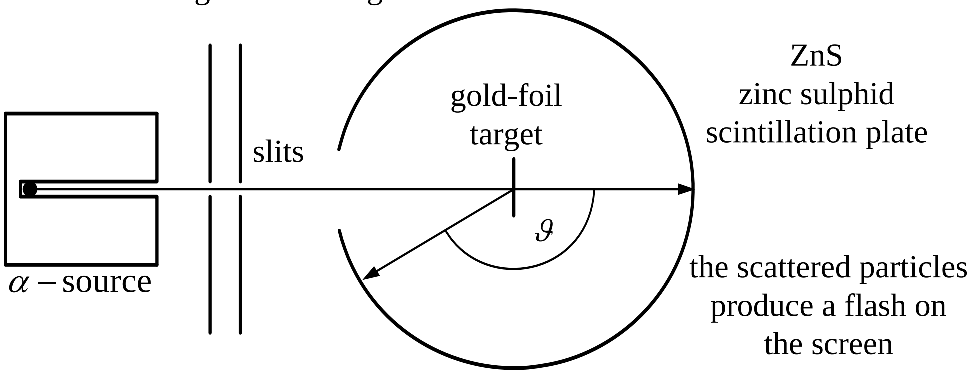
Nuclear physics

Discovering the nucleus:

In 1910 the available model of atomic structure was the so called “plum pudding model” developed by Thomson. He thought that the atom as a sphere of some positively charged substance, in which the electrons were embedded like plum in pudding.

Rutherford experiment (1911):
The first experiment designed to test Thomson’s model of the atom were carried out in 1911 by Rutherford. In the experiment alpha particles emitted from naturally radioactive element were used as projectiles and were directed at a thin gold target foil and measure the extent to which were deflected as they passed through the foil. The next figure is a schematic diagram of Rutherford’s experimental setup. A radioactive substance at the left emits alpha particles. Thick lead collimator stops all particles except those in a narrow beam. The beam passes through the foil target (consisting of gold, silver, or copper) and strikes screens coated with zinc sulphide, creating a momentary flash, or scintillation. Rutherford counted the numbers of particles deflected through various angles.



Most of the particles were scattered through rather small angles, but a very small fraction of them were scattered through large angles, approaching 180°. So the Thomson model was wrong and a new model was needed. Rutherford supposed that the positive charge, instead of being distributed through a sphere with atomic dimensions $10^{-10} m$ radius, is all concentrated in a much smaller region $10^{-14} m$. In other words, the atom is mostly empty space. Rutherford developed this model on the basis of his experiments and called the concentration of positive charge the nucleus. Later experiments showed that all nuclei are composed of positively charged protons and electrically neutral neutrons discovered in 1932 by Chadwick. In his experiment:

$$ {}^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{12}_{6}C + {}^{1}_{0}n $$

From this time we know the nuclei are made up of protons and neutrons. The number of protons in a nucleus (called the atomic number or proton number of the nucleus) is represented by the symbol *Z*; the number of neutrons (the neutron number) is represented by the symbol *N*. The total number of neutrons and protons in a nucleus is called its mass number *A*; thus $A = Z + N$.

The atomic number *Z* has a triple meaning; this is the number of the element in the periodic table, the charge of the nucleus in unit of the elementary charge (charge of an electron *e*) and the number of electrons in a neutral atom. Some nuclides that have the same *Z* but different *N*,

these nuclides are called isotopes of that element; they have different masses because they have different numbers of neutrons.
Examples: ${}^{1}_{1}H$ is called hydrogen, ${}^{2}_{1}H$ is called deuterium, ${}^{3}_{1}H$ is called tritium, the two common isotopes of uranium ${}^{235}_{92}U$, ${}^{238}_{92}U$.

