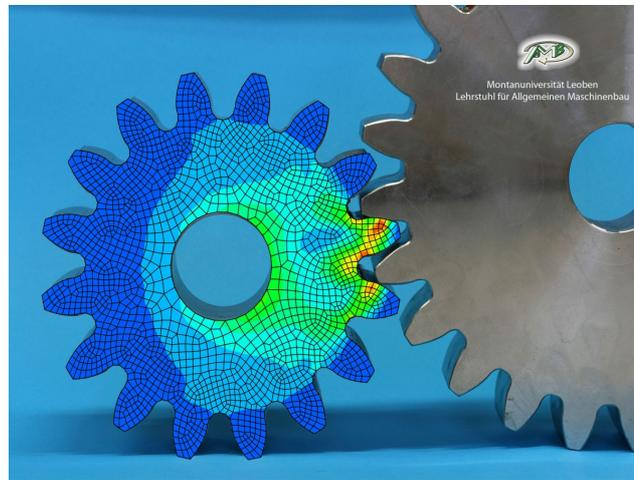


## CONTACT MODELS FOR MIXED FRICTION SIMULATION

*Dipl.-Ing. Jakob Moder, Univ.-Prof. Dipl.-Ing. Dr. mont. Florian Grün*  
Chair of Mechanical Engineering  
*Montanuniversität Leoben*

### SUMMARY

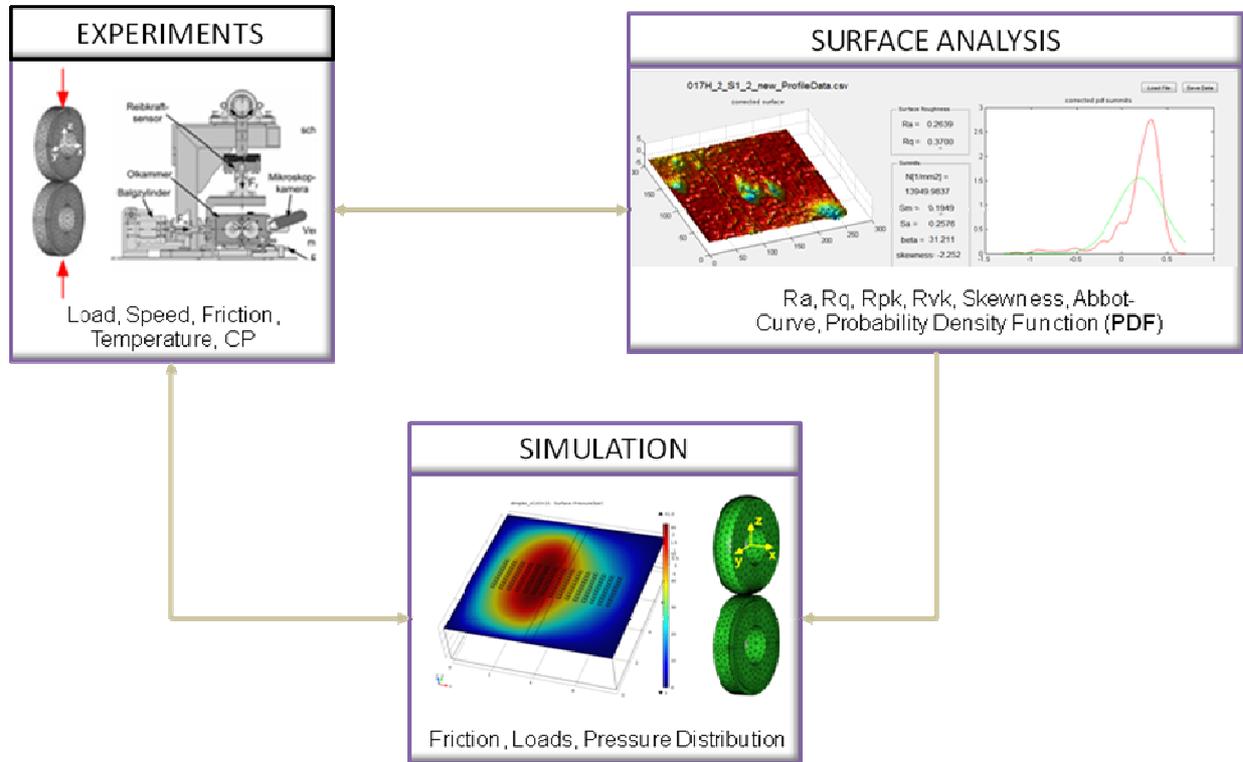
Due to legislative requirements companies are enforced to take on demanding engineering challenges in order to fulfil targets regarding CO<sub>2</sub> emissions and pollutions. Especially the automotive industry has come up with a range of technologies throughout the past decade to reduce the amount of exhaust gases.



