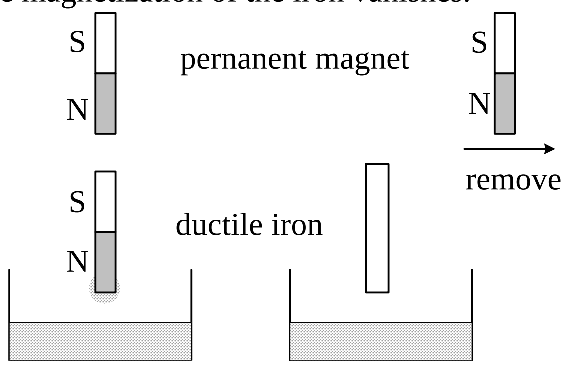
3 Magnetism

3.1 Basic magnetic phenomenon

1. It was observed at least 2000 years ago, that certain iron ores (magnetite, Fe_3O_4) would attract bits of iron. The interaction between the magnetite and the bit of iron is always attractive. This phenomenon is called magnetism; the magnetite is called permanent magnet. It was found also that when a steel rod was brought in contact with a natural magnet, the rod also became a magnet. The region of the body where the magnetism appears to be concentrated is called magnetic pole.
2. Due to experience a magnetized steel needle (thin rod) suspended by a string from its centre tended to line up in a north-south direction, like a compass needle. The Earth itself is a huge magnet. The end of the magnetized needle that points north is called North Pole or N-pole and the other end is the South Pole or S-pole. The north geographic pole of the Earth is a magnetic south pole. Two opposite poles attract each other, two like poles repel each other.
3. Ductile iron can be magnetized with a permanent magnet, bat if we remove the permanent magnet the magnetization of the iron vanishes.



   This phenomenon is called magnetic polarization.
4. Due to experience single isolated magnetic pole has never been found, poles always appear in pairs. If a bar magnet is broken in two, each broken end becomes a pole. Isolated magnetic pole or magnetic monopole does not exist. The simplest magnetic structure is the magnetic dipole.
5. Oersted discovered that a compass needle was deflected by a current-carrying wire. That is a moving charge or current can produce magnetic effects. This interaction is mutual, because in the surrounding of a magnet there is force acting on a current carrying wire, this force is the so called Ampère-force. We can intensity the magnetic effect of a current in a wire by forming the wire into a coil of many turns and by providing an iron core.

We define the space around a magnet or a current-carrying conductor as the site of a magnetic field. To describe the magnetic field we introduce the magnetic induction vector $\vec{B}$ (also called magnetic flux density or simply magnetic field).
The definition of $\vec{B}$ is based on the Ampere's Law. Let us place a small current element (differential element of a conductor) at the point P where we wish to determine the magnetic field, and measure the force acting on it. Denote the current of the conductor by I and its length by ds. If $\vec{\tau}$ directed into the direction of the current:

$$d\vec{r} = ds\,\vec{\tau}\ .$$

The experiences:

1. the force acting on the current element is always perpendicular to the elementary arc vector: $d\vec{F} \perp d\vec{r}$,
2. there is a special line through P, and taking the current element along this line there is no force acting on it: if $d\vec{r} \parallel e$, then $d\vec{F} = 0$,
3. if the differential element of the conductor makes α angle with the special line, the force is perpendicular to the plane of *e* and $d\vec{r}$, and its magnitude is proportional to the current *I*, the length of the current element ds , and $\sin\alpha$. That is the ratio is independent of the current element, depends only on the magnetic field at that point. The magnitude of the magnetic induction is defined as:

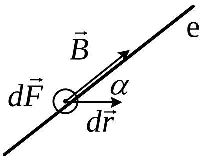
$$B = \frac{dF}{I\,ds\sin\alpha}$$

The direction of the magnetic induction is coincided with the special line *e* and the three vectors form a right-hand rule in this order.

