DE

Underdamped harmonic oscillator

Let

$$a = 2,\ b = 37,\ t_0 = 0,\ t_1 = 1, \tag{1}$$

$$ic_0 = 2,\ ic_1 = -1, \tag{2}$$

$$y''(t) + ay'(t) + by(t) = f(t). \tag{3}$$

The first order form of (3) is

$$\frac{d}{dt}\begin{pmatrix} y \\ v \end{pmatrix} = A\begin{pmatrix} y \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}(f(t)), \tag{4}$$

where $y' = v$.

Mathematica.
```
{a,b,t0,t1,ic0,ic1}={2,37,0,1,2,-1}
de={y''[t]+a y'[t]+b y[t]==f[t]}
ic={y[t0]==ic0,y'[t0]==ic1}
A={{0,1},{-b,-a}}
```

Octave.

1, 1

Exercise. Let $f(t) = 0$, (2) be true. How much is $y(t_1)$?

Mathematica.
```
desol=NDSolve[Join[de/.{f[t]->0},ic],y,{t,t0,t1}]
ans=(y[t1]/.desol)[[1]]
ans=Replace[y[t1],desol][[1]]
```

Octave.
```
pkg load symbolic
a=2; b=37; t0=0; t1=1; ic0=2; ic1=-1;
syms y(t)
de = diff(y, 2) + a*diff(y,1) + b*y == 0
sol_Gen = dsolve (de)
sol_Part = dsolve (de, y(0) == ic0, diff(y)(0) == ic1)
sym_ans = rhs(subs( solPart, t, t1))
display('subex 1')
num_ans = double(sym_ans)
```

Answers. A: 0.36684 B: 0.452681 C: 0.558608 D: 0.689322 E: 0.850623

Correct answer. D

2, 2

Exercise. $f(t) = 0$. The general solution of (3) is

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}, \;\; \Im(\lambda_1) > 0.$$

How much is $\Im(C_1)$, if (2) is satisfied?

Mathematica.
```
de2=Join[de/.{f[t]->0},ic]
sol2=DSolve[de2,y,t]
expSol=TrigToExp[sol2[[1,1,2,2]]]
```

Octave.

Answers. A: -0.0443479 B: -0.0547253 C: -0.0675311 D: -0.0833333 E: -0.102833

Correct answer. D

3, 3

Exercise. $f(t) = 0$. The general solution of (3) is

$$y(t) = e^{\alpha t}(C_1 \cos \omega t + C_2 \sin \omega t).$$

How much is C_1, if (2) is satisfied?

Mathematica.
```
de3=Join[de/.{f[t]->0},ic]
sol3=DSolve[de3,y,t]
cosSol=sol3[[1,1,2,2]]//Expand
```

Octave.
```
pkg load symbolic
a=2; b=37; t0=0; t1=1; ic0=2; ic1=-1;
syms y(t)
de = diff(y, 2) + a*diff(y,1) + b*y == 0
sol_Gen = dsolve (de)
sol_Part = dsolve (de, y(0) == ic0, diff(y)(0) == ic1)
display('subex 3')  # answer: 2
num_ans = double(2)
```

Answers. A: 1.06435 B: 1.31341 C: 1.62075 D: 2. E: 2.468

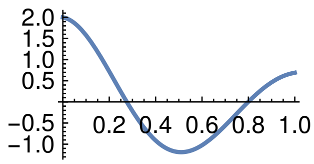
Correct answer. D

4, 4

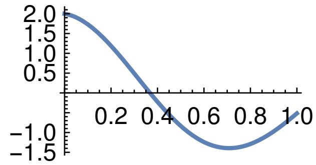
Exercise. $f(t) = 0$, (2), (3) true. Plot $y(t)$!

Mathematica.
```
de4=Join[de/.{f[t]->0},ic]
sol4=NDSolve[de4,y,{t,t0,t1}]
Plot[Evaluate[y[t]/.sol4],{t,t0,t1}]
```

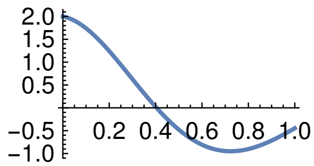
Octave.