Hofstadter measured the radius of several nuclei, and showed that a radius *R* of a nucleus depends on the mass number *A*. The empirical equation for the radii of most nuclei:

$$R = R_0 \cdot A^{\frac{1}{3}}, \text{ where } R_0 = 1,4-1,5\cdot 10^{-15} m .$$

$$V = \frac{4R^3\pi}{3} = \frac{4R_0^3 A\pi}{3} = \frac{4R_0^3\pi}{3} A .$$

$$\frac{A}{V} = \frac{3}{4R_0^3\pi} = \text{constant}$$

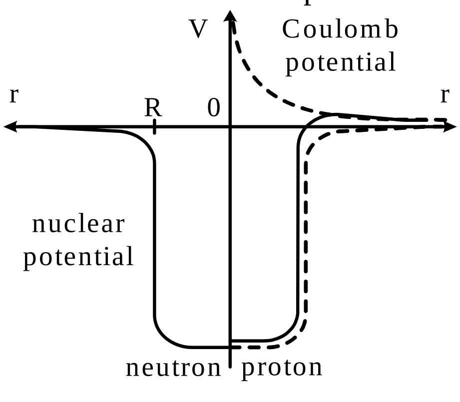
That is all nuclei have approximately the same density. This is a very important fact of the nuclear structure.

Nuclear interaction

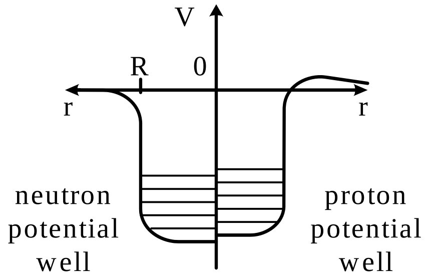
We know the composition of the nucleus: *Z* is the number of protons, and $N = A - Z$ the number of neutrons. In several cases the nucleus is sable structure. What kind of interaction exists inside the nucleus? This interaction is not gravitational (the gravitational interaction is very week), not electrical interaction (the neutrons are neutral, and what is more the protons repulse each other). The force that holds the protons and neutrons together in the nucleus is a new so called nuclear or strong interaction. What are the features of the nuclear interaction?

1. it does not depend on charge, charge independent: p-p, n-n, and p-n interactions are the same
2. it is always attractive, the potential energy is negative
3. the interaction is short range $1.4\ 10^{-15} m$, a given nucleon cannot interact simultaneously with all the other nucleons only a few in its vicinity. The nuclear interaction goes into saturation.
4. it is very strong, 100-times greater then the electrical interaction at the same distance

For most problems the nuclear interaction may be represented schematically by a potential energy as shown in the next figure. Beyond a certain distance the potential energy is practically constant. The left side shows the interaction in case of n-n, and n-p interactions. For p-p interactions, we must include the coulomb repulsion as well.



Each nucleon moves in an average field of force which is produced by the other nucleons and due to quantum mechanics the protons and the neutrons have discrete energies. They fill these energies in pairs in accordance with Pauli principle.

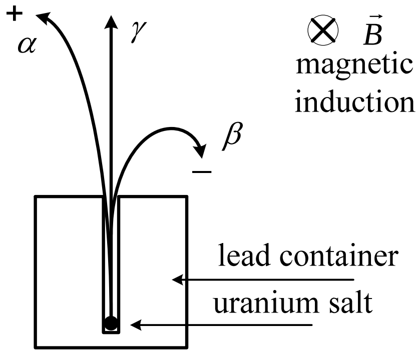


Radioactivity

Some nuclei have a combination of protons and neutrons which does not lead to a stable configuration. These nuclei are therefore unstable or radioactive.

It was discovered by Becquerel (1896) that there is a radiation from uranium salts seemed similar to x rays without any external effect. This phenomenon is called natural radioactivity. This radiation exposed the photo plate in its surrounding.

Later it was investigated by Rutherford, and Curies. In magnetic field the radiation was separated into three different parts. The emissions consist of positively and negatively charged particles and neutral rays; they were given the names alpha, beta, and gamma.



