Test2, Diff.Eq., 2018.05.02. NEPTUN: Name:
Signature:

1. (3+3+4 pints)
Compute $F(s)$!
a) $F(s) = \mathcal{L}(f(t)) = \mathcal{L}(\exp(-2t+7))$.
$F(s)=$

For what values of s does the improper integral defining the laplace transform exist?

b) Let $f(t) = t - 1$ and $g(t) = t$. What is their convolution $h = f * g$!

c) Let

$$\frac{d}{dt}\bar{y}(t) = \frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

How much is the Laplace transform $\bar{Y}(s)$ of the solution $\bar{y}(t)$?

4. ((1+2+2)+(2+2+1) points)
A) Let

$$\partial_t \phi(t,x) = \partial_x^2 \phi(t,x), \quad \phi(t, x+2\pi) = \phi(t,x), \quad \phi(0,x) = f(x),$$

where $f(x) = 5$ if $x \in [0,2]$, otherwise is 0.
1. Present an orthonormal basis of $L^2([-\pi,\pi], dx)$!

2. Express f by a linear combination of this basis!

3. Compute (with the help of Fourier series) $\phi(t,x)$!

B) $y'' + 4y' + 8y = 5(t+1)^2,\ y(0) = 2,\ y'(0) = 3$. How much is $Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$)

What is the partial fraction decompositionn of $Y(s)$?

What is $y(t)$?

2. ((2+2)+(4+2) points)
Which function is holomorf (complex differentiable)? Explain why!

- $f(z) = f(x+iy) = -x - iy$,

- $f(z) = f(x+iy) = x^2 + 3ixy - 4y^2$,

Finite elements, variational calculus.
Divide the $x \in [0,1]$ interval by the points $x_i = 0.2,\ 0.4$. Let $v(x)$ be a continuous and (on the subintervals) affine function whose values at the points $(x = 0,\ 0.2,\ 0.4, 1)$ are $(0, v_1, v_2, 0)$.

- Compute
$$Energy[v] = \tilde{E}(v_1, v_2) = \int_0^1 2(v')^2 - v' - (x^2-1)v\ dx.$$

- Write down the Euler-Lagrange equation for $Energy[v]$!

3. (3+4+3 points)
A) Let $f(x) = \sqrt[3]{x}$. What is f the linear approximation of f around $x_0 = 8$! Give an upper bound for $|f(8+\Delta x) - f(8) - f'(8)\Delta x|$ when $\Delta x \in [0, 0.1]$!

B) Let $y'(t) = (2 + y(t))(3+t),\ \ y(1) = 2$. Compute the second order Taylor polynom of $y(1+\Delta t)$!

C) Let $y'(t) = (y(t)+1)(t-1),\ \ y(3) = 2$. What is the prediction of Heun's method for $y(3.001)$?