Tribology has played an important role and will be significant in the future when it comes to creating environment-friendly machines. Especially machine elements, such as gears, camshaft or roller-bearings, which are referred to as non-conformal machine elements from here on, are heavily loaded and have to operate under severe conditions. Often this results in the energetically unfavourable mixed friction state, which means that the total external load is shared by the oil and micro contacts. The current work is focused on methods for modelling these micro contacts in order to take them into account in state of the art EHD (elastohydrodynamic) models. This enables an in depth analysis of highly loaded machine elements in order to optimize machine elements sustainably.

## INTRODUCTION

A comprehensive methodology, consisting of 3 major parts, has been developed throughout the last years at the Chair of Mechanical Engineering. Figure 1 illustrates the methodology.



**Figure 1: Applied Methodology**

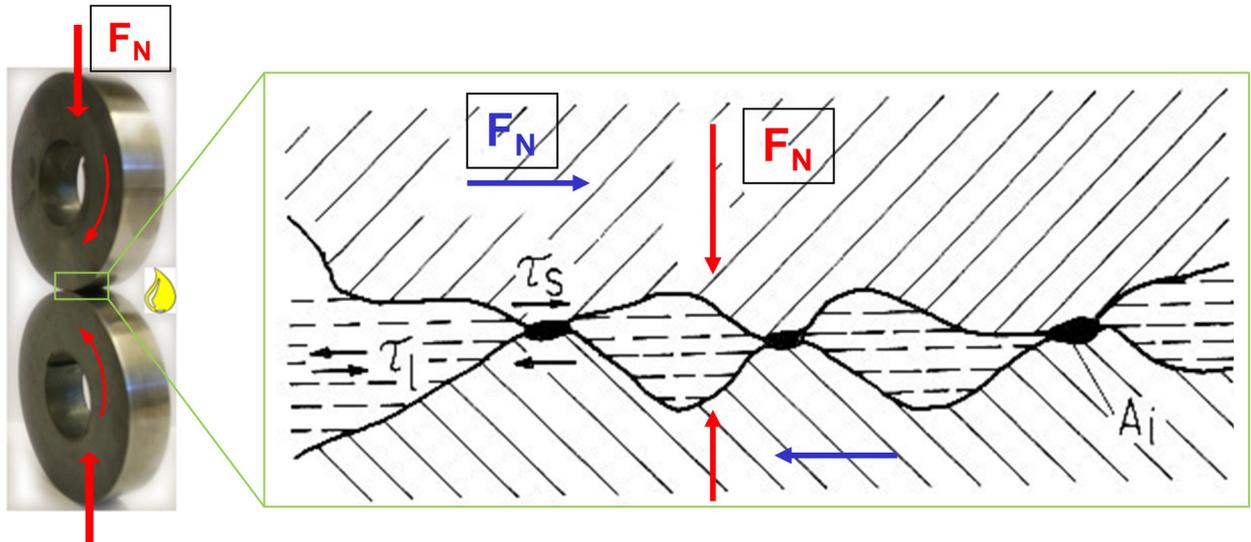
The methodology has been created to analyse the potential of different surface modification technologies (e.g. DLC coatings, micro structuring, superfinishing) in order to improve the tribological behaviour of non-conformal contacts. Discs are used as specimens, due to the straightforward manageability in terms of testing, machining and application of surface modifications. Nevertheless results can be transferred to actual machine elements.

