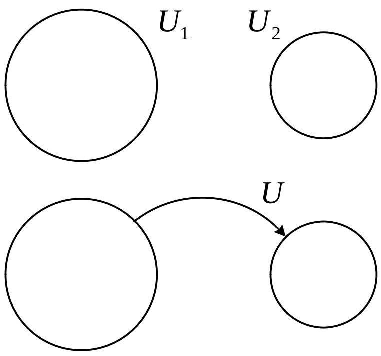
2 Electric current

The free electrons in an isolated metallic conductor are in random motion, their motion have no net direction. If we consider a hypothetical surface in the conductor, the rate at which electrons pass through it from right to left, is the same as from left to right, so the net rate is zero.
Consider now two charged conductors. Suppose that their potentials are different: Let $U_1 > U_2$.



If we connect the two conductors with a conductor wire, then charge flows through the wire until the potential of the united conductor equalize. The charge transfer directed from the higher potential to the lower one.
The motion of charge carriers during this rearrangement is temporary or transient. Due to the potential difference an electric field will be set up at every point within the wire, and this field will act on the charge carriers and give them a resultant motion. We can say that an ordered motion is superposed on to the chaotic motion of the carriers.

The electric current is defined as the ordered motion of electric charges. Due to agreement the direction of the current is the real or imaginary direction of motion of the positive charge carriers.
In a conductor, a charge bumps randomly, but due to field accelerates until collides with a stationary particle. The charged particle thus gives up some of its kinetic energy, accelerates again until it bumps, and so on. This is a random motion with a gradual drift in the direction of the field. The inelastic collisions with the stationary particles transfer energy to them: this increases their vibration energy and hence the temperature of the conductor

2.1 Current density and Current

First we introduce the current density vector $\vec{J}$. The magnitude of the current density vector numerically equals the net charge flowing through a unit perpendicular (to the flow) area in unit time. The direction of $\vec{J}$ is taken as the velocity of the ordered motion of the positive charge carriers (or opposite with the velocity of the ordered motion of the negative charge carriers).
Unit of the current density:

$$[J] = 1\frac{C}{m^2 s} = 1\frac{A}{m^2}$$

The current density vector consists of two parts:

$$\vec{J} = \rho\vec{v} + \vec{j}$$

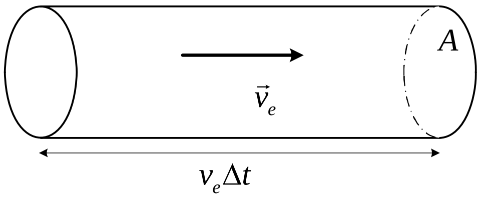
$\rho\vec{v}$: convection current density is associated with beams of charged particles or motion of charged insulator, ρ is the volume charge density and $\vec{v}$ is its velocity

$\vec{j}$: conduction current density

Consider now a crystallized or metallic conductor at rest. As the volume charge density $\rho = 0$, there is no convection current density. It is easy to see that the conduction current density:

$$\vec{j} = -e\, n_e \vec{v}_e\,,$$

where $e = 1.6 \cdot 10^{-19}\ C$ is the elementary charge (the magnitude of the charge of an electron), n_e is the number density of the free conducting electrons, $\vec{v}_e$ is the average drift speed of the electrons.



After a time interval Δt all the free electrons within the volume $Av_e\,\Delta t$ will have passed through the end face.
The charge will have passed:

$$\Delta Q = e\, n_e v_e \Delta t\, A$$

The current density:

$$\left|\vec{j}\right| = \frac{\Delta Q}{A\,\Delta t} = \frac{e\, n_e v_e \Delta t\, A}{A\,\Delta t} = e\, n_e v_e$$

$$\vec{j} = -e\, n_e \vec{v}_e$$

The direction of the current density is opposite that of the electron average drift speed.
Knowing the current density vector at each point along a surface *A*, electric current or simply current passing through this surface is:

$$I = \int_A \vec{J} \cdot d\vec{A}\,.$$

Current is a scalar algebraic quantity. The sign of the current is determined by the choice of the surface normal vector $d\vec{A}$. The current through an area is the net charge flowing through the area in unit time. This is the rate of flow of charge.

$$I = \frac{dQ}{dt}$$

The charge *Q* that passes a given cross-section in a given time interval is given by:

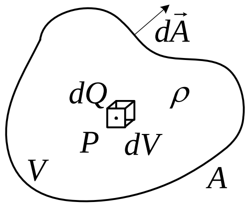
$$Q = \int_{t_1}^{t_2} I dt$$

Units:

$$[I] = 1\frac{C}{s} = 1A$$

$$[J] = 1\frac{A}{m^2}$$

Due to experiences the electric charge is conserved. Imagine a volume V boundered by a closed surface *A*.