$$\left\{d\vec{r}, \vec{B}, d\vec{F}\right\}$$

The Ampère-force in vector form for a differential current element:

$$d\vec{F} = I\,d\vec{r} \times \vec{B}$$



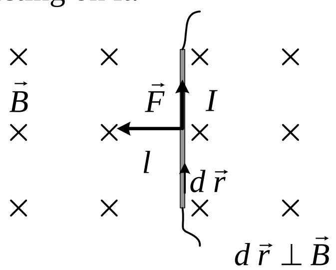
By integrating this formula for the length of conductor we can find the force acting on a finite thin conductor:

$$\vec{F} = I\int d\vec{r} \times \vec{B}\ .$$

The unit of the magnetic field *B:*

$$\left[B\right] = 1\frac{N}{Am} = 1\frac{Nm}{Am^2} = 1\frac{VAs}{Am^2} = 1\frac{Vs}{m^2} = 1tesla = 1T$$

Consider now a thin straight conducting wire whose length is *l* and its current is *I*. Immerse into homogeneous magnetic field $\vec{B}$ shown in figure. The crosses show the sense of $\vec{B}$, up out of the page. Determine the force acting on it.



$$\vec{F} = I \int\limits_{\substack{\text{lenght of}\\ \text{conductor}}} d\vec{r} \times \vec{B}$$

$$F = I\int B\,dr$$

$$F = BIl$$

If α is the angle between $d\vec{r}$ and $\vec{B}$:

$$F = BI\,l\sin\alpha\ .$$

As the conduction current density in a crystallised conductor:

$$\vec{j} = -e\,n_e \vec{v}_e$$

The current is $I = j\,A$, so:

$$F = B\,j\,Al = B(-e)v_e\,n_e\,Al$$

n_e is the number density of the free electrons and Al is its volume. $n_e\,Al$, is the number of the conduction electrons in this conductor part. That F is the force acting all the conduction electrons. The force acting on one electron in scalar and vector form:

$$F_{e^-} = (-e)v_e B$$

$$\vec{F}_{e^-} = (-e)\vec{v}_e \times \vec{B}$$

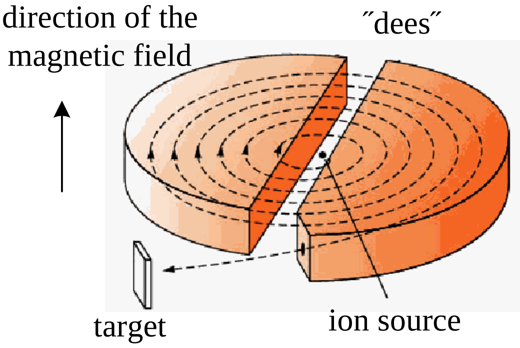
That is the force acting on an arbitrary moving point charge is the so called Lorentz-force:

$$\vec{F} = q\vec{v} \times \vec{B}$$

The magnetic field exerts a force on a moving charge. The force is perpendicular to both the velocity $\vec{v}$ and the magnetic field $\vec{B}$. The direction of the force is given by the right hand rule:

$$\{\vec{v}, \vec{B}, \vec{F}\}$$

The Lorentz-force has an important application to bend the path of the charged particles. This force will bend the path of a charge into a circle if the charge remains in the uniform magnetic field. This principle is used in cyclotron. The cyclotron was one of the earliest types of particle accelerators and is still used.



The applied electric field accelerates the particles between the "dees". The magnetic force bends the ion into a semicircle path in the dees.