α-radiation, or alpha decay: When a nucleus undergoes alpha decay, it transforms to a different nuclide by emitting an alpha particle (a helium nucleus, ${}^{4}_{2}He^{++}$): the penetration properties is small, a piece of paper is enough to stop radiation. The general formula: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$, and en example: ${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$

β-radiation or beta decay: When a nucleus undergoes beta decay, it transforms to a different nuclide by emitting an electron (or a positron). When the original element has decayed into a new chemical element in a process it is called as nuclear transmutation. The outgoing electron was not in the nucleus, it originated just at the moment from a decay of a neutron. The velocity of the electrons is very large $\sim 0.99c$, quite close to the velocity of light. The penetration properties is intermediate, a few mm aluminium plate stops the radiation. The general formula of negative beta decay:
${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e$, and an example: ${}^{3}_{1}H \rightarrow {}^{3}_{2}He + e$.

γ-radiation, or gamma decay: In this case the excited nucleus emits a very high frequency so called γ photon, the frequency is $\sim 10^{18} Hz - 10^{21} Hz$. The penetration properties is high, a few cm lead plate stops the radiation. The general formula of gamma decay:
${}_Z^A X^* \rightarrow {}_Z^A X + \gamma$.

The laws of radioactive decay

A radioactive nuclide spontaneously emits a particle, transforming itself in the process into a different nuclide. The radioactive decay provided the first evidence that the laws that govern the subatomic world are statistical. There is absolutely no way to predict whether any given nucleus in a radioactive sample will be among the small number of nuclei that decay during the next second. All have the same chance.

The experiences show that the time required for the number of radioactive nuclei to decrease to one-half the original number is always the same for a given nucleus. This time is called the half-life time $T_{1/2}$. The half time is characteristic for the given nucleus, and its value is about $(10^{-7}\,\text{s} - 10^{10}\,\text{year})$.

Let's denote the initial number of radioactive nuclei in the sample by *N_0*. Describe the variation of he numbers remaining after successive half-live times:

$$N_0 \underset{T_{1/2}}{\rightarrow} \frac{N_0}{2} \underset{T_{1/2}}{\rightarrow} \frac{N_0}{2\cdot 2} \underset{T_{1/2}}{\rightarrow} \frac{N_0}{2\cdot 2\cdot 2}.$$

The number of remaining nuclei as a function of time is $N(t) = \frac{N_0}{2^{\frac{t}{T_{1/2}}}} = N_0 \cdot 2^{-\frac{t}{T_{1/2}}}$, as $2 = e^{\ln 2}$

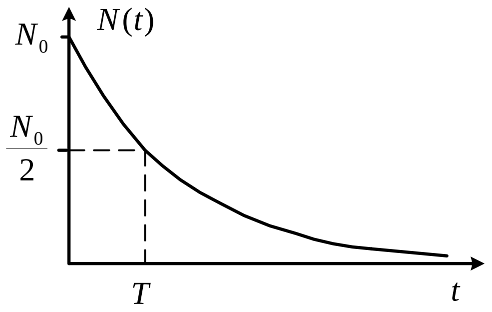
so $N(t) = N_0 (e^{\ln 2})^{-\frac{t}{T_{1/2}}} = N_0 e^{-\frac{\ln 2}{T_{1/2}} t}$. Introduce the decay constant $\lambda = \frac{\ln 2}{T_{1/2}}$,

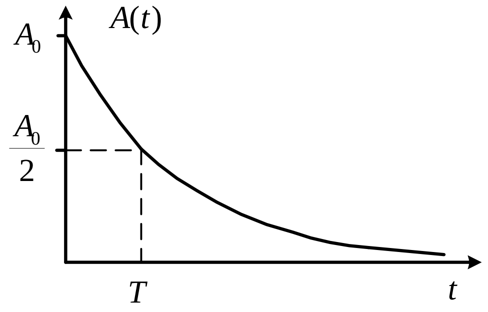
$$N(t) = N_0 e^{-\lambda t}.$$

We are often more interested the rate at which nuclei will decay. $A(t) = \left| \frac{dN}{dt} \right|$. So

$A(t) = \left| \frac{dN}{dt} \right| = N_0 \lambda e^{-\lambda t} = \lambda N(t)$, or $A(t) = A_0 e^{-\lambda t}$, where $A_0 = N_0 \lambda$, the activity in the initial moment. So the activity is the absolute value of the time rate of change of the variation of the number of remaining nuclei. Its unit is $1 \frac{\text{decay}}{s} = 1\,\text{becquerel} = 1\text{Bq}$.