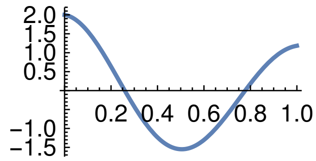
A



B



C



D

E: Sehol

Answers.

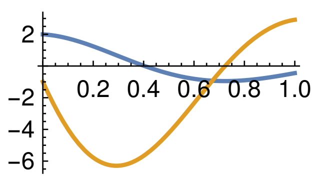
Correct answer. A

5, 5

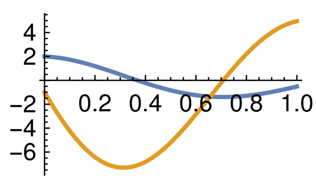
Exercise. $f(t) = 0$, (2), (3) is true. Plot $y(t), y'(t)$-t on a single figure!

Mathematica.
```
de5=Join[de/.{f[t]->0},ic]
sol5=NDSolve[de5,y,{t,t0,t1}]
Plot[Evaluate[{y[t],y'[t]}/.sol5],{t,t0,t1}]
```

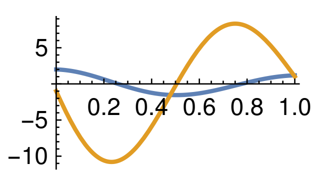
Octave.



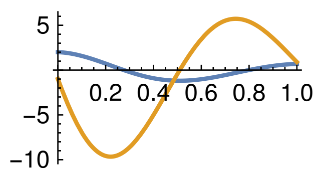
A



B



C



D

E: Sehol

Answers.

Correct answer. D

6, 6

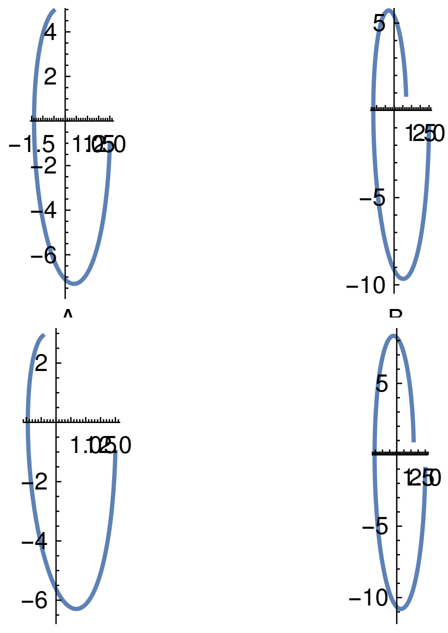
Exercise. $f(t) = 0$, (2), (3) are true. Plot the

$$\gamma : [t_0, t_1] \to \mathbb{R}^2, \qquad \gamma(t) = (y(t), v(t))^T$$

parametric curve!

Mathematica.
```
de6=Join[de/.{f[t]->0},ic]
sol6=NDSolve[de6,y,{t,t0,t1}]
ParametricPlot[Evaluate[{y[t],y'[t]}/.sol6],{t,t0,t1}]
```

Octave.



E: Sehol

Answers.

Correct answer. B

7, 7

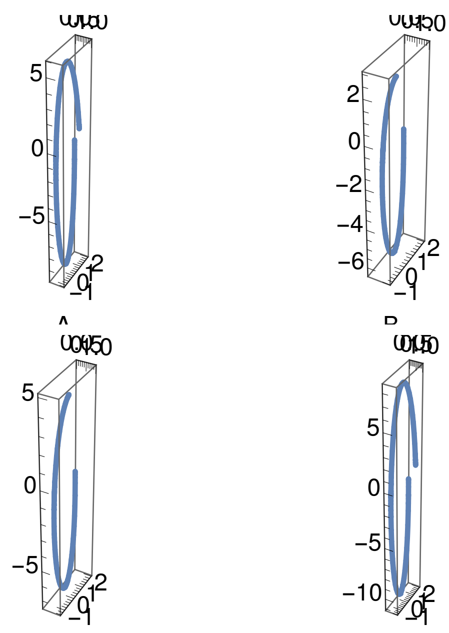
Exercise. $f(t) = 0$, (2), (3) are true. Plot the

$$\gamma : [t_0, t_1] \to \mathbb{R}^3, \qquad \gamma(t) = (t, y(t), v(t))^T$$

parametric curve!