In order to estimate which kind of surface processing technologies could be beneficial, simulations are carried out. They deliver information regarding pressure distribution, loads and friction. After a surface technology has been selected, the treated specimens are investigated by means of a laser-confocal microscope, which provides a 3D representation and various data off the disc surfaces. Finally, experiments are carried out in order to validate the simulation results and judge the suitability of the surface process for actual machine elements.

A key role in the simulation part is the accurate modelling of contact mechanics. However, since oil is always present in highly loaded contacts, complex interactions take place in this kind of tribological systems.

## MIXED FRICTION MODELLING APPROACH

The idea of mixed friction modelling in general is, that the external load  $F_N$  is carried partly by the oil and asperity contacts of the mating surfaces. Figure 2 illustrates this issue, by magnifying the contact area of a disc contact.



**Figure 2: Mixed friction model approach [1]**

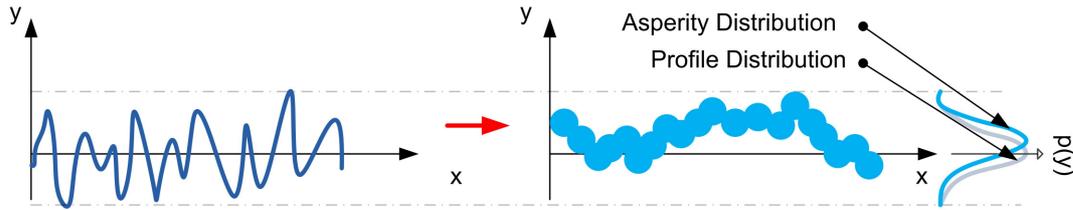
Due to the relative motion of the surfaces friction is present. The friction can be split into two parts:  $\tau_s$  which is resulting from the asperity contacts and  $\tau_l$  which occurs due to the presence of the fluid in the contact. Complex interactions can occur in this system, since the fluid directly affects the conditions for asperity contacts and vice-versa.

As a first step solely the contact mechanics problem will be dealt with, without taking into account the fluid and its properties.

## STATISTICAL CONTACT MODELS

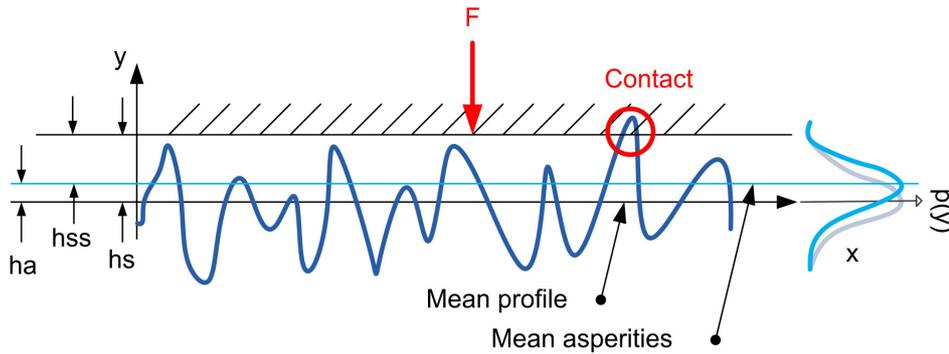
Statistical contact models make use of methods which describe technical surfaces in a statistical way, which means that contact problems can be dealt with in a relatively straightforward manner. In many cases analytical formulas can be applied in order to compute desired parameters.

The most popular model of this nature has been developed by Greenwood and Williamson [2]. It assumes that one surface is rough and that the other is perfectly smooth and rigid. The rough surface is transformed into spheres of a uniform diameter which behaves in a hertzian way. Figure 3 illustrates how an actual rough surface is transformed into the model.



**Figure 3: Model transformation**

Figure 4 shows a description of the model. It also contains various elements which are important for analytical formulas which will be presented later.