Thus the graph of activity versus time *A(t)* has the same shape as *N(t)*.



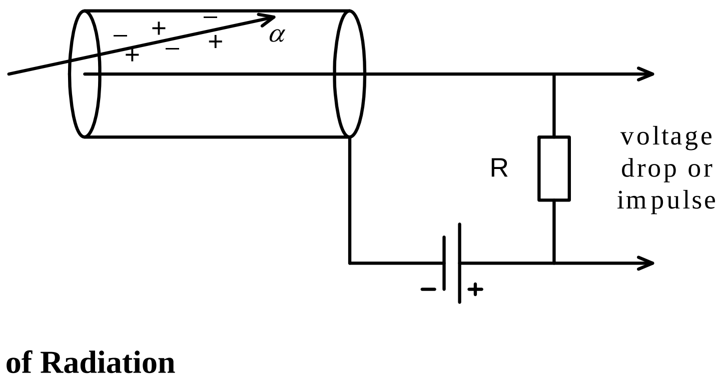


Radioactive decay series

The decaying nucleus is usually called the parent nucleus; the resulting nucleus is the daughter nucleus. When a radioactive nucleus decays, the daughter nucleus may also be unstable. In this case a series of successive decays occurs until a stable configuration is reached. During the decay the total number of neutrons and protons in a nucleus is called its mass number *A* is constant in beta-, and gamma decay, or decreases by 4 in case of alpha decay, there are four such radioactive decay series in nature at the end of the periodic table. Dividing the mass number by 4 there are 4 different possibilities for the remainder.
$A = 4n$, $A = 4n + 1$, $A = 4n + 2$, $A = 4n + 3$
The most important radioactive nuclide found on earth is the $^{238}_{92}U$ isotope ($A = 4n + 2$) which undergoes a series, terminating at a stable isotope of lead $^{206}_{82}Pb$. Radioactive decay series can be represented on a Segrè chart.

Detection and measurement of radioactive radiation
The alpha-, beta-, and gamma radiations are ionizing radiations, which causes ionization of atoms. The alpha-, and beta particles are charged and due to their electric field they are able to ionize the atoms and molecules. As they fly they produce ion pairs. To detect or measure the radiation we can detect or measure the ion pairs produced. The most known device is the so called Geiger-Müller counter.

The next figure shows the most frequently used device called Geiger-Müller counter. A thin, positively charged central wire is surrounded by a concentric, circular, conducting cylindrical shell and a potential difference is applied between them, creating a strong radial electric field. The counter contains a low-pressure inert gas. A particle of radiation entering the device through the window ionizes a few of the gas atoms. The resulting free electrons are drawn to the positive wire. As the electrons and ions accelerate towards the electrodes, they collide with gas atoms, and collide and ionize more atoms also. More free electrons are thereby created, and the process is repeated until the electrons reach the wire. The resulting "avalanche" of electrons is collected by the wire, generating a signal on a resistor. This impulse can be amplified and counted to record the passage of the original particle of radiation.



Biological Effects of Radiation

The effect of radiation on living tissue is a matter of public interest. As the radiation pass through matter, they lose energy creating ions—hence the term ionizing radiation, breaking molecular bonds and disturb the biochemical reactions. From now under radiation we include radioactivity (alpha, beta, and gamma radiations), neutrons and X-rays. Charged particles interact directly with the electrons in the material. X-rays interact by the photoelectric effect, or by Compton scattering. Neutrons cause ionization indirectly through collisions with nuclei.

The effect of the radiation depends on the absorbed energy.
Absorbed dose *D*. This is a measure of the radiation dose (as energy per unit mass) actually absorbed by a specific object, such as a patient's hand or chest.

$$D = \frac{d\overline{E}}{dm} = \frac{\text{energy of absorbed dose}}{\text{mass of living organism}}$$

Its SI unit is the gray (Gy). $\left[D\right] = 1\frac{J}{kg} = 1 gray = 1 Gy$.

