

CASE STUDY ON THE ESTIMATION ACCURACY OF THE PARAMETERS OF A ROCK PHYSICAL MODEL

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INTRODUCTION

Geophysics has several tools to obtain information about the Earth based on physical parameters. During the investigations the observation of seismic/acoustic waves has a key role. They are used at different scales: seismic measurements are essential for the determination of large scale geological structures, acoustic well logging is used to complete seismic measurements (calculation of acoustic impedance log and synthetic seismograms, improvement of seismic velocity model) and obtain information about porosity along the borehole, laboratory acoustic measurements provide velocity data on core samples under controlled stress states. All these methods are based on the measurement of the travel times of seismic/acoustic waves. If the velocity is also known, the length of propagation path (depth) can be calculated, or vice versa. Based on the velocities other important parameters, such as the elastic moduli can be determined which are important for example in the fracturing or production design.

Pressure has great influence beside others on acoustic velocities. Numerous laboratory measurements [1] [2] [3] [4] [5] focus on the investigation of the pressure dependence of the petrophysical parameters. Several studies found exponential relation. Birch [6] considers this phenomenon as the result of changing pore volume due to varying stress. Walsh and Brace [7] explain the increasing velocity under increasing pressure with the closure of microcracks. In the literature empirical relations can be found, authors published the regression parameters for the best fitting functions [8] [9] [10] [11]. Somogyi Molnár [12] introduced a new rock physical model describing the pressure dependence of longitudinal (P) wave velocity, which provides the physical connection between these quantities. Further investigations [13] [14] [15] demonstrated that the model can be extended to transverse (S) waves. In this paper, a complex velocity model is presented. To prove its applicability acoustic P and S wave velocity data measured on a sandstone core sample at the rock physical laboratory of the Department of Geophysics, University of Miskolc were processed in a joint inversion procedure. The rate of overdetermination – the number of data compared to the number of unknown model parameters – is an important characteristic of inverse problems. To test the reliability of the model, pressure dependent velocities were determined based on different rate of overdetermination.

THE VELOCITY MODEL

The suggested model is based on the idea formulated by Birch [6]. He assumed that the main reason for the increasing velocity under loading is the closure of pores. Increasing pressure pushes the grains closer to each other, the pore volume decreases,

the waves travel faster through the rock. Since the starting point of the model is the (pore) volume change, and it is an isotropic quantity, the model is applicable both for the P and S wave velocities. The first model law of the velocity model can be formulated by (1)

$$dV = -\lambda_v V d\sigma, \quad (1)$$

where dV is the change of unit pore volume, $d\sigma$ is the applied stress increase and λ_v is the proportionality factor, a new rock physical parameter. The negative sign represents that the increasing stress causes decrease in the pore volume.

Assuming linear relationship between the infinitesimal change of the appropriate wave velocity dv (substitutable with the longitudinal or shear wave) and dV the second model law is

$$dv = -\kappa dV, \quad (2)$$

where κ is a proportionality factor, a new material characteristic. The negative sign represents that the velocity is increasing with decreasing pore volume. Combining Equations (1) and (2) and solving the differential equation one can obtain

$$dv = \kappa \lambda_v V_0 \exp(-\lambda_v \sigma) d\sigma \rightarrow v = K - \kappa V_0 \exp(-\lambda_v \sigma), \quad (3)$$

where V_0 is the pore volume at stress-free state ($\sigma=0$), K is an integration constant. At stress-free state ($\sigma=0$) the propagation velocity v_0 can be measured and K can be computed from Equation (3) as $v_0 = K - \kappa V_0$. With this result and introducing the notation $\Delta v_0 = \kappa V_0$ Eq. (3) can be rewritten in the following form

$$v = v_0 + \Delta v_0 (1 - \exp(-\lambda_v \sigma)), \quad (4)$$

where v_0 is the propagation velocity at stress-free state. Δv_0 is called the velocity drop, which means the difference between the velocities measured at maximum and zero stresses. The parameter λ_v can be expressed as the logarithmic stress sensitivity of the velocity-drop [16]

$$S(\sigma) = -\frac{1}{\Delta v} \frac{d\Delta v}{d\sigma} = -\frac{d \ln(\Delta v)}{d\sigma} \rightarrow \lambda_v = -\frac{d \ln(\Delta v)}{d\sigma} = S. \quad (5)$$

By substituting the appropriate velocities the model equations describing the pressure dependence of longitudinal (α) and shear (β) waves can be obtained in the forms of (6)

$$\begin{aligned} \alpha &= \alpha_0 + \Delta \alpha_0 (1 - \exp(-\lambda_v \sigma)) \\ \beta &= \beta_0 + \Delta \beta_0 (1 - \exp(-\lambda_v \sigma)). \end{aligned} \quad (6)$$

Note that λ_v is the same for both types of waves therefore if both P and S wave velocity data are available they can be processed in a joint inversion procedure.