3.2 Torque acting on a plane current loop in homogeneous magnetic field

The Ampère's force acting on a differential current element:

$$d\vec{F} = I\,d\vec{r} \times \vec{B},$$

the torque acting on it is:

$$d\vec{M}_{torque} = \vec{r} \times d\vec{F} = \vec{r} \times \left(I\,d\vec{r} \times \vec{B}\right),$$

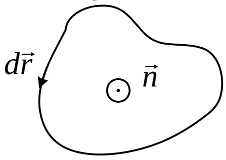
For a current loop the integral is taken for the loop:

$$\vec{M}_{torque} = I\oint \vec{r} \times \left(d\vec{r} \times \vec{B}\right).$$

It is conceivable that for a plane current loop the torque is independent of the shape of the loop and can be written as:

$$\vec{M}_{torque} = I\vec{A}\times\vec{B}$$

A is the surface of the loop and $\vec{n}$ is the normal vector, that is $\vec{A} = A\vec{n}$ is the surface vector.
Agreement: the direction of the normal vector of the loop is connected with a right hand rule to the direction of the current in the loop. Let the fingers of the right hand curl around the loop in the direction of the current, the extended right thumb will then point in the direction on $\vec{n}$.



On the figures the next symbols are used:

⊙ vector out of plane

⊗ vector into the plane

Introducing the magnetic dipole moment of the loop:

$$\vec{m} = I\vec{A}.$$

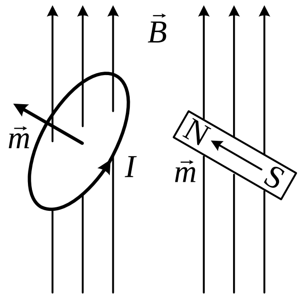
$$\vec{M}_{torque} = \vec{m}\times\vec{B}$$

The unit of the magnetic dipole moment:

$$[m] = 1Am^2$$

The torque $\vec{M}_{torque}$ tends to rotate the loop toward the equilibrium position when its magnetic moment $\vec{m}$ is parallel with the field $\vec{B}$. The torque is the greatest, when $\vec{m}$ and $\vec{B}$ is perpendicular and zero when they are parallel or anti parallel vectors.

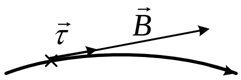
A current loop orienting itself in an external magnetic field reminds us of the action of a compass needle in such a field. Compass needles bar magnets have permanent magnetic dipole moment.



The magnetic dipole moment $\vec{m}$ of a permanent magnet depends on its magnetization and directed from S to N.

$$\vec{M}_{torque} = \vec{m}\times\vec{B}.$$

The magnetic field geometry can be represented by field lines just as we represented the electric field. The lines of magnetic induction are drawn so that a tangent to them at every point coincided with the direction of the vector $\vec{B}$.



line of induction

According to the agreement the lines of magnetic induction are drawn so that the number of lines on unit perpendicular cross-section area is equal to the magnitude of $\vec{B}$. The magnetic field line is not real, it is a concept developed as an aid to visualizing the field.

3.3 Magnetic flux and Gauss Low

The magnetic flux of induction refers always to an oriented surface and equals to the total number of lines of induction passing through the surface.
The elementary outward magnetic flux:

$$d\Phi = \vec{B}\, d\vec{A},$$

$$d\Phi = B\, dA \cos\alpha .$$

The total outward flux is the integral of $\vec{B}$ over the surface:

$$\Phi = \int_A \vec{B}\, d\vec{A}.$$

The unit of the magnetic induction flux:

$$[\Phi] = [B][A] = 1\frac{Vs}{m^2} m^2 = 1Vs$$

Magnetic field lines are very different from electric field lines because there are no magnetic monopoles. The magnetic field lines form closed loops.

Gauss'Law for magnetism states that the net magnetic flux over a closed surface is zero.
The integral form of Gauss'Law for magnetism:

$$\oint_A \vec{B}\, d\vec{A} = 0$$

This law states that the magnetic field has no sources and sinks. The absence of magnetic charges in nature results in the fact that the lines of $\vec{B}$ have neither a beginning nor an end.
Substituting a volume integral for the closed surface one in accordance with the Gauss-Ostogradskij theorem:

$$\oint_A \vec{B}\, d\vec{A} = \int_V \nabla \cdot \vec{B}\, dV = 0,$$

This condition must be observed for any arbitrarily chosen volume. This is possible only if the integrand at each point of the field is zero.
The differential form of Gauss'law for magnetism:

$$\nabla \cdot \vec{B} = 0$$

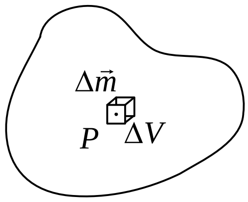
A magnetic field has a property that its divergence is zero everywhere.