Dose equivalent *H*. Although different types of radiation may deliver the same amount of energy to the body, they do not have the same biological effect. The dose equivalent allows us to express the biological effect by multiplying the absorbed dose by a numerical factor (from relative biological effectiveness). $H = DQ$, the SI unit $\left[H\right] = 1 sievert = 1 Sv$.
numerical factor Q

1	X-rays, gamma radiation, beta radiation
2.3	thermal (slow) neutrons
10	fast neutrons and protons
20	alpha radiation

It is well known that excessive exposure to radiation, including sunlight, X-rays, and all the nuclear radiations, can destroy tissues. In mild cases it results in a burn, as with common sunburn. Greater exposure can cause very severe illness or death by a variety of mechanisms, including massive destruction of tissue cells, alterations of genetic material, and destruction of the components in bone marrow that produce red blood cells.

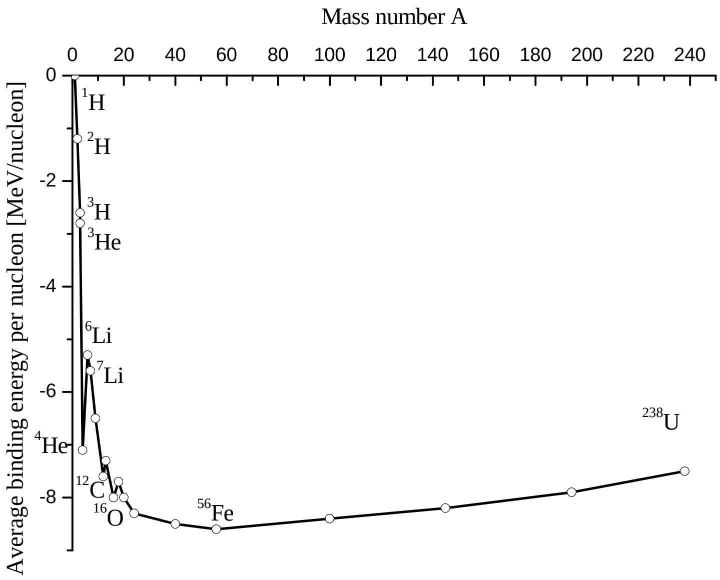
Mass defect and nuclear binding energy

At the end of the special theory of relativity we have already seen the experimental verification of the mass energy equivalence. It was the Cockroft – Walton experiment (1932). In a nuclear reaction: ${}^{1}_{1}H + {}^{7}_{3}Li \rightarrow {}^{4}_{2}He + {}^{4}_{2}He$ mass defect and kinetic energy growth appeared.

In general the mass $M(A,Z)$ of a nucleus is less than the total mass of its individual protons and neutrons. The difference between these masses is called the mass defect. In other words the mass of the components is greater than the mass of the nucleus.

$$\Delta m = M(A,Z) - Zm_p - (A-Z)m_n < 0$$

Due to the mass energy equivalence the binding energy of the nucleus is $E_b = \Delta mc^2 < 0$. In other words $\left|E_b\right|$ is the energy required to separate the nucleons composing a nucleus. As the mass of a nucleus can be measured its binding energy can be calculated. Introduce the average binding energy per nucleon $\varepsilon = \frac{E_b}{A} < 0$. On the next figure the average binding energy per nucleon is shown as a function of the mass number.



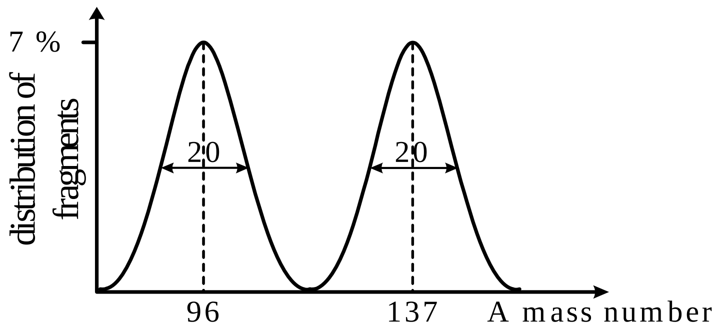
The binding energy per nucleon is practically constant (above $A=20,\ \varepsilon = -8\ \text{MeV/nucleon}$) suggest that each nucleon interacts only with its direct neighbours, independently of the total number of nucleons. The nuclear interaction goes into saturation. The curve has a minimum at ${}^{56}_{26}Fe$. Therefore there are two possibilities to release nuclear energy. Two light nuclei form a nucleus whose rest mass is less than the sum of the rest masses of the original nuclei. This process is called fusion. Energy is also liberated if a heavy nucleus is divided into two medium mass fragments, this process is called fission.