**Figure 4: Model description**

These elements are:

- Mean value of profile
- Mean value of asperity ( $h_a$ )
- Gap between mean of asperities and smooth surface ( $h_{ss}$ )
- External load ( $F$ )
- Distribution of asperities ( $p(y)$ )  $\phi_s(y)$

Another important value is the mean sphere radius of the rough surface, which is denoted as  $\beta$ , and the number of asperities in contact which is called  $n$ .

Let  $y$  be the vertical coordinate of the surface and  $w$  the displacement in this direction, then for a specific distance  $h_{ss}$   $w$  is:

$$w = y - h_{ss}$$

This results in a force acting on this asperity, which can be calculated as follows:

$$f = \frac{2}{3} E_r \beta^{1/2} w^{3/2} = \frac{2}{3} E_r \beta^{1/2} (y - h_{ss})^{3/2}$$

Taking into account that a technical surface has  $n$  asperities, the mean asperity pressure for the total contact can be computed as:

$$P = \frac{2}{3} n \beta^{1/2} E_r \int_{h_{ss}}^{\infty} (y - h_{ss})^{3/2} \phi_s(y) dy$$

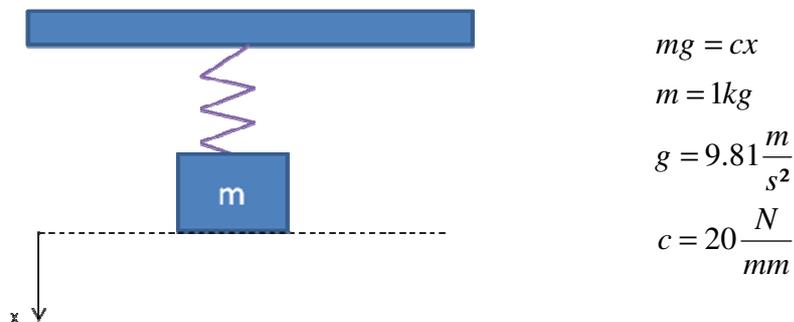
This method is very powerful considering that it is easy to handle and can still deliver accurate results, especially in situations where the load is moderate. However, non-linear material behaviour is not considered. The biggest drawback, however is, that each asperity is treated in an isolated way. Also it is not possible to evaluate local stresses in the contact.

## NUMERICAL CONTACT MODELS

Numerical contact mechanics has seen huge advances throughout the last couple of years due to the availability of huge amounts of computing power.

Nearly all methods are building on the Finite Element Method (FEM) which is based on the principle of minimum energy. The contact constraints are enforced by the formulation of an optimization problem.

In order to illustrate the problem, a spring mass system, as depicted in figure 5, is considered. Due to the gravity and the spring, the mass will stay in a position  $x^*$  for the steady state case.



**Figure 5: Spring mass system**

According to Newton the displacement of the mass is  $x^* = 0.4905mm$  .

Another way to solve this problem is the usage of energy equations.

The spring energy is defined as:

$$S(x) = \frac{1}{2}cx^2$$

And the gravitational energy:

$$G(x) = -mgx$$

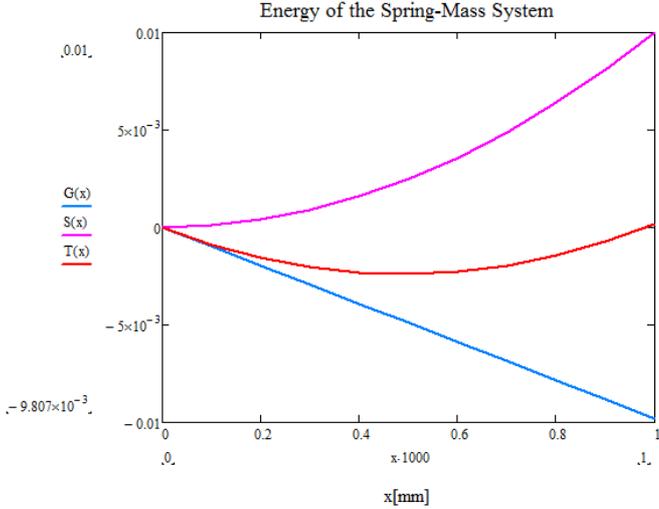
Therefore the total energy of the system is:

$$T(x) = G(x) + S(x)$$

Following the principle of minimum energy the stationary solution is:

$$\delta T(x) = 0$$

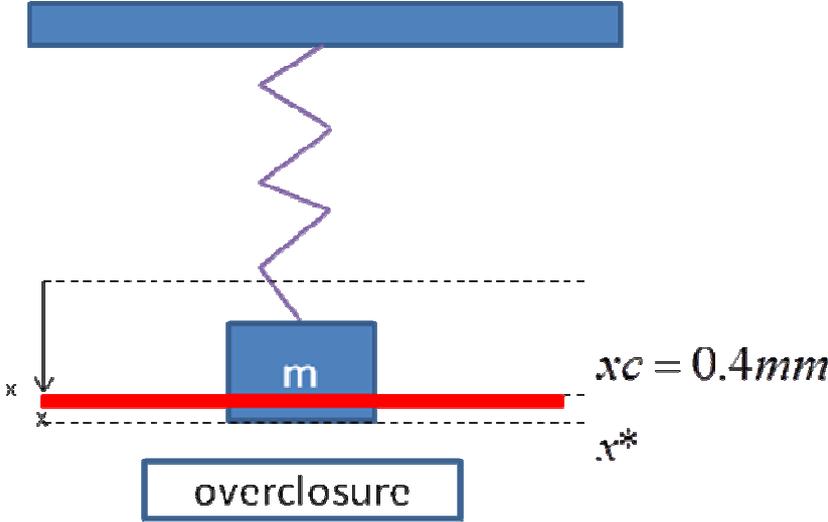
Figure 6 displays the total, spring and gravitational energy depending on the coordinate  $x$ .



**Figure 6: Energy as a function of  $x$**

It can be seen that a minimum of the total energy (red curve) is at  $x^* = 0.4905mm$ .

Now lets consider the same spring mass system, however this time a rigid plane is introduced to establish a contact problem. Figure 7 shows the configuration.



**Figure 7:Spring mass system with rigid plane**

In order to solve this contact problem the total energy equation will be turned into a constrained problem:

$$T(x) = G(x) + S(x) + \frac{1}{2} \epsilon p^2(x)$$

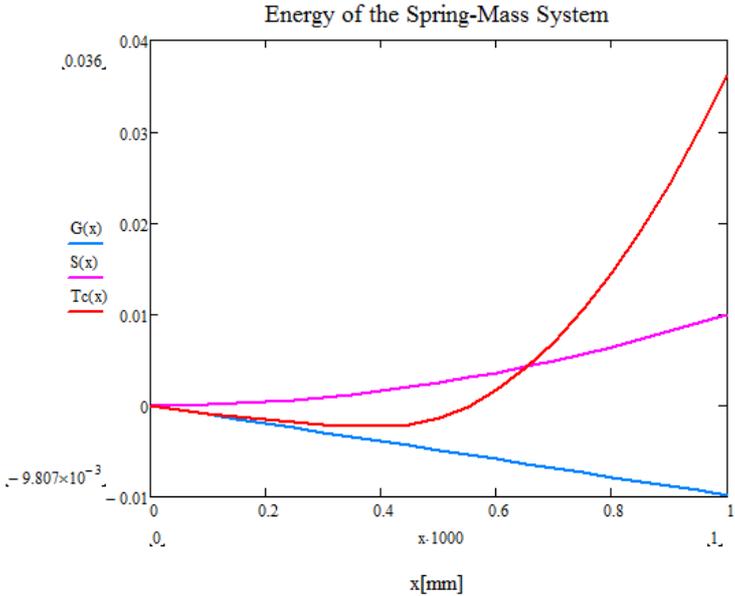
This is also called the penalty method, because it punishes overclosure, such as displayed in figure 7, by adding energy to the system.  $\epsilon$  is a constant and  $p(x)$  a function of  $x$ :

$$p(x) = \langle x - xc \rangle$$

The brackets  $\langle x \rangle$  have been introduced by Macaulay and simply mean:

$$\langle x \rangle = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

For the current problem this results in the application of penalty energy only when overclosure is present. The corresponding energy graph is depicted in figure 8.