LABORATORY MEASUREMENTS

The rock physical laboratory of the Department of Geophysics, University of Miskolc is equipped with a load frame (maximum compression force 300 kN), a triaxial cell (maximum operating pressure 80 MPa, maximum axial load 400 MPa, diameter of the pressure stamp 35 mm) and an ultrasonic 2-channel device able to measure P and S wave travel times (Figure 1). Piezoelectric crystals for acoustic measurements are built in the pressure stamps of the cell, the transducers have 1 MHz eigenfrequency and are sealed against the confining pressure. Detailed technical data can be found in [17].

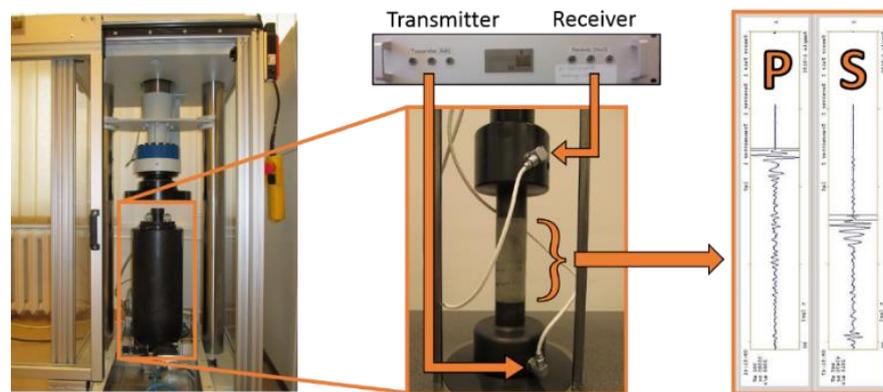


Figure 1.

Load frame and pressure cell; ultrasonic device and a sandstone sample between transmitter and receiver built in the pressure stamps; P and S wave arrivals.

P and S wave travel times were measured with pulse transmission technique [18] on cylindrical sandstone samples under uniaxial load. In this paper, the investigations on sample A are presented (Figure 2.). The workflow of the measurement is shown in Figure 3. The loading of the sample was carried out by the Freely Programmable Interface module of the software DION 7, which provides a completely automatic measurement. As it can be seen the sample was loaded linearly with 0.05 kN/s by a ramp function. After each ramp a 340 s break was embedded for the relaxation of the sample. The P and S wave travel times were measured at each stress steps. The axial pressure was increased from 5 MPa to 91 MPa. By knowing the length of the sample the P and S wave velocities were calculated. We applied 256-fold stacking to increase the signal-to-noise ratio and 32 dB gain. The measured values are plotted with red dots in Figure 4.

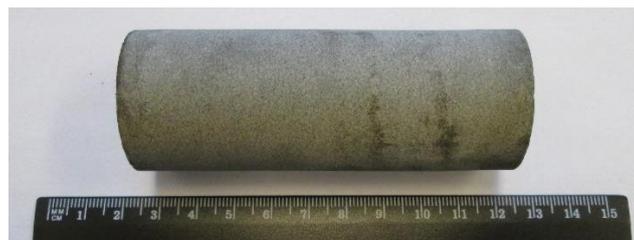


Figure 2.

Sample A: diameter 35 mm, length 93.45 mm, density 2.56 g/cm³.

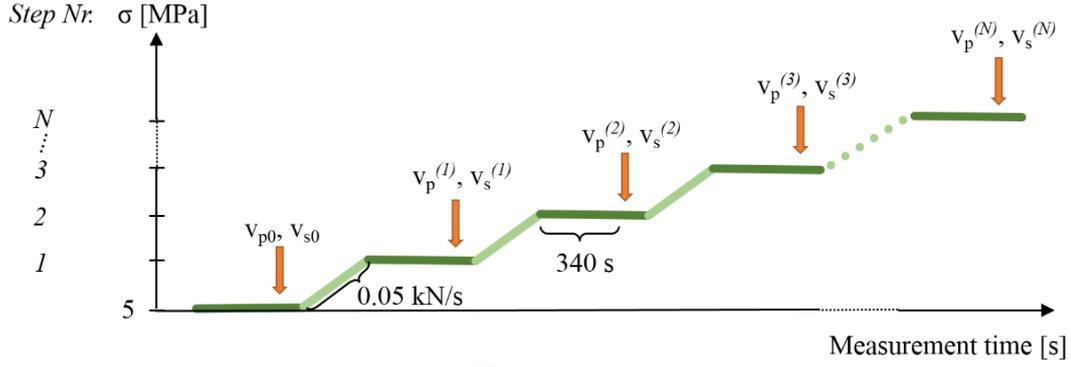


Figure 3.