3.4 Magnetization or magnetic polarization

The magnetic dipole moment of a nucleon (proton or neutron) is much more less then the magnetic dipole moment of an electron. So the magnetic dipole moment of an atom or molecule is due to the resultant magnetic dipole moment of the electrons.
The magnetic moment of an electron consists of two parts:

1. Magnetic dipole moment of the electrons due to their orbital motions. (The orbital motion of electrons creates tiny atomic current loops, which produce magnetic fields.)
2. The electrons have an intrinsic "spin" magnetic dipole moment. (It is connected to the intrinsic "spin" angular momentum.)

In vacuum the magnetic induction vector $\vec{B}$ would be enough to describe the magnetic field. In presence of some chemical substance a new vector must be introduced to characterize the magnetization of the material. This vector is called magnetization.



Denote by $\Delta\vec{m}$ the resultant magnetic dipole moment of the volume ΔV, then:

$$\vec{M} = \lim_{\Delta V \to 0} \frac{\Delta\vec{m}}{\Delta V}$$

The magnetization of a material is expressed as the net magnetic dipole moment of a unit volume.

Unit of magnetization:

$$[M] = 1\frac{Am^2}{m^3} = 1\frac{A}{m}$$

It is suitable to introduce the magnetic field strength $\vec{H}$ as the linear combination of $\vec{B}$ and $\vec{M}$ by the next definition:

$$\vec{H} = \frac{\vec{B}}{\mu_o} - \vec{M}\,.$$

The magnetic field strength *H* has the same unit as the magnetization *M:*

$$[H] = 1\frac{A}{m}$$

μ_o is a universal constant called the permeability of vacuum, and its value:

$$\mu_o = 4\pi \cdot 10^{-7}\,\frac{Vs}{Am}$$

The introduction of the magnetic field strength is useful, because we can write a very simple law for $\vec{H}$.

The connection between the magnetization $\vec{M}$ and the magnetic induction is called constitutive equation. The experiments show that in the first approximation (for non-ferromagnetic materials) $\vec{B}$ is proportional to $\vec{M}$. But it is a customary practice to associate the magnetization not with the magnetic induction but with the magnetic field strength. That is:

$$\overrightarrow{M} = \chi\overrightarrow{H}$$

χ is a quantity characteristic of a given magnetic material called the magnetic susceptibility.

Let’s apply the first approximation:

$$\vec{H} = \frac{\vec{B}}{\mu_o} - \vec{M}$$

$$\vec{B} = \mu_o\left(\vec{H} + \vec{M}\right)$$

$$\vec{B} = \mu_o\left(\vec{H} + \chi\vec{H}\right)$$

$$\vec{B} = \mu_o\left(1 + \chi\right)\vec{H}$$

The dimensionless quantity $\mu' = 1 + \chi$ is called the relative permeability of a substance.

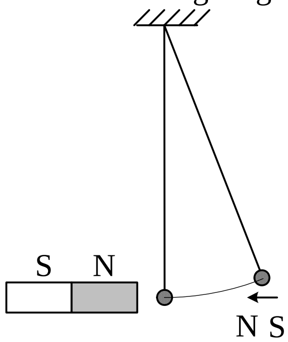
$$\vec{B} = \mu_o\mu'\vec{H}$$

3:5 Magnetic properties of materials

Materials may be classified by their response to externally applied magnetic fields as diamagnetic, paramagnetic, or ferromagnetic. (These are the most important.)

