3a. (1+2+1+2 point)
$y' = (y^2 - 1)(2 - y)$.
Find the fixed points of the DE!

Write down the linearized versions of the DE around the fixed points!

If $y(0) = 1.34$, how much are
$\lim_{x\to\infty} y(x) =$ $\lim_{x\to-\infty} y(x) =$

Plot the solution curves of the DE!

3b. (4 pont) How much is

$$\exp\left[t\begin{pmatrix}5 & 6 & 0\\ 0 & 5 & 0\\ 0 & 0 & 7\end{pmatrix}\right]$$

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(2+2+(2+4) point)
1a. $y' = e^{t^2},\ y(4) = 6$. Express $y(7)$ using definite integration

1b. Let $f(x) = \sqrt[2]{x}$. Determine the linear approximation of f around $x_0 = 8$! Find an upper bound for the error of the linear approximation, i.e. estimate $|f(8 + \Delta x) - f(8) - f'(8)\Delta x|$, if $\Delta x \in [0, 0.1]$!

1c.

$$\begin{pmatrix}y_1'\\ y_2'\end{pmatrix} = \begin{pmatrix}(y_2 - 2)(1 - y_1)\\ (y_2 - 3)(y_1 - 4).\end{pmatrix}$$

Find the fixed points of the DE!!

Write down the linearized versions of the DE around the fixed points!

2. (5+2+3 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 3y_1 \\ 4y_1 + 5y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Find the eigenvalues and eigenvectors of A !

Find the general solution of the DE!

Find the particular solution of the DE!

(5×2 pont)
What is the relation between A and the diagonal matrix D consisting of the eigenvalues?

How much is e^{xA}?

Express the paricular solution with e^{xA} !

2b) Rewrite the following DE as a first order time-independent system!

$$\frac{d^2}{dt^2} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1' - ty_2^2 \\ y_2' - t^2 y_1' - t \end{pmatrix}$$

Determine the algebraic form of $e^{-2+i\pi/3}$!