4a. (4 pont)
$y' = (y^2 + 4)y.$
Keresd meg a DE fixpontjait!
Find the fixed points of the DE!
Ird fel a fixpontok koruli linearizalt kozelito DE-t!
Write down the linearized approximation of the DE at the fixed points!
Ha $y(0) = 1.34$, mennyi
If $y(0) = 1.34$, then how much are
$\lim_{x\to\infty} y(x) =$ $\quad\quad$ $\lim_{x\to-\infty} y(x) =$
Vazold a DE megoldasgorbeit!
Plot the solution curves of the DE!
4b. (6 pont)

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} (y_2 - 2)y_1 \\ (y_1 - 4)(y_2 + 5). \end{pmatrix}$$

Keresd meg a DE fixpontjat!
Find the fixed points of the DE!
Ird fel a fixpont koruli linearizalt kozelito DE-t!
Write down the linearized approximation of the DE at the fixed points!
1a. Mi az $y'(t) = 4 + \delta(t),\ y(3) = 4$ DE megoldasa?
What is the solution of this DE?
1b. Mi az $y''(t) = 4 + \delta(t),\ \ y(3) = 4,\ y'(3) = 5$ DE megoldasa?
What is the solution of this DE?
1c1. Mi az $y''(t) = -4y(t) + \delta(t)$ DE retardalt fundamentalis megoldasa?
What is the retarded fundamental solution of $y''(t) = -4y(t) + \delta(t)$?
1c2. Mi az $y''(t) = -4y(t) + f(t),\ y(t) = f(t) = 0,$ ha $t << 0$, DE megoldasa?
What is the solution of $y''(t) = -4y(t) + f(t),\ y(t) = f(t) = 0,$ if $t << 0$, ?
2. (5+2+3 pont) Legyen

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_1 - 3y_2 \\ 3y_1 + 2y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{illetve} \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Keresd meg A sajatertekeit es sajatvektorait!
Ird fel a DE altalanos megoldasat!
Szamold ki a DE partikularis megoldasait!
2. (5+2+3 pont) Let

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_1 - 3y_2 \\ 3y_1 + 2y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{illetve} \quad \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Find the eigenvectors and eigenvalues of A !
Find the general solution of th DE!
Compute the particular solutions!
3a1. Mennyi e^{tA} ?
How much is e^{tA} ?
3a2. Mi az $\frac{d}{dt}\bar{y}(t) = A\bar{y}(t) + \bar{f}(t),\ \bar{y}(t) = \bar{f}(t) = 0,$ ha $t << 0$, DE megoldasa?
Solve the following DE:
$\frac{d}{dt}\bar{y}(t) = A\bar{y}(t) + \bar{f}(t),\ \bar{y}(t) = \bar{f}(t) = 0,$ ha $t << 0$
3b. Let

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2y_1 - 3y_2 \\ 2y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Compute e^{tA} ?
Mi az elozo DE partikularis megoldasa az $(y_1(0), y_2(0))^T = (4, 5)$ kezdeti feltetel mellett?
What is the particular solution, if $(y_1(0), y_2(0))^T = (4, 5)$?

1 Possible variations

4a

Solve 4a if y' is

$$(y-4)y, \quad y^2+4, \quad ,e^y-77, \quad 1-\frac{0.5}{1+y^2}, \quad y(1-y)(4+y)$$

4b

Solve 4b when $\frac{d}{dt}\bar{y}$ is

$$\begin{pmatrix} y_2y_1 \\ (y_1+4)(y_2+5) \end{pmatrix}, \quad \begin{pmatrix} y_2-3 \\ (y_1+4)(y_2+5) \end{pmatrix}, \quad \begin{pmatrix} y_1^2+1 \\ (y_1+4)(y_2+5) \end{pmatrix}, \quad \begin{pmatrix} e^{y_1}y_2 \\ (y_1+3)(y_2+5) \end{pmatrix}.$$

1a

Solve the following DE:

$$y'(t)=3+\delta(t+2), \ \ y(3)=7, \qquad y'(t)=3+5\delta(t-5), \ \ y(3)=7.$$

1b

Solve the following DE:

$$y''(t)=-\delta(t-2), \ \ y(0)=7, \ y(3)=7, \qquad y''(t)=-\delta(t-2), \ \ y(0)=0, \ y'(3)=0.$$

1c1+1c2

Solve these problems when the homogeneous part of the DE is

$$y''=4y, \qquad y''=-4y'-5y.$$

2+3a1+3a2

Solve these exercises when the coefficient matrix A is

$$\begin{pmatrix} -5 & 0 \\ 5 & 6 \end{pmatrix}, \quad \begin{pmatrix} -5 & 6 \\ 6 & -5 \end{pmatrix}, \quad \begin{pmatrix} -5 & 6 \\ -6 & -5 \end{pmatrix}, \quad \begin{pmatrix} -5 & 0 & 0 \\ 5 & 6 & 0 \\ 0 & 0 & 7 \end{pmatrix}.$$

3b

Solve these exercises when the coefficient matrix A is

$$\begin{pmatrix} -5 & 6 \\ 0 & -5 \end{pmatrix}, \quad \begin{pmatrix} -5 & 0 & 0 \\ 5 & -5 & 0 \\ 0 & 0 & 7 \end{pmatrix}, \quad \begin{pmatrix} -5 & 0 & 0 \\ 1 & -5 & 0 \\ 0 & 1 & -5 \end{pmatrix}$$