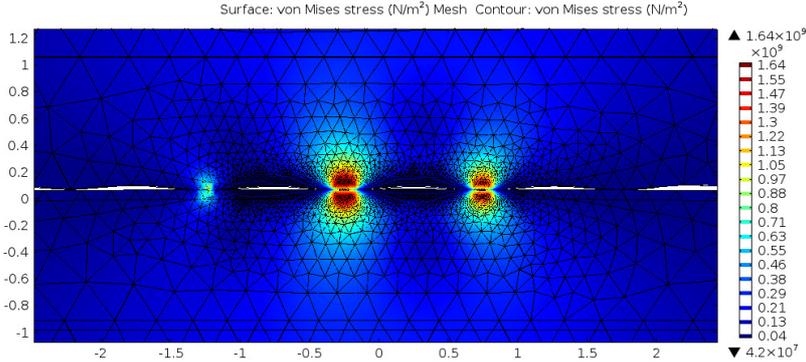
**Figure 8: Energy as a function of x for the constrained problem**

Again the solution is derived by calculating the first derivative of the total energy. The solution yields:

$$x^* \cong 0.41 \text{ mm}$$

Interestingly there is a difference between the solution and the position of the plane of 0.01mm. This is due to the nature of the penalty method, which permits a small overclosure. However, by using a very high value for  $\epsilon$  the error can be minimized.

This principle can also be applied to much more complex geometries by using the finite element method, as figure 9 shows. In this case a mesh size of about  $1 \mu\text{m}$  is necessary to resolve the problem.



**Figure 9: Finite element method applied on a contact problem**

In this scenario a smooth surface and a rough surface are loaded against each other. High stress concentrations typically occur at the asperities. In this case the mises stress is evaluated. Numerical contact models come with several advantages:

- ◆ Accurate modelling of real surfaces
- ◆ Consideration of non-linear material behaviour possible
- ◆ Inter-asperity interactions are considered
- ◆ Local stress evaluation

However, the big disadvantage is the high numerical cost which is necessary for the resolution of contact problems. This results from the fact that the problem is highly non-linear because neither the contact area nor the contact stress are known a-priori.

## **CONCLUSION AND OUTLOOK**

The importance of accurate modelling of mixed friction has been pointed out, whereby the focus has been laid on the functionality of different contact models. Statistical models can be used for straightforward modelling of contacts by using certain parameters of surfaces. They provide accurate results for moderate loadings and are often used for modelling conforming contacts like journal bearings. However, limitations such as isolated asperity treatment or the lack of information of local stresses present drawbacks.

On the other hand a numerical method, which is based on the finite element method combined with the penalty method, has been presented. Surfaces can be modelled accurately and inter-asperity interactions are taken into account. Furthermore local stresses can be evaluated. The biggest drawback is the high computational cost, which is in the nature of this kind of methods. Both methods can be integrated into state of the art EHD solvers which have been presented in [3] and [4] in order to deepen the understanding of phenomena taking place in heavily loaded non-conforming machine elements.

## **ACKNOWLEDGEMENT**

Financial support by the Austrian Federal Government (in particular from Bundesministerium für Verkehr, Innovation und Technologie and Bundesministerium für Wissenschaft, Forschung und Wirtschaft) represented by Österreichische Forschungsförderungsgesellschaft mbH and the Styrian and the Tyrolean Provincial Government, represented by Steirische Wirtschaftsförderungsgesellschaft mbH and Standortagentur Tirol, within the framework of the COMET Funding Programme is gratefully acknowledged.

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