If the charge density is ρ the charge in the volume is:

$$Q = \int_V \rho dV .$$

The time rate of change of this charge:

$$\frac{dQ}{dt} = \frac{d}{dt}\int_V \rho dV .$$

The net charge leaving the volume along the closed surface is given by:

$$\oint_A \vec{J} \cdot d\vec{A} .$$

As it is customary to choose the outward direction for the surface element vector $d\vec{A}$, according to the law of conservation of charge:

$$\frac{d}{dt}\int_V \rho dV = -\oint_A \vec{J} \cdot d\vec{A}$$

This relation is called the integral form of continuity equation.

As the volume is fixed the order of integration with respect to coordinates is interchangeable with the differentiation respect to time. And transform the other side in accordance with the Gauss-Ostogradsky theorem:

$$\int_V \frac{\partial \rho}{\partial t} dV = -\int_V \nabla \cdot \vec{J}\, dV$$

$$\int_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} \right) dV = 0 .$$

This must be observed upon an arbitrary volume *V*. This is possible only if at every point of space:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

This last equation is the differential form of the continuity equation.

2.2 Electromotive Force EMF

If only electric field acts on the charge carriers, the charge flows through the wire from the higher potential to the lower potential (positive carriers) and this would lead to the equalization of potentials and stopping of the current.
To maintain a current for sufficiently long time it is necessary the aid of forces of non-electrostatic origin, called extraneous forces. The extraneous forces move the positive charge carriers from the lower potential back to the higher potential.
These extraneous forces may be of chemical nature in a battery or cell, or magnetic nature in an alternating-current generator. (Extraneous force appears as magnetic field varies with time.

(This will be discussed later.) The device in which extraneous forces act maintaining the potential difference is called a seat of electromotive force. It has two terminals, the higher potential is called positive terminal the lower potential is called negative terminal.
The symbol of the extraneous force is: $\vec{F}^*$.
The strength of extraneous field is:

$$\vec{E}^* = \frac{\vec{F}^*}{q}$$

The work done by the extraneous forces within the seat of emf between the two terminals:

$$W^*_{\overset{\frown}{-+}} = \int_{-}^{+} \vec{F}^* \cdot d\vec{r} = \int_{-}^{+} q\vec{E}^* \cdot d\vec{r}$$

The electromotive force emf is defined as the work done by the extraneous forces on a unit positive a charge as it moves between the terminals:

$$\mathcal{E}_{\overset{\frown}{+-}} = \frac{W^*_{\overset{\frown}{-+}}}{q} = \int_{-}^{+} \vec{E} \cdot d\vec{r} \,.$$

Unit:

$$[\mathcal{E}] = 1\frac{J}{C} = 1V = 1volt \,.$$

We suppose that the emf is independent of the path taken within the seat of emf. A section of a circuit on which no extraneous force $\vec{E}^* = 0$ acts is called homogeneous (metallic conductor). The section in which the charge carriers experience extraneous force is called inhomogeneous (seat of emf or current source), $\vec{E}^* \neq 0$. In the metallic conductor the current flows from the higher potential to the lower one, in the seat of emf the current flows from the lower potential to the higher one.

