(1) Let us take the following mapping $\phi : \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3 & 3 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$. How much is the signed area of the image of the unit square?

A) 3, B) 6, C) 4, D) 2, E) 5

A

1

(2) Let us take the following mapping $\phi : \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 \\ z & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$. How much is z, if ϕ is not invertible?

A) 10, B) 8, C) 11, D) 9, E) 7

B

1

(3) Compute $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 0 & 3 \end{pmatrix}^{-1}$. How much is $a + 2b + 3c + 4d$?

A) $\frac{23}{12}$, B) $\frac{25}{12}$, C) $\frac{13}{12}$, D) $\frac{17}{12}$, E) $\frac{19}{12}$

C

1

(4) Compute $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 1 & 2 \end{pmatrix}^{-1}$. How much is $a + 2b + 3c + 4d$?

A) $\frac{9}{4}$, B) $\frac{5}{4}$, C) $\frac{7}{4}$, D) $\frac{11}{4}$, E) $\frac{3}{4}$

C

1

(5) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$. How much is $a + 2b + 3c + 4d$?

A) $\frac{13}{5}$, B) $\frac{11}{5}$, C) $\frac{12}{5}$, D) $\frac{14}{5}$, E) $\frac{9}{5}$

E

1

(6) One of the eigenvectors of the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. What is the corresponding eigenvalue?

A) 0, B) 2, C) -1, D) 3, E) 1

C

1

(7) One of the eigenvectors of the matrix $\begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. What is the correspondig eigenvalue?

A) 8, B) 7, C) 10, D) 6, E) 9

D

1

(8) One of the eigenvalues of the matrix $\begin{pmatrix} 4 & 4 \\ 0 & 5 \end{pmatrix}$ is 5. Compute the correspondig eigenvector $\begin{pmatrix} x \\ y \end{pmatrix}$. How much is x/y ?

A) -3, B) -2, C) 1, D) -1, E) 0

D

1

(9) Suppose that the dynamics of a two state stochastic system is generated by the matrix $\begin{pmatrix} \frac{1}{5} & \frac{9}{10} \\ \frac{4}{5} & \frac{1}{10} \end{pmatrix}$. Find the steady state probability vector $\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$. How much is p_1/p_2 ?

A) $\frac{15}{8}$, B) $\frac{7}{8}$, C) $\frac{13}{8}$, D) $\frac{9}{8}$, E) $\frac{11}{8}$

D

1

(10) Let $f(x) = 5x + 8$. Compute $f^{-1}(x) = cx + d$. How much is $2c + 3d$?

A) $-\frac{24}{5}$, B) $-\frac{26}{5}$, C) $-\frac{23}{5}$, D) $-\frac{22}{5}$, E) $-\frac{21}{5}$

D

1

(11) Let $f(x) = 2x - 3$ and $x_0 = 4$. Compute $f^{11}(x_0)$.

A) 2052, B) 2055, C) 2054, D) 2053, E) 2051

E

1