1.

Finite Elements, variational formulation.

Divide the $[0,1]$ intervall to 4 subintervals by the points $x_i = 0.3,\ 0.5,\ 0.7$. Let $v(x)$ be the continuous function which is affine on the subintervals and its values at $x = 0,\ 0.3,\ 0.5,\ 0.7,\ 1$ are $0, v_1, v_2, v_3, 0$.

- Write down the EL equation for the $Energy[u]$ functional, where $Energy$ is given by the next item!
- Compute
$$Energy[v] = \int_0^1 (v')^2 + (1-x^2)v\,dx$$
approximatly or exactly!
- What is the weak formulation of the problem?

2.

Finite differences.

Find numerical equations for an approximate solution of the DE

$$u''(x) + xu(x) = x(1-x), \quad u(0) = u(1) = 0.$$

Approximate the function u by the vector $\vec{u}_i = u(i\Delta x),\ i = 1, \ldots, 4,\ \Delta x = 1/5$.

- Express $u''(x)$ by $u(x \pm \Delta x), u(x)$!
- Write down the corresponding finite difference approximation of the DE as an inhom.lin. equation for $\vec{u}$!

3.

1. Solve the $y' - 7y = f(t)$ DE!
   - Find and plot the retarded Green function G!
   - Use G to express the solution of the DE under the conditions $y(t) = f(t) = 0$ for $t \ll 0$!
   - Use G to express the solution of the DE for $t > 0$ under the initial condition $y(0) = 8$!
2. Solve the $y'' + 4y' + 8y = f(t)$ DE!
   - Find and plot the retarded Green function G!
   - Use G to express the solution of the DE under the conditions $y(t) = f(t) = 0$ for $t \ll 0$!

4.

1. Use the definition of the Laplace tr. for the computation of $F(s) = \mathcal{L}(f(t)) = \mathcal{L}(e^{-5t+7})$.
2. $F(s) = \mathcal{L}(f(t)) = \mathcal{L}(H(-5-t)e^{-5t})$ (Here H is the Heaviside function.)
3. Compute the $h = f * g$ and $h = g * f$ convolutions of $f(t) = 4t^2$ and $g(t) = 5$!
4.
   - $y'' - 4y = (t-1)^2,\ y(0) = 5,\ y'(0) = 7$. How much is $Y(s)$? ($\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$)
   - Write down the partial fraction decomposition of $Y(s)$! (Do not compute the coefficients!)
   - How much is $y(t)$?
5. Let
$$\frac{d}{dt}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{pmatrix} 1+t \\ t+t^2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}.$$
   Express $\bar{Y}(s)$!