Mechanics and Thermodynamics Problems

1. Compare your average speed in the following two cases: a) You travel *24 km* at a speed of *40 km/h* and then *24 km* at a speed of *100 km/h.* b) You travel *3 hours* at a speed of *40 km/h* and then *3 hours* at a speed of *100 km/h*. (3/38)

2. The *x(t)* function of an object moving in a straight line is given by $x = 3t - 4t^2 + t^3$,where *x* is in meters and *t* is in seconds. What is the position vector of the object at $t_1 = 2\ s$ and $t_2 = 4\ s$? What is the displacement vector between these two moments? What is the average velocity of the particle during the previous interval? Construct the *x* versus *t* graph. What is the instantaneous velocity at $t_2 = 4\ s$? Give the acceleration time function. (6,9/38)

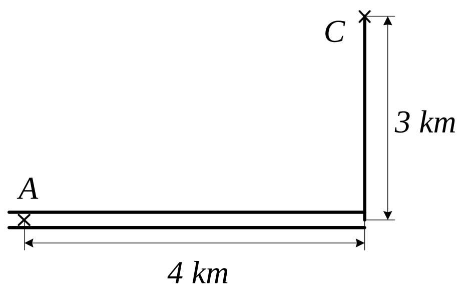
3. A particle A moves along the line $y = d$ with *3 m/s* constant speed and $d = 30\ m$. Its velocity vector directed parallel to the positive *x*-axis. A second particle B starts at the origin with zero initial speed and constant acceleration $a = 0.4\ m/s^2$ at the same instant that particle A passes the *y*-axis. What angle α between *a*, and the positive *y*-axis would result in a collision between these two particles?(7/53)

4. A particle moves so that its position vector as a function of time in SI units is: $\vec{r}(t) = \vec{i} + 4t^2\ \vec{j} + t\,\vec{k}$. Write expressions for its velocity and acceleration as functions of time. What is the shape of the particle's trajectory? (6/53)

5. A particle moves in a plane so that its coordinate versus time functions are: $x(t) = a\sin\omega t$, $y(t) = b\sin\left(2\omega t + \frac{\pi}{2}\right)$, where $a = 4\ cm$, $b = 3\ cm$, $\omega = constant$. Find the equation of the trajectory and plot the curve. (FF/62)

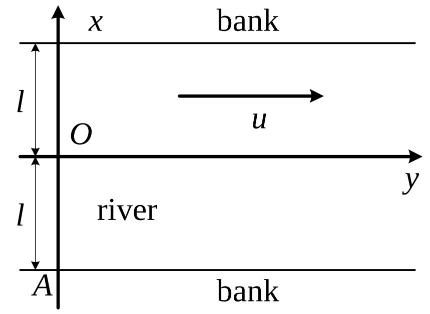
6. Two perpendicular roads cross each other. There are two cars travelling towards the cross. One car travels with a speed of *60 km/h*, the other with a speed of *40 km/h*. At a given moment the first car is *200 m* away from the cross the second is *500 m* away the cross. When will be the distance minimum between them? What is the minimal distance? Where are the cars at this moment? (FF/9)

7. A motorcyclist would like to travel from point A to point C. On the road its maximum speed is *50 km/h* and on the field it is only *25 km/h*. Where does he have to leave the road for minimal time of motion? What is this minimal time? (FF/10)



8. A man in a boat wants to cross the river *2l* wide. See figure. His rowing speed relative to the water is $c = constant$ and perpendicular to the river bank (rowing speed in still

water). The speed of the river current changes in the perpendicular direction relative to the bank. The function is give by: $u = u_0\left(1-\frac{x^2}{l^2}\right)$, where u_0 = *constant*. Determine the equation of the trajectory of the boat? Where will it reach the opposite bank of the river, if it starts from point A? (FF/58)



9. P moves in the *x*-*y* plane. Its coordinates are given by the next functions: $x = c_1 t^2, \quad y = c_2 - c_3 t^2$, where c_1 = *15 m/s²*, c_2 = *4 m*, c_3 = *20 m/s²*. Give the path, speed and tangential acceleration of the particle. How much time is needed for the particle to travel the distance between the two coordinate axes? (FF/16)