3.5.1 Diamagnetism

A diamagnetic material freely suspended in a strong magnetic field will be weakly repelled.



The sample will be magnetized in the direction opposite to $\vec{B}$. The effect is very weak. $\chi < 0$

$$|\chi| = 10^{-5} - 10^{-6}$$

$$\mu' = 1 + \chi \approx 0{,}9999...$$

$$\vec{B} = \mu_0 (1+\chi)\vec{H} = \mu_0 \mu' \vec{H}$$

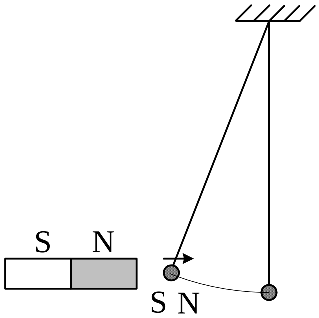
$\mu' < 1$ so the induction is less than the induction in vacuum $\vec{B_0} = \mu_0 \vec{H}$.

The atoms of a diamagnetic substance have no intrinsic magnetic dipole moment. However, a magnetic dipole may be induced by the action of an external field. These induced dipoles point in a direction opposite to that of the field that caused them. Two electrons orbit in opposite directions in the same diamagnetic atom, their two magnetic moments cancel. When a magnetic field is applied, one electron is slowed down, and the other is speeded up. The two magnetic moments no longer cancel.

Diamagnetic materials: copper, bismuth, gold, silver…

3.5.2 Paramagnetism

A sample of paramagnetic material placed near a pole of a strong magnet will be weakly attracted. It will be magnetized in the same direction as the external field $\vec{B} \uparrow\uparrow \vec{M}$.



$$\chi > 0$$

$$\chi \approx 10^{-3} - 10^{-6}$$

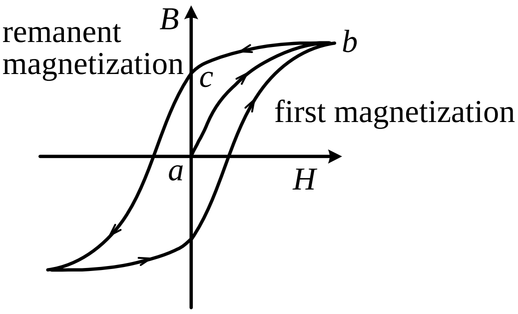
$$\mu' = 1.000....$$

$$\vec{B} = \mu_0(1+\chi)\vec{H} = \mu_0\mu'\vec{H} \ .$$

$\mu' > 1$ the induction in greater than the induction in vacuum $\vec{B}_0 = \mu_0\vec{H}$.
The atoms of a paramagnetic substance have permanent magnetic dipole moments, and these dipoles tend to line up with an external magnetic field. The aligning process seriously interfered with thermal agitation effects. The alignment is inversely proportional with the absolute temperature.
Paramagnetic materials: aluminium, helium, oxygen, air, platinum, sodium, uranium, chromium…

3.5.3 Ferromagnetism

The iron, nickel, cobalt and their alloys and compounds are called ferromagnetics. They are strongly magnetic substances. The relation between $\vec{H}$ and $\vec{M}$ (or between $\vec{H}$, and $\vec{B}$) is nonlinear, and the magnetization curves for ferromagnetic materials do not retrace themselves as we increase and decrease the external magnetic field. The figure shows the magnetization or hysteresis loop.



