This very important experiment was taken in 1887 by Michelson and Morley, and was a significant role in the birth of special theory of relativity.

The experimental device is called Michelson-Morley interferometer.

The experiment was repeated several times and despite the orbital velocity the earth appeared to be at rest relative to the ether. This and all similar experiments yielded consistently negative results and the ether concept has been discarded.

This negative result baffled physicists of the time, and to this day the Michelson-Morley experiment is the most significant “negative result” experiment ever performed.

Einstein was who discarded the concept of ether and the absolute frame of reference and he stated the postulates of the special theory of relativity.

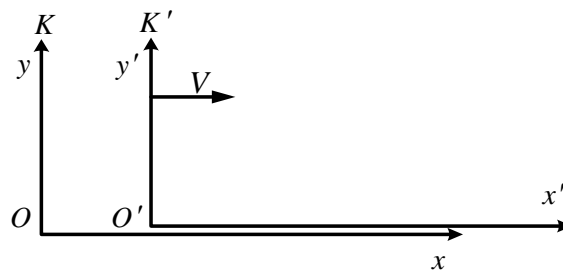
1. There is no absolute or preferred inertial system. The laws of physics are the same in all inertial frames. It is impossible to find difference between them by any physical experiment.
2. Ether does not exist; the light waves propagate isotropic way without medium. The speed of light is the same in all inertial system  $c=300\ 000\ \text{km/s}$ .

The Special Theory of Relativity was worked out in 1905 by Einstein. The word special means that it deals only with inertial reference frames.

There is a contradiction between the special relativity and the Galilean relativity.

Consider the well known Galilean coordinate transformation equations between two inertial frames, K and K'. Let the x-axes of the two frames lie along the same line, but let the origin O' of K' move relative to the origin O of K with constant velocity V. The origins coincide at time t=0.

V = constant , and t = 0, when O = O'



Each observer records the same event, which might be the detonation of tiny flashbulb, and assigns space and time coordinates to the event, namely  $x, y, z, t$  and  $x', y', z', t'$ . What are the relations between these two sets of numbers?

$$\left. \begin{array}{l} x = x' + Vt' \\ y = y' \\ z = z' \\ t = t' \end{array} \right\} \left. \begin{array}{l} x' = x - Vt \\ y' = y \\ z' = z \\ t' = t \end{array} \right\} \text{the Galilean transformation equations}$$

$$v_x = \frac{dx}{dt} \quad v'_x = \frac{dx'}{dt} = \frac{dx}{dt} - V$$

$$\left. \begin{array}{l} v'_x = v_x - V \\ v'_y = v_y \\ v'_z = v_z \end{array} \right\} \text{velocity transformation}$$

This equation is in contradiction with principle of special theory of relativity, that the speed of light is constant. The cause of this is that the same time scale is applied  $t = t'$ . The time is not the same for each observer. The Galilean transformation equations are correct in the important region  $V \ll c$ , but fail as  $V \rightarrow c$ . The corresponding equations are the so called Lorentz transformation equations, without proving:

$$\left. \begin{array}{l} x' = \gamma(x - Vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \frac{V}{c^2}x\right) \end{array} \right\} \left. \begin{array}{l} x = \gamma(x' + Vt') \\ y = y' \\ z = z' \\ t = \gamma\left(t' + \frac{V}{c^2}x'\right) \end{array} \right\} \gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \text{ Lorentz-factor}$$

The transformation equations have sense until  $V < c$  ( $\gamma$ -must be real number). In the Special theory of Relativity the speed of light is an upper limit for the speeds of material objects.

If  $V \ll c$  or  $c \rightarrow \infty$ , the Lorentz equations reduce to Galilean equations. So for slow object the classical or Newtonian mechanics is correct.

As the transformations are linear in the variables, they can be changed to differentials or finite differentia. It means instead of  $x$  we can use  $dx$  or  $\Delta x$ .

$$\left. \begin{aligned} \Delta x' &= \gamma(\Delta x - V \Delta t) \\ \Delta y' &= \Delta y \\ \Delta z' &= \Delta z \\ \Delta t' &= \gamma\left(\Delta t - \frac{V}{c^2} \Delta x\right) \end{aligned} \right\} \left. \begin{aligned} \Delta x &= \gamma(\Delta x' + V \Delta t') \\ \Delta y &= \Delta y' \\ \Delta z &= \Delta z' \\ \Delta t &= \gamma\left(\Delta t' + \frac{V}{c^2} \Delta x'\right) \end{aligned} \right\}$$

$$\left. \begin{aligned} dx' &= \gamma(dx - V dt) \\ dy' &= dy \\ dz' &= dz \\ dt' &= \gamma\left(dt - \frac{V}{c^2} dx\right) \end{aligned} \right\} \left. \begin{aligned} dx &= \gamma(dx' + V dt') \\ dy &= dy' \\ dz &= dz' \\ dt &= \gamma\left(dt' + \frac{V}{c^2} dx'\right) \end{aligned} \right\}$$

The Lorentz equations have several interesting consequence, velocity transformation, time dilatation, and length contradiction.

Velocity transformation:

At given  $v'_x$  calculate  $v_x$ .

$$v_x = \frac{dx}{dt} = \frac{\gamma(dx' + V dt')}{\gamma\left(dt' + \frac{V}{c^2} dx'\right)} = \frac{v'_x + V}{1 + \frac{V}{c^2} v'_x}$$

If  $v'_x = c$ , then

$$v_x = \frac{c + V}{1 + \frac{V}{c^2} c} = \frac{c + V}{\frac{c + V}{c}} = c.$$

Time dilatation:

Let measure a time interval in  $K'$   $\Delta t'$ , for the observer in  $K$  this interval is  $\Delta t$ , and  $\Delta t > \Delta t'$ , which is called time dilatation, we often verbalize it as the “moving clocks run slow”.