Workflow of the automatic measurement including the loading rate (0.05 kN/s) and the relaxation time (340 s). Orange arrows show the velocity measurements.

The measured P and S wave velocity data were processed by means of quality checked joint inversion method. Since the problem was overdetermined (number of data $N >$ number of model parameters M), the Gaussian Least Square Method was used [19]. After determining the rock physical parameters ($\alpha_0, \Delta\alpha_0, \lambda_v, \beta_0, \Delta\beta_0$) of the model equations (6) - where λ_v connects the two data sets - velocities were calculated for the stresses in the region of interest by using equations (6). The results are shown in Figure 4. As it was mentioned, dots mean the measured values, the lines represent the values calculated by inversion. It can be seen the measured and calculated data are in good accordance.

The errors are the elements of the main diagonal of the covariance matrix in parameter space (the number of parameters $M=i=1,2,\dots,5$ in the given problem)

$$\sigma_i = \sqrt{\text{cov}(\mathbf{m})_{ii}} . \quad (7)$$

To characterize the accuracy of inversion estimates the RMS value was calculated with the following formula [19]

$$\text{RMS} = \sqrt{\frac{1}{N} \sum_{k=1}^N \left(\frac{d_k^{(m)} - d_k^{(c)}}{d_k^{(c)}} \right)^2} \cdot 100 [\%], \quad (8)$$

where $d_k^{(m)}$ is the measured data at the k-th pressure and $d_k^{(c)}$ is the k-th calculated data which can be computed by the model equations (6). The mean spread showing the reliability of the suggested rock physical model was also determined by the formula

$$S = \sqrt{\frac{1}{M(M-1)} \sum_{i=1}^M \sum_{j=1}^M (\text{corr}(\mathbf{m})_{ij} - \delta_{ij})^2} , \quad (9)$$

where δ is a Kronecker-delta symbol and $\text{corr}(\mathbf{m})$ is the correlation matrix in parameter space, which provides the strength of linear relationships between each pair of model parameters.

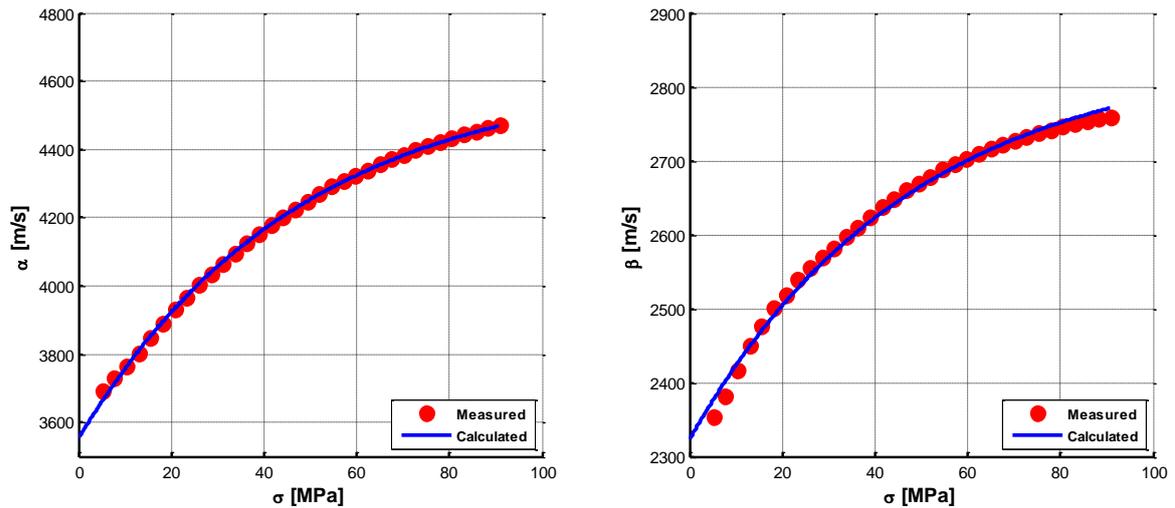


Figure 4.
Measured and calculated velocity data in the range of 5 and 91 MPa.