2.3 Stationary or steady-state electric field

All physical quantities are constant in time, they can depend only on the position, but the charges can move in a stationary way. Due to experiences the stationary electric field is conservative.

$$\oint_g \vec{E} \cdot d\vec{r} = 0$$

$$\nabla \times \vec{E} = 0 \quad \rightarrow \vec{E} = -\nabla U$$

In case of stationary or steady state current the charge distribution in space remains constant so in the continuity equation:

$$\frac{d}{dt}\int_V \rho dV = 0$$

It means that the integral form of the stationary field is:

$$\oint_A \vec{J} \cdot d\vec{A} = 0$$

The differential form:

$$\nabla \cdot \vec{J} = 0$$

The boundary conditions at the interface between two different media:

$$E_{t1} = E_{t2}$$

The tangential component of the electric field is continuous.
And as $\vec{J}$ has no sources $\left(\nabla \cdot \vec{J} = 0\right)$:

$$J_{n1} = J_{n2}$$

The normal component of the current density vector is continuous.

2.4 Connection between electric field and current density, differential form of Ohm's Law

To describe the mechanism of current flow, consider a conduction electron in a metal. Its momentary velocity consists of two parts:

$$\vec{v} = \vec{V} + \vec{u}\ .$$

$\vec{u}$ is the velocity of the chaotic or disordered motion, $\vec{V}$ is the velocity of the ordered motion.
As we have already seen at thermodynamics:

$$u_{rms} = \left|\overline{\vec{u}}\right| \sim \sqrt{T}\ .$$

The average drift speed of the electrons in a typical conductor $\sim 10^{-4}\,\dfrac{m}{s}$ and the average speed of the random motion is $\sim 10^{6}\,\dfrac{m}{s}$ so $u_{rms} >> V$ inequality is valid.

The conduction electrons in a metal are accelerated by electric forces until they collide inelastically with lattice ions. Denote the mean free path by $\overline{\lambda}$, this is the average distance between two collisions. So the average time between collisions:

$$\overline{\tau} = \frac{\overline{\lambda}}{u_{rms}}\ .$$

During this time in a steady field $\vec{E}$ the electrons have a constant acceleration:

$$\vec{a}_e = \frac{\vec{F}_e}{m_e} = \frac{-e\left(\vec{E} + \vec{E}^*\right)}{m_e}\ ,$$

m_e is the mass of the electron.

Immediately after a collision, the velocity of the ordered motion is zero. $\vec{V} = 0$
Before the next collision:

$$\vec{V}_{\max} = \vec{a}_e\overline{\tau} = \frac{-e}{m_e}\frac{\overline{\lambda}}{u_{rms}}\left(\vec{E} + \vec{E}^*\right)$$

The average velocity of the ordered motion (drift):

$$\vec{v}_e = \frac{\vec{0} + \vec{V}_{\max}}{2}$$

$$\vec{v}_e = -\frac{e}{2m_e}\frac{\overline{\lambda}}{u_{rms}}\left(\vec{E} + \vec{E}^*\right)$$

As the conduction current density in a crystal:

$$\vec{j} = -e\,n_e\,\vec{v}_e\text{, so}$$

$$\vec{j} = \frac{e^2\overline{\lambda}}{2m_e}\frac{n_e}{u_{rms}}\left(\vec{E} + \vec{E}^*\right)$$

Introducing the electrical conductivity γ :

$$\gamma = \frac{e^2 \bar{\lambda} n_e}{2 m_e u_{rms}}$$

The resistivity of the substance is:

$$\rho = \frac{1}{\gamma}.$$

The differential form of Ohm's Law:

$$\vec{j} = \gamma\left(\vec{E} + \vec{E}^*\right)$$

$$\rho \vec{j} = \vec{E} + \vec{E}^*$$

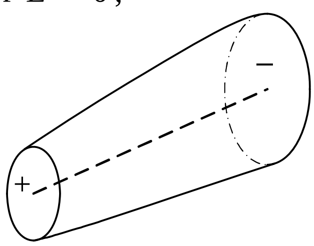
- In case of metals with increasing temperature u_{rms} increases, n_e nearly constant so γ the conductivity decreases.
- In case of semiconductors with increasing temperature U_{rms} increases but n_e increases faster, finally γ the conductivity increases.

The differential Ohm's Law $\vec{j} = \gamma\left(\vec{E} + \vec{E}^*\right)$ is not a strict proportionality between $\vec{j}$ and $\vec{E}$, because in some cases γ and $\vec{j}$ are not independent.