10. A particle moves in a counter-clockwise circular path of radius *0.6 m* centred at the origin of the *x*-*y* plane with a period of revolution of *2 s*. What are the *x*- and *y* components of the velocity when *x = 0.48 m* in the first quadrant! What will be the two components of the acceleration at the same instant? (46/56)

11. A point moves along a circle having a radius of *r = 20 m* with a constant tangential acceleration of $a_t = 5\ m/s^2$. How much time is needed after motion begins for the normal acceleration a_n of the point to be (1) equal to the tangential acceleration, (2) double the tangential acceleration.

12. A point moves along a circle with the relationship between the distance and the time is given by $s = A + Bt + Ct^2$ where *B = – 2 m/s* and $C = 1\ m/s^2$. Find the speed of the point, and its tangential, normal and total acceleration in *t = 3 seconds* after motion begins. The normal acceleration of the point at *t' = 2 seconds* is $a'_n = 0.5\ m/s^2$.

13. A point moves along a circle with a radius of *r = 2 m*. The relationship between the distance and the time is given by the equation: $s = Ct^3$, where $C = 0.1\ m/s^2$. Find the normal and the tangential accelerations of the point at the moment when its speed is *v' = 0.3 m/s*.

14. The polar coordinates of a particle are given by: $\rho = be^{kt}, \quad \varphi = \omega t$, where *b, k* and ω are positive constants. Find the velocity and acceleration vectors as function of time. Find also the speed and the magnitude of the acceleration at time *t = 0*.

15. A particle moves along the plane of a plane polar coordinate system as $\rho = bt, \quad \varphi = \omega t$, where *b* and are ω positive constants and $t \geq 0$. What is the shape of the trajectory? How do the speed and the magnitude of the acceleration depend on time? (FF/64)

16. A particle moves along a circle of radius *R*. When the time is zero its initial speed is v_o. The velocity makes a constant α angle with the acceleration vector. How does speed depends on time? (FF/43)

17. A point starts from rest with *2* m/s^2 tangential acceleration along a circle whose radius is *10 m*. What is the speed, the magnitude of the acceleration the angular speed and angular acceleration *10 seconds* after starting? What is the distance travelled? When will be equal the tangential and normal acceleration? (FF/40)

18. A *5 kg* particle moves so that its velocity time function is: $v_x = 6t, \quad v_y = 4$, measured in *m/s*. When the time is zero the particle is in the origin of the coordinate system. What is the shape of the trajectory, give the *x(y)* function and draw the curve. Determine the coordinates of the velocity and acceleration when the *x* coordinate of the particle is *27 m*. Find the force acting on the particle at that moment.

19. In the water of a lake a boat is started by a push to obtain a *6 m/s* initial speed. After starting we let the particle to move alone. Its mass is *49.75 kg*. The drag force (resistance to the motion of the body) is proportional to its speed and the proportionality constant is *0.5 kg/s*. Find the speed and the travelled distance as a function of time. (FF/86)

20. A particle of mass *m* moves in the *x-y* plane of the coordinate system. Its coordinates versus time functions are given by: $x(t) = a\cos\omega t$, $y(t) = b\sin\omega t$, where *a, b,* and $\omega > 0$ are constants. Find the work done by the force acting on the particle in the time interval $(0, \pi/4\omega)$.(FF/185)

21. The mass point moves in the next force field: $F_x = ky^2, \quad F_y = kxy$, $k > 0$, constant. Find the work done by the field as the body moves from point *A(0, r)* to *B(r, 0)*. a) along the circle of radius *r* and whose centre is the origin, b) along the *AO* and *OB* straight lines. Find the equation of the field lines. (FF/194)

22. A mass *m* on a frictionless table is attached to a hanging mass *M* by a cord through a hole in the table, see the figure. Find the condition (*v* and *r*) with which *m* must spin for *M* to stay at rest. (27/90)

23. A *4.0 kg* block is put on top of a *5.0 kg* block. In order to cause the top block to slip on the bottom one, held fixed, a horizontal force of *12 N* must be applied to the top block. The assembly of blocks is now placed on a horizontal, frictionless table, see the figure.