EFFECT OF OVERDETERMINATION RATIO

During laboratory measurements some limiting factors must be considered. At small pressures the first arrivals can be often hardly detected, especially in case of transverse waves. The signal-to-noise ratio can be increased by higher stacking number (sum of more measurements), but it has limitations. Another aspect is to avoid the making of new microcracks or total failure of the sample due to overloading. In other words samples have to be handled with care and a reliable model which is able to predict the velocities for wider pressure intervals can be a good solution.

The aim of this study was to examine the effect of rate of overdetermination (N/M) on inversion results. Therefore data from different pressure ranges were used as input data vector for the inversion. Based on the “actual” measured data the model parameters were estimated by joint inversion and the velocities were calculated for the pressure range 0-150 MPa in each cases. Results are shown in Figures 5 and 6. For the better visibility graphs do not contain the measured data, only the velocities calculated by inversion. The estimated model parameters with the estimation errors in the parenthesis, as well as the RMS and mean spread are summarized in Table 1. In Case 1, P and S wave velocities belonging to the pressure ranges 5-31 MPa, 5-50 MPa, 5-70 MPa and 5-91 MPa were processed, accordingly the N/M rate was increased from 2.2 to 6.8 by involving measured data from higher pressures. It can be seen in Figure 5 and in Table 1 as well, that the model parameters determined from less measured data slightly overestimate the velocities. Their estimation errors – especially that of the velocity drops – decrease with increasing N/M ratio. Of course, it is expected that the higher overdetermination of the problem reduces the error of the determined parameters. Comparing the results obtained for the pressure interval 5-31 MPa and 5-91 MPa, it can be seen that the estimation error of $\Delta\alpha_0$ reduces from 17 % to 0.7 %, in case of $\Delta\beta_0$ reduces from 17 % to 1.3 %. Since the picking of first arrivals in the low pressure intervals is uncertain because of the higher noise, and that the 11 measured data are mostly on the linear interval of the velocity change, the

applied velocity model can be considered reliable even for the case when less data in the lower pressure interval are available. It is important if the strength of the sample is unknown and one would like to avoid the overloading and failure of the sample and the measurement is planned to stay in the “safe” interval.

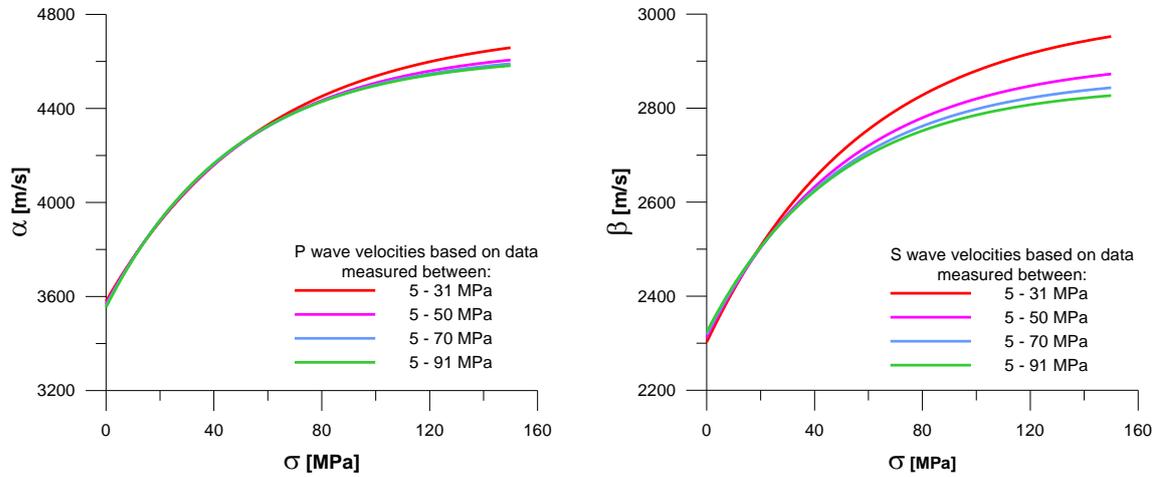


Figure 5.

Velocities estimated by inversion based on data measured in different pressure intervals.

In Case 2, the measured points at the beginning phase of loading (5-23 MPa) were excluded, because in this region the picking of first arrivals is not always easy and this interval can contain more uncertainty. Velocity data for pressure intervals 23-50 MPa, 23-68 MPa and 23-91 MPa were inverted (N varies from 11 to 27). With increasing data number the velocities determined by the velocity model slightly decrease as it can be seen in Figure 6. Comparing the results for N/M ratios 2.2 and 3.6 from Case 1 and 2, it can be seen that the estimation errors are lower if data from the non-linear phase are inverted. According to the results plotted with blue and green lines in Figure 6 it can be concluded that the intermediate number of data had already provided acceptable results, especially for P wave.