Remark:
The resistance of a large group of metals and alloys at a temperature of the order of several kelvins vanishes in jump. It is called superconductivity discovered by Kammerling and Onnes (1911).
In case of lead $T_{cr} = 7K$ and $\frac{1}{\gamma} = 0$

2.5 The integral form of Ohm's Law for a homogeneous conductor

Consider a homogeneous conductor $\vec{E}^* = 0$,



The potential difference:

$$U = \int_{+}^{-} \vec{E} \cdot d\vec{r}$$

The current passing through:

$$I = \int_{A} \vec{J} \cdot d\vec{A}$$

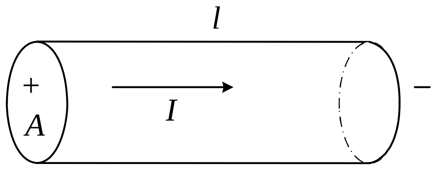
The Ohm's Law was discovered experimentally: the current passing through a homogeneous conductor is proportional to the potential difference across its terminals. Their ratio is called the electric resistance of the conductor *R*.

$$R = \frac{U}{I} = \frac{\int_{+}^{-} \vec{E} \cdot d\vec{r}}{\int_{A} \vec{J} \cdot d\vec{A}}$$

Unit of resistance:

$$[R] = 1\frac{V}{A} = 1\Omega = 1ohm\,.$$

Determine the resistance of a homogeneous cylindrical conductor of length *l*, cross-sectional area *A*, and resistivity ρ:



$$U = \int_{+}^{-} \vec{E} \cdot d\vec{r} = \int_{+}^{-} E\,dr = E\int_{+}^{-} dr = E\,l$$

$$I = \int_{A} \vec{J} \cdot d\vec{A} = \int J\,dA = J\,A\text{, and}$$

Due to the differential Ohm's Law $\rho\vec{j} = \vec{E}$, and $\vec{j} = \vec{J}$:

$$R = \frac{U}{I} = \frac{E\,l}{J\,A} = \frac{\rho\,J\,l}{J\,A} = \rho\frac{l}{A}$$

If the cross-sectional area and the resistivity is the function of the distance measured along the cylindrical conductor then:

$$R = \int_{g} \rho(s)\frac{ds}{A(s)}\,.$$

The integral is taken for the whole length of the conductor.

The resistivity depends on the:

- substance,
- presence of residual mechanical stresses,
- temperature.

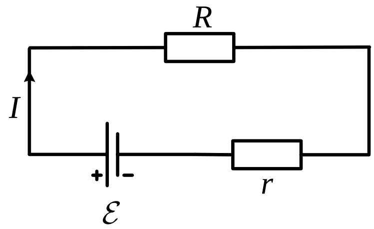
For many materials the relationship is nearly linear over a large temperature range:

$$\rho = \rho_0\left[1 + \alpha\left(T - T_0\right)\right]$$

Here ρ_0 is the resistivity at a reference temperature T_0 (often taken as *0 °C* or *20 °C*), and ρ is the resistivity at temperature *T*. The factor α is called temperature coefficient of resistivity. The resistivity of metallic conductors increases with increasing temperature so $\alpha > 0$. In case of carbon $\alpha < 0$, the resistivity decreases with increasing temperature. The resistivity of the alloy manganin is practically independent of temperature. Manganin (*Cu 84%, Mn 12%, Ni 4%)* $\alpha \approx 0$.

2.6 Integral form of Ohm's Law for a closed circuit

Consider now a closed circuit consist of a seat of emf and a consumer. Demote the emf by $\mathcal{E}$ and the resistance of the seat of emf by *r*, called internal resistance. The figure shows the internal resistance r, and the emf separately, although they occupy the same region of space. The continuous resistance of the external circuit is denoted by R and symbolized by a rectangle, the straight lines are conductors having negligible resistance.



Take the differential Ohm's Law and integrate for the closed circuit:

$$\oint_g \rho \vec{j} \cdot d\vec{s} = \oint_g \left(\vec{E} + \vec{E}^*\right) \cdot d\vec{s} \,,$$

g is directed as the current in the circuit. Divide the closed curve g into two parts, where g_2 is inside the seat of emf, and g_1 is outside. The right side:

$$\oint_g \vec{E} \cdot d\vec{s} + \oint_g \vec{E}^* \cdot d\vec{s} = \int_{+g_1}^{-} \vec{E}^* \cdot d\vec{s} + \int_{-g_2}^{+} \vec{E}^* \cdot d\vec{s}$$

As the stationary electric field is conservative: $\oint_g \vec{E} \cdot d\vec{s} = 0$.