Nuclear fission

In 1932 Chadwick discovered the neutron, and it was used as a nuclear projectile. In 1937 Hahn and Strassmann bombarded uranium with thermal or slow neutrons and they had found two intermediate radioactive fragments and very high radioactivity. Some typical reactions:

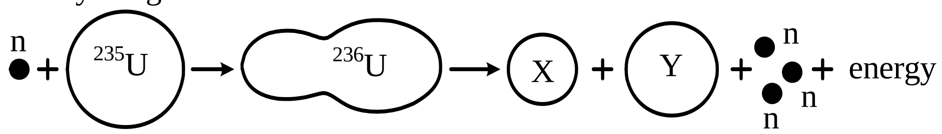
$$ {}^{235}_{92}U + n \rightarrow {}^{236}_{92}U \rightarrow {}^{139}_{56}Ba + {}^{94}_{36}Kr + 3n + energy $$
$$ {}^{235}_{92}U + n \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{54}Xe + {}^{90}_{38}Sr + 2n + energy $$
$$ {}^{235}_{92}U + n \rightarrow {}^{236}_{92}U \rightarrow {}^{137}_{55}Cs + {}^{96}_{37}Rb + 3n + energy $$

Nuclear fission is a decay process in which an unstable nucleus splits into two fragments of comparable mass, some neutrons appear and energy is released during the process. These two properties of fission make it very important for practical application. The fission fragments always unstable nuclei have to many neutrons for stability so they are very radioactive. The distribution of the fragments in percent is shown on the next figure. The most probable mass numbers, occurring in about 7% of the events, are around $A \approx 95$ and $A \approx 137$. The width at the half maximum is about 20.



The formula of a general reaction:
$^{235}_{92}U + n \to \, ^{236}_{92}U \to \, ^{96}X + \, ^{137}Y + 3n + energy$.
If we consider large number of reaction the average number of neutrons is about
$\overline{n} \simeq 2.5 \dfrac{\text{neutron}}{\text{fission}}$. The total kinetic energy of the fission fragments is about 200 MeV.

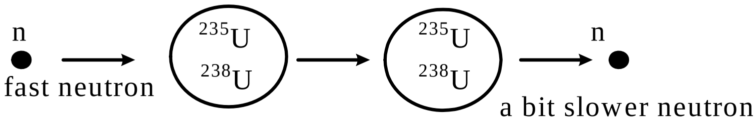
What is the mechanism of the fission? The ^{235}U nucleus absorbs the thermal neutron and the neutron gives its energy to the nucleus. The resulting ^{236}U nucleus is in a highly excited state and oscillates strongly. During the oscillation two globs appear, both globs contain protons and are positively charged and thus they repel each other. If the globs move apart enough, the electric repulsion rips apart the nucleus. At the end of the process the two fragments eject neutrons. The resulting fragments have too many neutrons, therefore they are β^- radioactive isotopes and very dangerous.



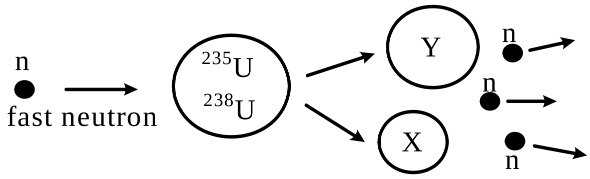
Chain reaction

Discovery of the facts that 200 MeV of energy is released when uranium undergoes fission and other neutrons are liberated during fission, suggested the possibility of chain reaction that is a self-sustaining series of events. The reaction can be either rapid (as in a nuclear bomb) or controlled (as in a nuclear reactor). Introduce the multiplication factor *k*. During *n* fission in average $\sim 2.5 \cdot n$ number neutron appear if n' will cause fission again then the multiplication factor *k* is defined as $k = \dfrac{n'}{n}$. We speak about chain reaction if the multiplication factor *k* is one or greater then one $k \geq 1$.
$k < 1$ the reaction is subcritical
$k = 1$ the reaction is critical
$k > 1$ the reaction is supercritical
Consider now two pure metal blocks ^{235}U , and ^{238}U , and follow the life of the released neutrons after the fission. These neutrons are fast neutrons.