Length or Lorentz contradiction:

Let measure the length of a rod in  $K'$   $\Delta x'$ , for the observer in  $K$  this length is measured  $\Delta x$ , and  $\Delta x < \Delta x'$ , and it is called length contradiction.

## Relativistic dynamics

Due to the postulates of the special theory of relativity, the laws of physics are the same in all inertial frames.

1. The Maxwell equations are exact equations, because they are invariant under the Lorentz coordinate transformation.
2. The Newton's laws of motion are invariant under the Galilean coordinate transformation, but in case of relativistic mechanics we have to apply the Lorentz transformations, and they are not invariant under Lorentz coordinate transformation.

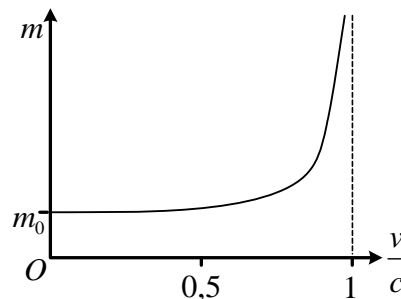
This requires generalization in the laws of motion, and the definitions of momentum and energy.

In the special theory of relativity we can apply the equation:  $\frac{d\vec{p}}{dt} = \vec{F}$ , the force is the time rate of change of momentum, where  $\vec{p} = m\vec{v}$ , but the mass is not constant any more. At around 1901 Kaufmann measured the dependence of the inertial mass (or momentum) of an object on its velocity. Exactly he measured the electron charge to mass ratio for different velocities of the electron. In 1901 he was able to measure a decrease of the charge-to-mass ratio, thus demonstrating that mass or momentum increases with velocity.

The variation of mass with velocity is given by the relativistic mass formula:

$$m = m_0 \gamma = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}}, \text{ where } m_0 \text{ is the rest mass, that is, the mass measured when the particle}$$

is at rest with respect to the observer, and  $\gamma$  is the Lorentz factor.



$m$  is the relativistic mass,  $m_0$  is the rest mass. If,  $V \rightarrow c$ , then  $m \rightarrow \infty$ .

The relativistic equation of motion

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt} \cdot \vec{v} + m \cdot \frac{d\vec{v}}{dt}$$

Since we generalized the Newton's II law to bring it into accord with the principle of relativity, it is not surprising that the energy concept also requires generalization.

In classical or Newtonian mechanics the definition of the momentary power of force is

$$P = \vec{F} \cdot \vec{v}, \text{ and the power law } P = \frac{dT}{dt}, \text{ where } T = \frac{1}{2}mv^2, \text{ the kinetic energy.}$$

This equation states that the time derivative of the kinetic energy is equal to the power delivered by the force. This is the so called power law.

In case of relativistic mechanics the form of the power law:

$$P = \frac{d}{dt}(mc^2).$$

It means that we can keep the original form of the power law, if we introduce the relativistic kinetic energy as:

$$T = mc^2 + K, \text{ where } K \text{ is constant. It is a trivial agreement, that if } v=0, \text{ then } T=0.$$

$$0 = m_0c^2 + K, \quad K = -m_0c^2.$$

Thus the relativistic kinetic energy:

$T = mc^2 - m_0c^2 = m_0\gamma c^2 - m_0c^2 = m_0c^2(\gamma - 1)$ . Of course if  $v$  is much smaller than  $c$ , we can obtain the classical kinetic energy expression  $T = \frac{1}{2}mv^2$ .

$$mc^2 = T + m_0c^2$$

$$E = mc^2, \text{ total or relativistic energy}$$

$$E_0 = m_0c^2, \text{ rest energy}$$

$$T, \text{ kinetic energy}$$

$$E = T + E_0$$

The rest energy is a new concept. If  $v=0$ , then  $T=0$ , and  $E = E_0$

The best known result of the special theory of relativity is the so called mass energy equivalence.  $E = mc^2$ . That is the conservation of total energy is equivalent to the conservation of relativistic mass. Mass and energy are equivalent they are connected by a universal constant the speed of light.

In classical physics we had two separate conservation principles: conservation of mass, and conservation of energy. In relativity these merge into one conservation principle, that of the conservation of mass-energy.

The most important experimental verification of the mass energy equivalence is the Cockroft – Walton experiment (1932).

After discovering the particle accelerators, resting Li atoms were bombarded by accelerated protons. This was the first artificial splitting of a nucleus. It was also the first transmutation using artificially accelerated particles.

The following reaction took place:  ${}^1_1\text{H} + {}^7_3\text{Li} \rightarrow {}^4_2\text{He} + {}^4_2\text{He}$

In the reaction it was found for the rest masses:  $m_o^{Li} + m_o^H > 2m_o^{He}$ , so a mass defect appeared

$$\Delta m_o = 2m_o^{He} - m_o^{Li} - m_o^H < 0.$$

But at the same time the kinetic energy of the two He nuclei is greater than the kinetic energy of the proton and Li nucleus:  $T_{He_1} + T_{He_2} > T_H + T_{Li}$ , that is kinetic energy growth appeared

$$\Delta T = T_{He_1} + T_{He_2} > T_{Li} + T_H > 0$$

Due to experience in the reaction mass converted into kinetic energy  $\Delta T + \Delta m_o c^2 = 0$ .

This reaction was the first experimental proof of Einstein's  $E = mc^2$ . We can say that some part of rest energy was converted into kinetic energy.