Find (*a*) the maximum horizontal force *F* that can be applied to the lower block so that the blocks will move together, and (*b*) the resulting acceleration of the blocks. (17/89)

24. A small coin is placed on a flat, horizontal turntable. The turntable is observed to make three revolutions in *3.14 s*. (*a*) What is the speed of the coin when it rides without slipping at a distance *5.0 cm* from the centre of the turntable? (*b*) What is the acceleration (magnitude and direction) of the coin in part (*a*)? (*c*) What is the frictional force acting on the coin in part (*a*) if the coin has a mass of *2.0 g*? (*d*) What is the coefficient of static friction between the coin and the turntable if the coin is observed to slide off the turntable when it is more than *10 cm* from the centre of the turntable? (29/90)

25. An airplane is flying in a horizontal circle at a speed of *480 km/h.* If the wings of the plane are tiled 45^o to the vertical, what is the radius of the circle the plane is flying? (40/91)

26. Show that the periods of two conical pendulums of different lengths that are hung from a ceiling and rotate with their bobs an equal distance below the ceiling are equal. (28/90)

27. Calculate the work done by the force shown on the graph, displacing a particle from $x = 1\ m$ to $x = 3\ m$. The curve is given analytically by $F = a/x^2$, where $a = 9\ N\cdot m^2$. (7/104)

28. A *2.0 kg* block is placed against a compressed spring on a frictionless incline, as shown in figure. The spring, whose force constant is *1960 N/m*, is compressed *20 cm*, after which the block is released. How far up the incline will it go before coming to rest? Assume that the spring has negligible mass and that the spring and block are not attached to each other. Measure the final position of the block with respect to its position just before being released. (20/128)

29. A *1.5 MW* railroad locomotive accelerates a train from a speed of *10 m/s* to *25 m/s* at full power in *6.0 min*. (*a*) Neglecting friction, calculate the mass of the train. (*b*) Find the speed of the train as a function of time in seconds during the interval. (*c*) Find the force accelerating the train as a function of time during the interval. (*d*) Find the distance moved by the train during the interval. (37/106)

30. A *2.0 kg* block is forced against a horizontal spring of negligible mass, compressing the spring by *15 cm*. When the block is released from the compressed spring, it moves *60 cm* across a horizontal tabletop before coming to rest. To force constant of the spring is *200 N/m*. What is the coefficient of sliding friction between the block and the table? (21/105)

31. A force acts on a *3.0 kg* particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where *x* is in meters and *t* is in seconds. Find the work done by the force during the first *4.0 s*. (19/105)

32. A mass point *m* starts sliding down from the top of a hemisphere. The surface of the hemisphere is frictionless. Determine the normal force exerted by the surface to the mass point as a function of the angle measured from vertical to the momentary position. What is the angle where the particle leaves the surface? (29/129)

33. What force corresponds to a potential energy $U = ax^2 + bxy + z$? (24/128)

34. A *30 g* bullet initially travelling *500 m/s* penetrates *12 cm* into a wooden block to stop. What average force does it exerts? Find the speed of the bullet as it penetrates *5 cm*. Give the speed of the bullet as a function of the penetrating distance. (14/105)

35. Two children are playing a game in which they attempt to hit a small box on the floor using a spring-loaded marble gun placed horizontally on a frictionless table, see the next figure. The first child compresses the spring *1.0 cm* and the marble falls *20 cm* short of the target, which is *2.0 m* horizontally from the edge of the table. How far should the second child compress the spring so that the same marble falls into the box? (21/128)

36. A steel ball of mass *0.50 kg* is fastened to a cord *70 cm* long and is released when the cord is horizontal. At the bottom of its path the ball strikes a *2.5 kg* steel block initially at rest on a frictionless surface. The collision is elastic. Find (*a*) the speed of the ball and (*b*) the speed of the block just after the collision. (18/166)

37. A mass point is moving in the *x*-*y* plane on an elongated cycloid whose coordinate time functions are given by: $x = \frac{r}{2}\left(1 + 2\omega t - \cos\omega t\right), \quad y = \frac{r}{2}\sin\omega t$ where ω and *r* are positive constants. Find the rectangular coordinates of the velocity and acceleration. Find the speed and tangential acceleration.