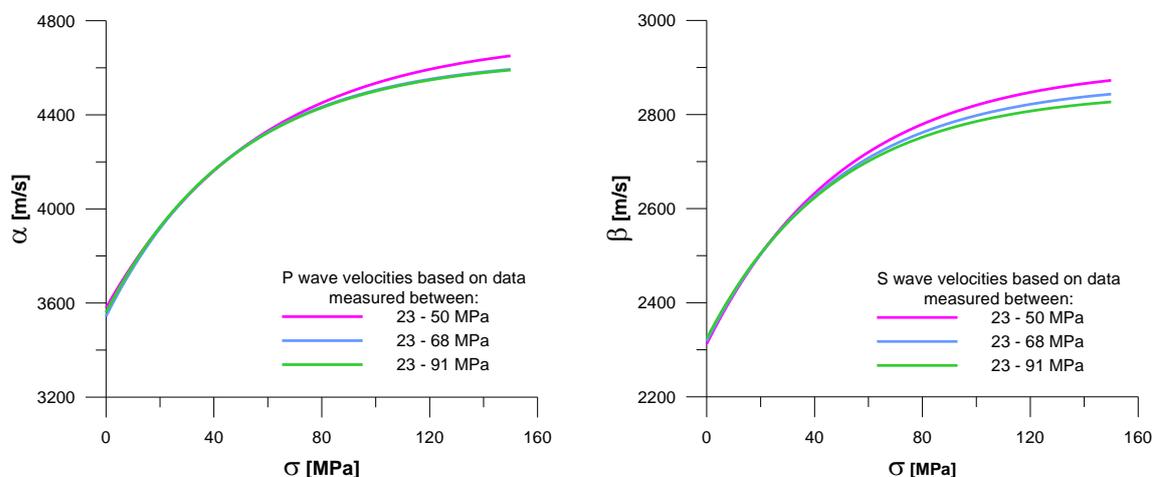


Figure 6.

Velocities estimated by inversion based on data measured in different pressure intervals.

Table 1.

Estimated model parameters and their estimation errors, RMS and mean spread.

#	N	N/M	Pressure [MPa]	α_0 [m/s]	$\Delta\alpha_0$ [m/s]	λ_v [1/MPa]	β_0 [m/s]	$\Delta\beta_0$ [m/s]	RMS [%]	S [-]
CASE 1	11	2.2	5-31	3579 (± 10)	1168 (± 203)	0.0172 (± 0.0042)	2302 (± 9)	704 (± 123)	0.25	0.80
	18	3.6	5-50	3568 (± 7)	1097 (± 43)	0.0194 (± 0.0014)	2312 (± 6)	593 (± 26)	0.26	0.70
	26	5.2	5-70	3560 (± 6)	1080 (± 15)	0.0204 (± 0.0007)	2318 (± 5)	551 (± 11)	0.26	0.58
	34	6.8	5-91	3553 (± 5)	1074 (± 7)	0.0211 (± 0.0004)	2323 (± 5)	526 (± 7)	0.27	0.47
CASE 2	11	2.2	23-50	3579 (± 9)	1154 (± 24)	0.0176 (± 0.0009)	2355 (± 5)	543 (± 12)	0.04	0.88
	18	3.6	23-68	3543 (± 6)	1099 (± 4)	0.0208 (± 0.0004)	2344 (± 4)	505 (± 4)	0.05	0.60
	27	5.4	23-91	3523 (± 5)	1097 (± 3)	0.0220 (± 0.0003)	2345 (± 4)	485 (± 4)	0.08	0.56

CONCLUSIONS

New rock physical model describing the pressure dependence of P and S wave velocities was presented. P and S wave velocities were determined on a sandstone sample under uniaxial load. Since these measurements have limitations, i. e. the determination of first arrivals can be uncertain at low pressures, and we have to avoid the failure of the sample at high pressures, the effects of rate of overdetermination on inversion results were investigated. In Case 1 measured data from higher pressures were involved stepwise, in Case 2 the beginning phase of loading was excluded. Based on the investigations it can be concluded that the presented velocity model predicts reliable velocities even if data up to ($\sigma < 50$ MPa) or only in (23 MPa $< \sigma < 68$ MPa) an intermediate pressure level are measured.

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