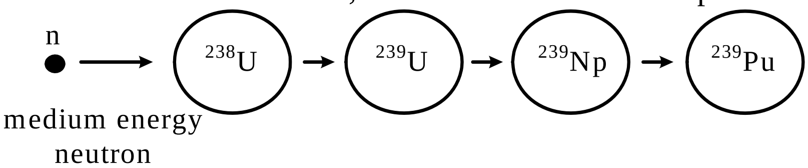
In most cases the neutrons lose some energy in the collision with the uranium nuclei:



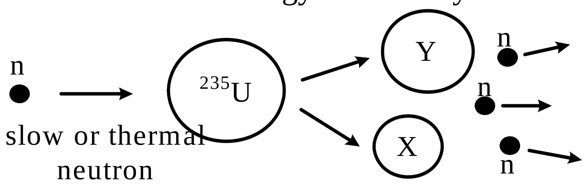
The fission is very rare in case of fast neutrons:



As the neutrons slow down at medium energy the ^{238}U nucleus capture the neutron, and removes the neutron from the fission chain, this is called resonance capture



Finally the neutron slow down to thermal energy and usually cause fission of ^{235}U :



The neutron slow down or will be captured in 10^{-5} s.

1. Consider a pure metal block ^{238}U . The fast neutrons cause fission very rare, and the medium energy neutrons are captured by the nucleus there is no possibility of chain reaction.
2. Consider a pure metal block ^{235}U . The fast neutrons cause fission very rare, they slow down to thermal energy and cause fission. If the size of the block is smaller than 10 cm the neutron loss or leakage is so high that there is no chain reaction. If the size is greater than 10 cm the neutron loss or leakage is small $k > 1$ and chain reaction occurs.
3. The natural uranium contains 0.7% of ^{235}U isotope and 99.3% of ^{238}U isotope. This is the cause that the natural uranium is not fissionable $k < 1$ due to the resonance capture, and also this is the cause that the natural uranium mines do not burned out.

The first controlled chain reaction was made in 1942 in USA by Szilárd and Fermi. In this very small nuclear reactor they applied natural uranium with graphite moderator. The role of the moderator is to slow down the neutrons but keep the neutron loss small. The neutrons left the uranium fuel slow down and the thermal neutrons entered the uranium and caused fission. The graphite block was cooled by water and heat from nuclear fission was passed to the water. Finally a steam generator and a turbine and an electric generator were applied. The gained electric power was 200W.

Classification by moderator material:

Graphite ^{12}C , slow down the neutrons and do not cause neutron loss

Heavy-water $^{2}H_{2}0$, slow down the neutrons and do not cause neutron loss, too expensive

Light-water (ordinary water) $H_{2}0$, slow down the neutrons and but cause a bit neutron loss.

In case of natural uranium and ordinary water moderator, the multiplication factor $k < 1$. Artificially enriching the uranium fuel so that it contains $\sim 3\%$ ^{235}U with ordinary water moderator, the multiplication factor will be $k > 1$.

The nuclear reactor

A nuclear reactor is a system in which a controlled nuclear chain reaction is used to liberate energy. In a nuclear power plant, this energy is used to generate steam, which operates a turbine and turns an electrical generator.