In the conductor $\vec{E}^* = 0$, so $\int_{+g_1}^{-} \vec{E}^* \cdot d\vec{s} = 0$

The last term is just the electromotive force , emf: $\int_{-g_2}^{+} \vec{E}^* \cdot d\vec{s} = \mathcal{E}_{\overset{\frown}{-+}}$

Using the next expressions: $j = \dfrac{I}{A}$, $\quad \vec{j} \cdot d\vec{s} = I \dfrac{ds}{A}$, the left side:

$$\oint_g \rho \vec{j} \cdot d\vec{s} = \int_{+g_1}^{-} \rho \vec{j} \cdot d\vec{s} + \int_{-g_2}^{+} \rho \vec{j} \cdot d\vec{s} = I \int_{+g_1}^{-} \rho \frac{ds}{A} + I \int_{-g_2}^{+} \rho \frac{ds}{A} \,.$$

As $\int_{+g_1}^{-} \rho \dfrac{ds}{A} = R$, and $\int_{-g_2}^{+} \rho \dfrac{ds}{A} = r$.

$$\mathcal{E}_{\overset{\frown}{-+}} = I(R + r)$$

Use $\mathcal{E}$ for the electromotive force:

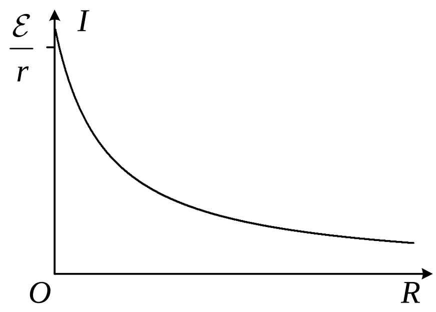
$$\mathcal{E} = I(R + r) \rightarrow I = \frac{\mathcal{E}}{R + r}$$

This is Ohm's Law for a complete circuit. The current equals the emf of the source divided by the total circuit resistance, external plus internal.

The terminal potential difference under closed circuit conditions:

$$U_K = IR = \mathcal{E} - Ir$$

The terminal potential difference is less then the emf $\mathcal{E}$ if $I \neq 0$. The variation of the current I as a function of R the external resistance is shown on the next figure:

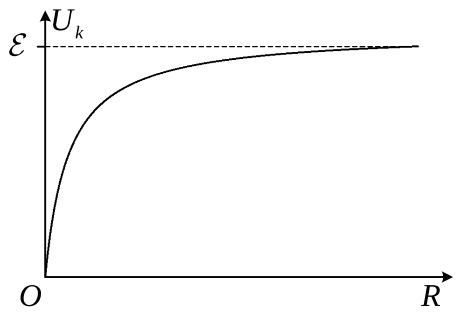


If the terminals of a source are connected by a conductor of zero (or negligible) resistance, the source is said to be short circuited. The short-circuit current is:

$$I_s = \frac{\mathcal{E}}{r}.$$

The terminal voltage:

$$U_K = IR = \mathcal{E}\frac{R}{R+r}$$



- if $R = 0$ or $R << r$, then it is a short-circuit, and $U_K = 0$,
- if $R = \infty$ or $R >> r$, then it is an open-circuit, and $U_K = \mathcal{E}$.

The terminal potential difference of a battery is less than $\mathcal{E}$ unless the battery has no internal resistance (*r = 0*), or if it is an open circuit (*R = ∞*), then the terminal potential difference is equal to $\mathcal{E}$.

2.7 Multiloop Circuits, Kirchoff's Rules

The calculation of multiloop circuits or network is considerably simplified if we use two rules formulated by Kirchoff.
First we define some concepts:

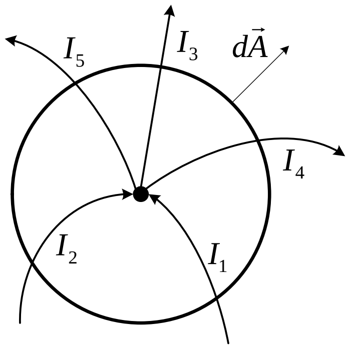
- The branch point or junction in a network is a point where three or more conductors are joined.
- The branch is a part of the network whose end points are junctions but there is no inner junction on it. In a branch the current is the same at all points. The circuit elements in a branch are connected series.
- A loop is any closed conducting path.
- The consumers are connected parallel if their terminals are at the same potentials.