38. A block of mass $m_1 = 2.0\ kg$ slides along a frictionless table with a speed of *10 m/s*. Directly in front of it, and moving in the same direction, is a block of mass $m_2 = 5.0\ kg$ moving at *3.0 m/s.* A mass less spring with a spring constant $k = 1120\ N/m$ is attached to the backside of m_2, as shown in figure. When the blocks collide, what is the maximum compression of the spring? Assume that the spring does not bend and always obeys Hooke's law. (31/167)

39. Determine the period of a conical pendulum whose length is $l = 2\ m$ and the cord makes with 30^o the vertical.

40. Two pendulums each of length *l* are initially situated as shown in figure. The first pendulum is released and strikes the second. Assume that the collision is completely inelastic and neglect the mass of the strings and any frictional effects. How high does the centre of mass rise after the collision? (20/166)

41. A *20 kg* body is moving in the direction of the positive *x*-axis with a speed of *200 m/s* when, due to an internal explosion, it breaks into three parts. One part, whose mass is

10 kg, moves away from the point of explosion with a speed of *100 m/s* along the positive *y*-axis. A second fragment, with a mass of *4.0 kg*, moves along the negative *x*-axis with a speed of *500 m/s*. (*a*) What is the velocity of the third *6.0 kg* fragment? (*b*) How much energy was released in the explosion? Ignore effects due to gravity. (40/167)

42. A ball with an initial speed of *10 m/s* collides elastically with two identical balls whose centres are on a line perpendicular to the initial velocity and which are initially in contact with each other, see figure. The first ball is aimed directly at the contact point and all the balls are frictionless. Find the velocities of all three balls after the collision. (Hint: The directions of the two originally stationary balls can be found by considering the direction of the impulse they receive during the collision.) (41/167)

43. The two masses on the right of figure are slightly separated and initially at rest: the left mass is incident with speed v_o. Assuming head-on elastic collisions: (*a*) if $M \le m$, show that there are two collisions and find all final velocities; (*b*) if $M > m$, show that there are three collisions and find all final velocities. (32/167)

44. A mass point *P* of mass *10 kg* moves along the *x*-axis. There are two forces acting on it, the first is a linear restoring force, the force constant is *250 N/m*. The second is a damping force which is proportional to the velocity and the proportionality factor is $b = 60$ *Ns/m*. Initially *P* is in $x = 8$ *m* and its velocity is zero. What is the $x(t)$ function? (FF/115)

45. A point of mass $m = 10$ *kg* moves along the *x*-axis. There are two forces acting on it. The value of the linear restoring force is *20 N* when the displacement is *1 m*. There is a damping force which is proportional to the velocity. When the time is zero the body is at rest. After three whole oscillations the amplitude is one tenth of the original one. What is the period? (FF/116)

46. A body moves, in a media in which there is a damping force, because of a linear restoring force. The damping force proportional to the speed of the body. Initially the body is in the origin and its speed is *1 m/s.* The time of period is *2s*. The quotient of two amplitudes if the elapsed time is the period is $e^{1/2}$. Describe the motion. (FF/117)

47. A block is on a piston which is moving vertically with a simple harmonic motion of period *1.0 s*. (*a*) At what amplitude of motion will the block and piston separate? (*b*) If the piston has amplitude of *5.0 cm*, what is the maximum frequency for which the block and piston will be in contact continuously? (12/239)

48. A block is on a horizontal surface which is moving horizontally with a simple harmonic motion of frequency *2.0 Hz*. The coefficient of static friction between block and plane is *0.50*. How great can the amplitude be if the block does not slip along the surface? (25/239)