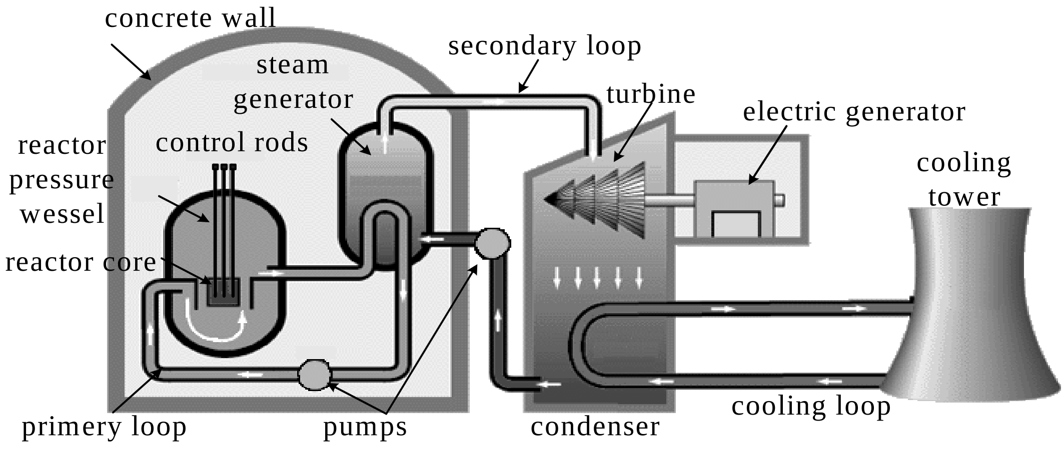
In a typical reactor, the uranium fuel is in the form of uranium oxide pellets, inserted metal tubes. The ordinary water moderator surrounds of these fuel rods, forming the reactor core. This geometric arrangement increases the probability that a fast neutron, produced in a fuel rod, will find itself in the moderator and slow down without captured. Once the neutron has reached thermal energies, wander back into a fuel rod and produce a fission event. The energetic fission fragments heat the water surrounding the reactor core. The steam generator is a heat exchanger that takes heat from this highly radioactive water and generates nonradioactive steam to run the turbines. A so called pressurized water reactor PWR is shown on the next figure. In such case the pressure in the reactor chamber is about 125×10^5 Pa, and the working temperature of the surrounding water is about 300^0C.

The water moderator has a triple role in the reactor operation:

1. slow down the fast neutrons to thermal energy
2. cool the hot fuel rods
3. transfer the heat to the water in the secondary loop

This type of reactor is so called partly self-controlled, because if due to some problem the temperature increasing in the core the water boiling up and goes to steam, and the moderation of fast neutrons decreases so the number of fission events decreases. The precision rate of the reaction is controlled by inserting or withdrawing control rods made of elements cadmium whose nuclei absorb neutrons and with boron solute in water.

The reactor is started by withdrawing the cadmium control rods and dilution of the boron in the water and the multiplication factor is on $k = 1.001$. As the power reached the planned value the multiplication factor is regulated to $k = 1.000$ by the cadmium rods.



An unavoidable feature of reactor operation is the accumulation of radioactive wastes; they release additional energy in thermal form. The burned out fuel rods from reactor are immersed in water and stored under cooling.

If the chain reaction is fast and uncontrolled, it is called atomic bomb; its destructive ability is many thousands of times that of previously existing bombs.

Nuclear fusion

As we have already seen the other possibility to release nuclear energy when two light nuclei form a nucleus whose rest mass is less than the sum of the rest masses of the original nuclei. This process is called fusion. The fusion reaction in the Sun is a multistep process in which hydrogen is burned to form helium. In case of a proton-proton chain reaction a helium nucleus is formed from four protons in a three steps process. Such reaction is take place in the interior of the Sun and in many stars.

$${}_{1}^{1}H + {}_{1}^{1}H \rightarrow {}_{1}^{2}H + e^{+} + \text{energy}$$

$${}_{1}^{2}H + {}_{1}^{1}H \rightarrow {}_{2}^{3}He + \gamma + \text{energy}$$

$${}_{2}^{3}He + {}_{2}^{3}He \rightarrow {}_{2}^{4}He + {}_{1}^{1}H + {}_{1}^{1}H + \text{energy}$$

About 50 millions K temperature is necessary to start the proton-proton chain reaction. Serious attempts are being made for several years over the world to control the fusion of hydrogen. The momentary and uncontrolled fusion is the hydrogen bomb.