2.7.1 Kirchoff's I. Rule

In case of stationary or steady-state current the conservation of charge is described by the simplified continuity equation:

$$\oint_A \vec{J} \cdot d\vec{A} = 0$$

Consider the next closed mathematical surface:



$$I > 0 \text{ if } \vec{J} \cdot d\vec{A} > 0$$
$$I < 0 \text{ if } \vec{J} \cdot d\vec{A} < 0$$

In the example: $I_3 + I_4 + I_5 - I_1 - I_2 = 0$.

In general:

$$\sum_{i=1}^{n} I_i = 0$$

The algebraic sum of the currents at a junction is zero.

2.7.2 Kirchoff's II. Rule

The algebraic sum of the potential differences in any loop must equal zero.

$$\sum_{i=1}^{n} U_i = 0$$

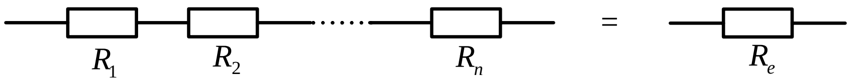
2.7.3 Problem-solving strategy:

- draw a large circuit diagram, label all quantities,
- in all branches set up arbitrary current directions,
- write Kirchoff's I. Rule for the junctions (only the independent equations),
- choose any closed loop in the network and designate an arbitrary direction to traverse the loop,
- write Kirchoff's II. Rule for these loops: go around the loop in the designated direction, adding the potential differences. An emf. is connected as positive when it is traversed from + to – terminals, and negative when form – to +. An *IR* product is positive if we passes through the resistor in the same direction as the assumed current and *IR* product is negative in opposite case.
- write as many equations as unknowns,
- finally solve the equations simultaneously to determine the unknowns.

2.8 Applications of Kirchoff's Rules

2.8.1 Combinations of Resistors

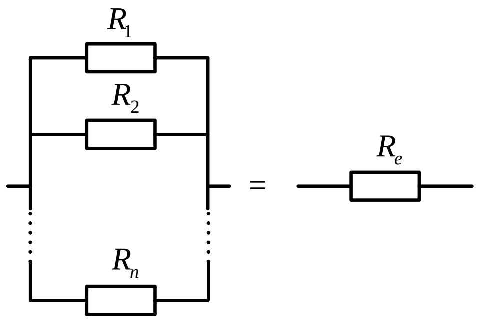
a) Series combination:



$$R_e = \sum_{i=1}^{n} R_i$$

The equivalent resistance of any number of resistors in series equals the sum of their individual resistances.

b) Parallel combination:

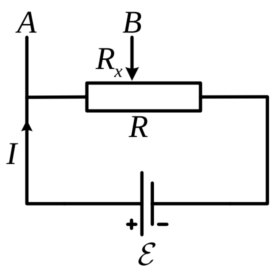


$$\frac{1}{R_e} = \sum_{i=1}^{n} \frac{1}{R_i}$$

For any number of resistors in parallel, the reciprocal of the equivalent resistance equals the sum of the reciprocals of their individual resistances.

2.8.2 The Potential Divider

If the potential divider is open:

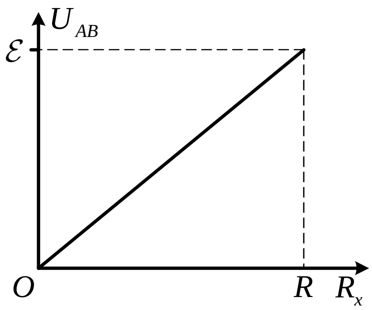


$$I = \frac{\mathcal{E}}{R}$$

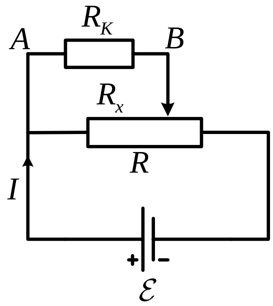
$$U_{AB} = IR_x = \mathcal{E}\frac{R_x}{R}$$