49. A *0.10 kg* block slides back and forth along a straight line on a smooth horizontal surface. Its displacement from the origin is given by $x = (10\ cm)\cos\left[(10\ rad/s)t + \frac{\pi}{2} rad\right]$ (*a*) What is the oscillation frequency? (*b*) What is the maximum speed acquired by the block? At what value of *x* does this occur? (*c*) What is the maximum acceleration of the block? At what value of *x* does this occur? (*d*) What force must be applied to the block to give it this motion? (15/239)

50. A body oscillates with simple harmonic motion according to the equation $x = (6.0\ m)\cos\left[(3\pi\ rad/s)t + \frac{\pi}{3} rad\right]$ What is (*a*) the displacement, (*b*) the velocity, (*c*) the acceleration, and (*d*) the phase at the time *t = 2.0 s*? Find also (*e*) the frequency and (*f*) the period of the motion. (16/239)

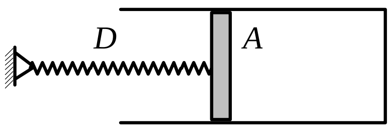
51. For the system shown in figure, the block has a mass of *1.5 kg* and the spring constant *k = 8.0 N/m*. Suppose that the block is pulled down a distance of *12 cm* and released. If the friction force is given $by -b\frac{dx}{dt}$, where *b = 0.23 kg/s*, find the number of oscillations made by the block during the time interval required for the amplitude to fall to one-third of its initial value. (53/241)

52. Two springs are joined and connected to a mass *m* as shown in figure. The surfaces are frictionless. If the springs each have force constant *k*, show that the frequency of oscillation of *m* is $v = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$.(21/239)

53. The springs are now attached to *m* and to fixed supports as shown in figure. Show that the frequency of oscillation in this case is $\nu = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$. (22/239)

54. The weight of a piston in a frictionless vertical cylinder is *200 N*. The cross-section of the cylinder is *100 cm*^{2}. The temperature of the gas in the container is *12* ^oC. The piston is *60 cm* above the bottom of the cylinder. The atmospheric pressure is *1 bar*. Find the sinking distance of the piston if the temperature increases to *17* ^oC and a *100 N* body is taken onto it. (FF/425)

55. The pressure of the air in a cylinder at *300 K* is *100 kPa*. The cross sectional area of the cylinder is *100 cm*^{2} and the volume of the gas is *1 dm*^{3}. The atmospheric pressure is *100 kPa*. The piston is connected to a spring whose force constant is *5 kN/m*. (See figure.) Find the pressure of the air in the cylinder if the temperature is increased to *600 K*. (There is no friction between the piston and the cylinder.) (FF/426)



56. The pressure of the gas in a rigid container is *40 bar* and its temperature is *37* ^oC. What will be the final pressure if the temperature decreases to *12* ^oC and half of the gas is deflated? (FF/427)

57. The pressure of the gas in a *20 dm*^{3} container is *10 MPa* and its temperature is *0* ^oC. We deflate *0.86 kg* gas. Find the pressure if the temperature is the same. For what temperature need to be heated to reach again the original *10 MPa* pressure. The density of the gas at *0* ^oC and at *100 kPa* is *1.43 kg/m*^{3}. (FF/428)

58. The pressure of the nitrogen in a *5 dm*^{3} container is *0.1 MPa*. What is the final pressure of the gas if *1.5 kJ* heat is absorbed by the system? The adiabatic exponent of the nitrogen is *1.4*. (FF/450)

59. The work done by the ideal gas as it expands at constant pressure is *200 J*. How much heat is absorbed by the system if the adiabatic exponent of the gas is *1.4*? (FF/451)

60. The initial pressure of a gas is *0.1 MPa* and its volume is *10* dm^3. What is the final pressure of the gas if it is compressed isothermally and the system gives off *3.14 kJ* heat? (FF/452)

61. The initial pressure of the air is *0.1 MPa*. It is compressed adiabatically to a final volume of *1* m^3 and a final pressure of *0.2 MPa*. What is the change of the internal energy of the system? The adiabatic exponent of the gas is *1.4*. (FF/455)