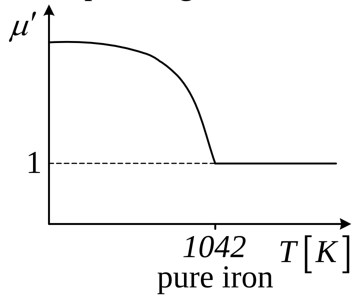
The line *ab* represents the first or initial magnetization and *M* (or *B*) reaches saturation. At point *c* the sample is magnetized even though there is no external field. Some alignment of dipoles remains. The result is the familiar permanent magnet. The *ac* is the measure of remanence or remanent magnetization. The hysteresis loop shows, that μ' is not constant, but depends on *H*.

$$\frac{\mu'}{\chi} \geq 10^3$$

Permanent magnets are made of hard magnetic material which has a large remanence. An electromagnet is made of a soft magnetic material which has a small remanence so it is strong by temporary magnet.

In ferromagnetic material there are regions of the crystal (magnetic domains) in which the alignment of the atomic dipoles is essentially perfect. The volumes of the domains are 10^{-9}-10^{-12} cm^3, and contain about 10^{15} atoms. In a domain there is special interaction called exchange coupling between adjacent atoms, coupling their magnetic moments together in rigid parallelism. This is a purely quantum effect, and cannot be explained in terms of classical physics. If there is no external field the domains are so oriented that they cancel each other. When an external magnetic field is applied to a specimen the domains that are suitably oriented grow in preference to others. The orientation of the magnetic dipole moment of a domain may turn into the favourite direction suddenly. This effect is called Barkhausen effect.

When the temperature exceeds the so called Curie temperature the coupling effect disappears, and the ferromagnetic material become paramagnetic.



The figure shows how μ' decreases as T increases.
Curie temperature for:

cobalt:1348 K (1075 °C),
nickel: 633 K (360 °C),
iron: 1042 K (769 °C).

3.6 Ampère's Law for the Magnetic Field

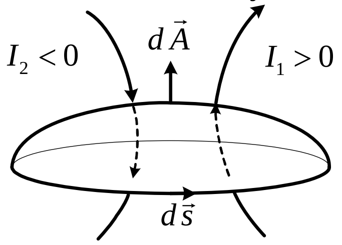
The experience shows that the line integral of the magnetic field intensity vector $\vec{H}$ over a closed curve is proportional to the algebraic sum of the currents boundered by the closed curve. The proportionality factor is a dimensionless quantity and its value depends on the permeability of vacuum. The proportionality factor is just 1, if the value is:

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am} = 1.265 \cdot 10^{-6} \frac{Vs}{Am}.$$

Due to experience this relation is valid in presence of magnetic materials as well. The integral form of Ampère's Law for magnetic field:

$$\oint_g \vec{H} \cdot d\vec{s} = \sum_{i=1}^{n} I_i$$

The circulation of the magnetic field intensity $\vec{H}$ along a closed path is equal to the net current through the area boundered by the path. The direction of the closed surve and the direction of the surface element vector is connected by the right hand rule.



The right hand fingers point in the direction of curve and by the thumb is indicated $d\vec{A}$.
Applying this rule can be obtained the sign of the currents: $I_1 > 0$, $I_2 < 0$.
If the current is distributed along a surface the current can be expressed as the flux of the current density, consequently Ampère's Law may be expressed in the form:

$$\oint_g \vec{H} \cdot d\vec{s} = \int_A \vec{J} \cdot d\vec{A}$$

Transforming the left-hand side integral according to Stokes's theorem:

$$\int_A \left(\nabla \times \vec{H}\right) \cdot d\vec{A} = \int_A \vec{J} \cdot d\vec{A}$$

$$\int_A \left(\nabla\times\overrightarrow{H}-\overrightarrow{J}\right)\cdot d\overrightarrow{A}=0$$

This must be true for any surface; this is possible only if the integrand is zero.

$$\nabla\times\overrightarrow{H}-\overrightarrow{J}=0$$

The differential form of Ampère's Law:

$$\nabla\times\overrightarrow{H}=\overrightarrow{J}\ .$$

So the magnetic field is not a conservative field. This is why we cannot ascribe to a magnetic field a scalar potential.