$$0 \le U_{AB} \le \mathcal{E}$$

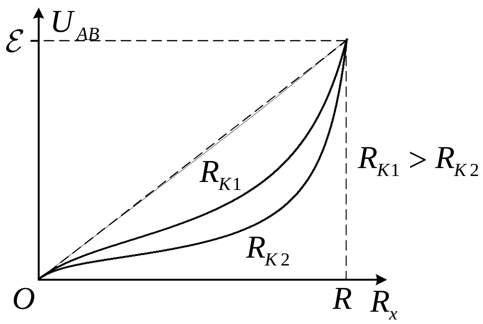
The value of U_{AB} can be varied by moving the sliding contact of the resistor. This voltage is linear function of R_x.



The potential divider is loaded if the variable potential difference is applied on a consumer R_K .

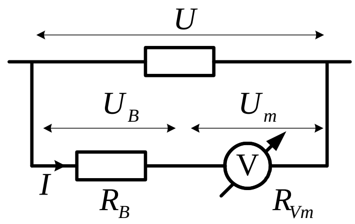


In case of loaded divider the potential difference is not a linear function of the R_x resistance.



2.8.3 Broaden the range of the potential difference can be measured by a voltmeter, adding a bolbin (a resistor of large resistance R_b) in series.

A voltmeter is always measures the potential difference or voltage between two points, and its terminals must be connected to these points, that is parallel with the resistance or other circuit components on which the voltage is to be measured. Voltmeters should therefore have a high resistance. The range of a voltmeter may be extended by connecting a resistor R_B in series with the meter, as in figure, so that only some fraction of the total potential difference appears across the meter itself, and the remainder across R_B . The series resistor is called bobbin. Denote the full-scale reading of the voltmeter by U_m .



Suppose that we need a voltmeter with a range of U, based on a meter U_m, and internal resistance R_{Vm}. Calculate the bobbin resistance R_B.
Due to Kirchoff's II. Rule:

$$U_m + U_B - U = 0.$$

Due to the serial connection of R_B, and R_{Vm}:

$$I = \frac{U_m}{R_{Vm}} = \frac{U_B}{R_B}$$

$$U_B = U_m \frac{R_B}{R_{Vm}}$$

$$U = U_m + U_m \frac{R_B}{R_{Vm}} = U_m \left(1 + \frac{R_B}{R_{Vm}}\right)$$

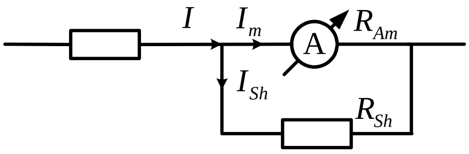
$$n = \frac{U}{U_m} = 1 + \frac{R_B}{R_{Vm}}$$

The bobbin resistance R_B:

$$R_B = (n-1) R_{Vm}$$

2.8.4 Broaden the range of the current can be measured by an ammeter, adding a shunt (a resistor of small resistance R_s) placed in parallel.

An ammeter is always placed in series with the resistance or other circuit components through which the current is to be measured. Ammeters should therefore have a low resistance compared with that of the rest of the circuit. An ammeter can be adapted to measure currents larger than its full-scale reading by connecting a resistor parallel with it, as shown in figure, so that some of the current bypasses the meter. The parallel resistor is called shunt.



Suppose that we need an ammeter with a range of I, based on a meter I_m, and internal resistance R_{Am}. Calculate the shunt resistance R_{Sh}.
Due to Kirchoff's I. Rule:

$$I = I_m + I_{Sh}$$

Due to Kirchoff's I:. Rule:

$$I_m R_{Am} = I_{Sh} R_{Sh}$$

$$I_{Sh} = I_m \frac{R_{Am}}{R_{Sh}}$$

$$I = I_m + I_m \frac{R_{Am}}{R_{Sh}} = I_m \left(1 + \frac{R_{Am}}{R_{Sh}}\right)$$