Mathematica.
```
de7=Join[de/.{f[t]->0},ic]
sol7=NDSolve[de7,y,{t,t0,t1}]
ParametricPlot3D[Evaluate[{t,y[t],y'[t]}/.sol7],{t,t0,t1}]
```

Octave.



E: Sehol

Answers.

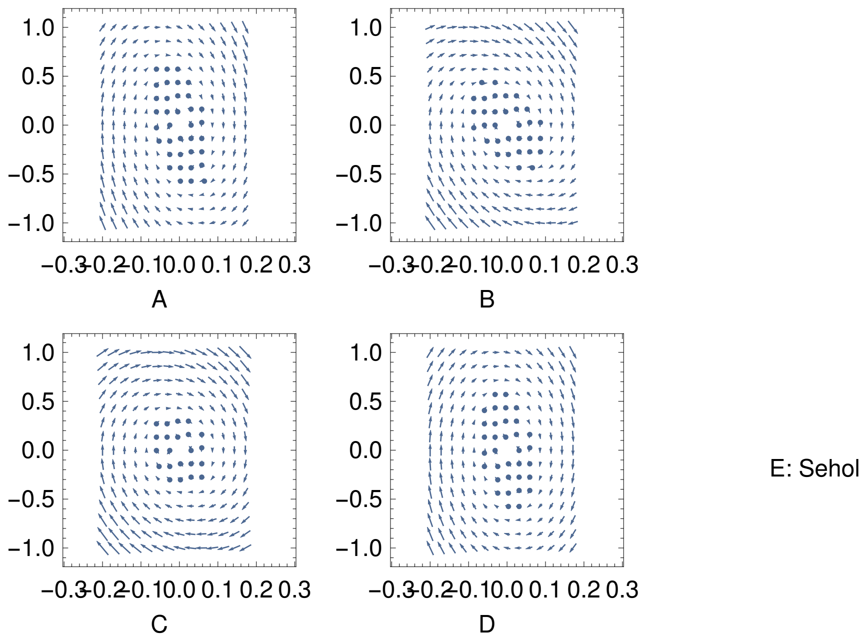
Correct answer. A

8, 8

Exercise. Plot the $(y, v)^T \rightarrow A(y, v)^T$ vector field!

Mathematica. `VectorPlot[A.{y,v},{y,-.2,.2},{v,-1,1},`
`VectorPoints->15,VectorStyle->Arrowheads[0.02]]`

Octave.



Answers.

Correct answer. A

9, 9

Exercise. Compute $U = \exp(1.4A)$! How much is U_{11} ?

Mathematica. `ans9=MatrixExp[1.4 A]`
`ans9[[1,1]]`

Octave.

Answers. A: -0.0400776 B: -0.0494558 C: -0.0610284 D: -0.0753091 E: -0.0929314

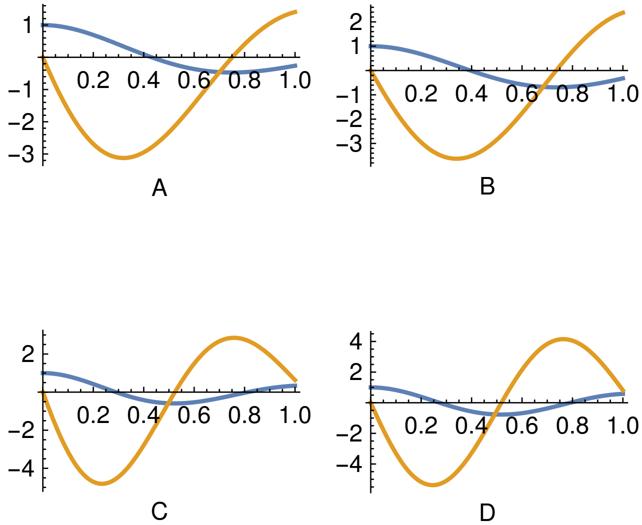
Correct answer. E

10, 10

Exercise. Compute $U(t) = \exp(tA)$-t and plot its first column's functions!

Mathematica. `U=MatrixExp[t A]`
`Plot[{U[[1,1]],U[[2,1]]},{t,t0,t1}]`

Octave.