3.7 Boundary conditions at the interface between two different magnetics

It is provable using the Ampère's Law for magnetism, that the tangential components of the magnetic field strength at the interface of two media is the same (continuous):

$$H_{1t}=H_{2t}$$

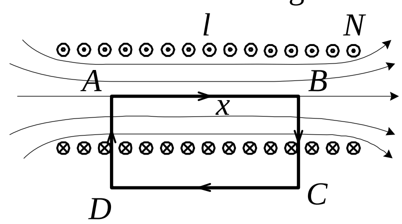
It is also provable using the Gauss'Law for magnetism, that the normal component of the magnetic induction at the interface between two magnetics is the same (continuous):

$$B_{1t}=B_{2t}\ .$$

3.8 The magnetic field inside a long cylindrical solenoid

Ampère's law is particularly useful when we must to compute the magnetic field produced by current distributions having certain geometrical symmetries. Consider now the magnetic field at the centre of a very long solenoid. Suppose that the turns are so closely spaced that they touch each-other. *N* is the number of turns, *l* is the length of the solenoid and *d* is its diameter. Suppose that *l* >> *d*, so we neglect the fringing effect. The number of turns per unit length are $n=\frac{N}{l}$, each truns carrying a current *I*. The field inside a long solenoid is nearly uniform.

Apply Ampère's law for the closed curve shown on figure:



$$\oint_g \overrightarrow{H}\cdot d\vec{s}=\sum_{i=1}^{n} I_i\ ,$$

$$\int_A^B \overrightarrow{H}\cdot d\vec{s}+\int_B^C \overrightarrow{H}\cdot d\vec{s}+\int_C^D \overrightarrow{H}\cdot d\vec{s}+\int_D^A \overrightarrow{H}\cdot d\vec{s}=\frac{N}{l}x\,I$$

$$\int_A^B \overrightarrow{H}\cdot d\vec{s}=H\,x\text{ ,due to }\overrightarrow{H}\uparrow\uparrow d\vec{s}$$

$$\int_B^C \overrightarrow{H}\cdot d\vec{s}=\int_D^A \overrightarrow{H}\cdot d\vec{s}=0\text{, due to }\overrightarrow{H}\perp d\vec{s}$$

$$\int_C^D \vec{H} \cdot d\vec{s} = 0 \text{, due to } H \approx 0 \text{,}$$

$$H\,x = \frac{N}{l} x\, I$$

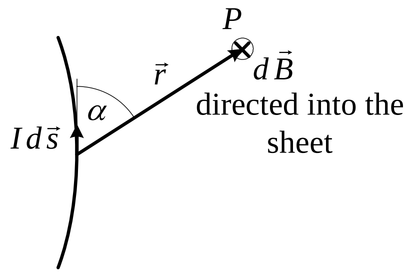
$$H = \frac{NI}{l}.$$

Magnetic induction due to along solenoid:

$$B = \mu H \qquad B = \mu \frac{NI}{l}.$$

3.8 The Biot-Savart Low for a current element

It can be proved that the elementary contribution to the magnetic field due to a thin wire segment through which a current flows is:



$$d\vec{B} = \frac{\mu}{4\pi} \frac{Id\vec{s} \times \vec{r}}{r^3}$$

The above equation is called the law of Biot-Savart. To find the total magnetic field $\vec{B}$ any point in space due to current in a circuit, we have to integrate the expression:

$$\vec{B} = \frac{\mu I}{4\pi} \int_g \frac{d\vec{s} \times \vec{r}}{r^3} \quad \text{,or} \quad \vec{B} = \frac{\mu I}{4\pi} \oint_g \frac{d\vec{s} \times \vec{r}}{r^3}$$

The direction of $d\vec{B}$ is perpendicular to the plane of $d\vec{s}$ and $\vec{r}$ ($\vec{r}$ directed from the current element to the point in question), and form a right hand system:

$$\left\{ d\vec{s}, \vec{r}, \vec{B} \right\}.$$