$$n = \frac{I}{I_m} = 1 + \frac{R_{Am}}{R_{Sh}}$$

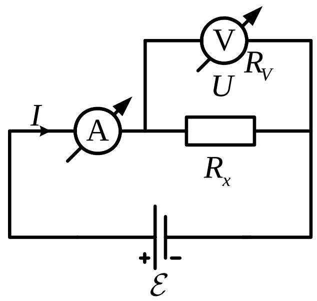
The shunt resistance R_{Sh}:

$$R_{Sh} = \frac{R_{Am}}{n-1}.$$

2.8.5 The measurement of resistance by ammeter – voltmeter method

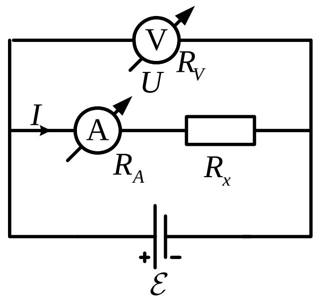
Using the defining equation $R_x = \frac{U}{I}$, measure I using an ammeter in series and U with a voltmeter in parallel. The two possibilities:

a) if $R_x << R_V$ (internal resistance of voltmeter)



$$R_x \approx \frac{U}{I}$$

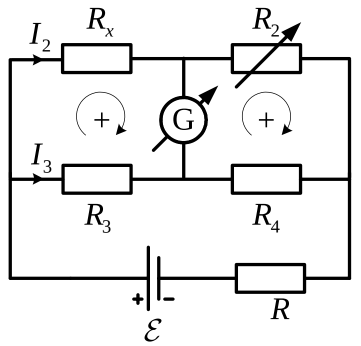
b) if $R_x >> R_A$ (internal resistance of ammeter)



$$R_x \approx \frac{U}{I}$$

In case of digital multimeters an operational amplifier is used to increase the internal resistance of the voltmeter to *~10 MΩ*. This resistance is so large that does not disturb the circuit. The current measurement is really a potential difference measurement on a high accuracy resistor.

2.8.6 The measurement of resistance by the Wheatstone Bridge



This arrangement of resistors is called Weatstone bridge. R_x is the unknown resistance.
One resistor is varied until the current in the galvanometer *G* is zero. The bridge is then said to be balanced. $I_G = 0$
The loop equations:

$$I_2R_x - I_3R_3 = 0 \quad I_2R_x = I_3R_3$$

$$I_2R_2 - I_3R_4 = 0 \quad I_2R_2 = I_3R_4$$

$$R_x = R_2 \cdot \frac{R_3}{R_4}$$

This method is suitable for $1\Omega - 10^6\,\Omega$.

2.9 Work and Power in stationary current circuit

Consider a consumer in a circuit, having a current *I* and potential difference U_{12} between the two terminals. As charge passes through this circuit element the electric field does work on the charge. In time t the amount of charge passes through the consumer, in case of stationary current is $Q = I\,t$, so the work is:

$$W = QU_{12} = U_{12}I\,t$$

By means of this work the electric field transfers energy into this portion of the circuit. The rate of the energy transfer is the power:

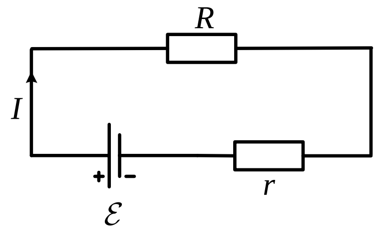
$$P = \frac{W}{t} = U_{12}I$$

In case of a pure resistance:

$$U_{12} = I\,R$$

$$P = U_{12}I = I^2R = \frac{U_{12}^2}{R}$$

This is the power input to the resistor. The circulating charges give up energy to the atoms of the resistor when they collide with them and the temperature of the resistor increases. We say that energy is dissipated in the resistor at rate IR^2.
Consider the next circuit: The source has an emf $\mathcal{E}$ and internal resistance *r*. The external resistance is *R*.



The terminal potential difference across the resistance *R* is:

$$U_{12} = \mathcal{E} - I\,r \quad /\cdot I$$

$$U_{12}\,I = \mathcal{E}\,I - I^2 r$$

$U_{12}\,I$ is the input power to the resistor *R,*

$\mathcal{E}\,I$ is the rate of conversion of nonelectrical to electrical energy (power output of the source),

$I^2 r$ is the rate of energy dissipation in the source.