E: Sehol

Answers.

Correct answer. C

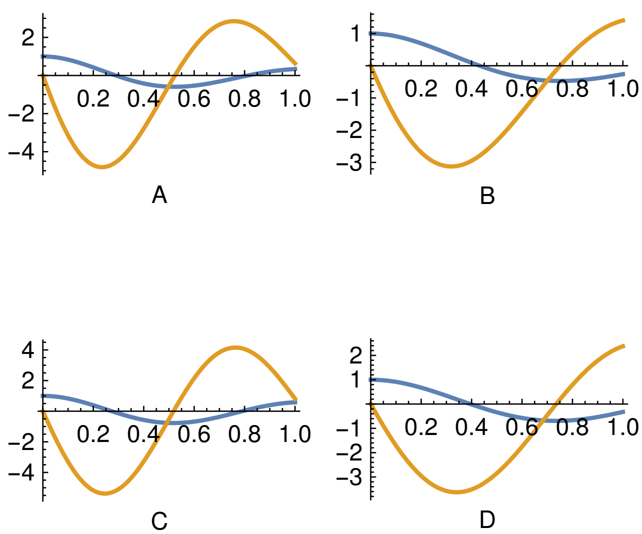
11, 11

Exercise. $f(t) = 0$, (2) true, $y(0) = 1$, $y'(0) = 0$. Plot $(y(t), y'(t))^T$!

Mathematica. `de11=Join[de/.{f[t]->0},ic]`

```
sol11=NDSolve[de11,y,{t,t0,t1}]
Plot[Evaluate[{y[t],y'[t]}/.sol11],{t,t0,t1}]
```

Octave.



E: Sehol

Answers.

Correct answer. A

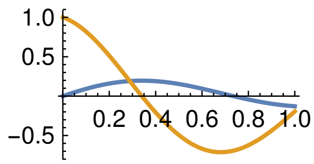
12, 12

Exercise. Compute $U(t) = \exp(tA)$-t and plot its second column's functions!

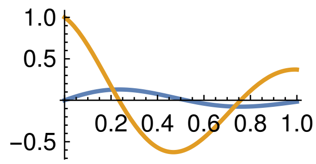
Mathematica. `U=MatrixExp[t A]`

```
Plot[{U[[1,2]],U[[2,2]]},{t,t0,t1}]
```

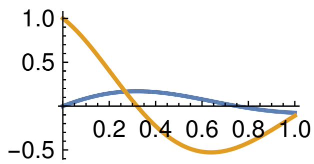
Octave.



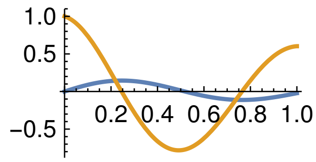
A



B



C



D

E: Sehol

Answers.

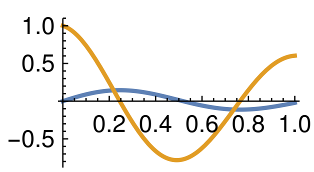
Correct answer. B

13, 13

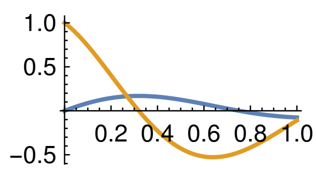
Exercise. $f(t) = 0$, (2) true, $y(0) = 0$, $y'(0) = 1$. Plot $(y(t), y'(t))^T$!

Mathematica.
```
de13=Join[de/.{f[t]->0},ic]
sol13=NDSolve[de11,y,{t,t0,t1}]
Plot[Evaluate[{y[t],y'[t]}/.sol13],{t,t0,t1}]
```

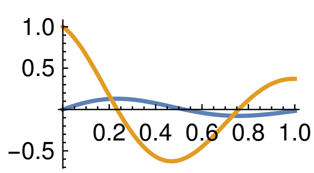
Octave.



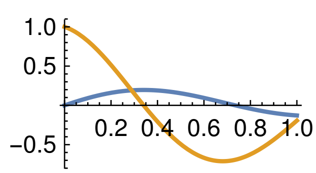
A



B



C



D

E: Sehol

Answers.

Correct answer. C