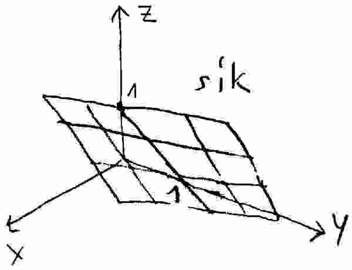
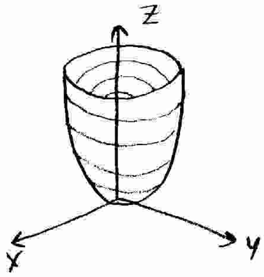
11) Rajzold le a következő függvényeket!

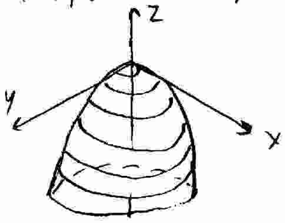
$f(x,y) = x - y + 1$



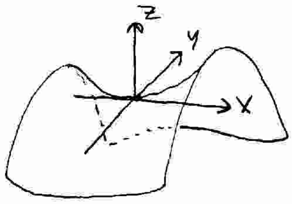
$f(x,y) = x^2 + y^2$



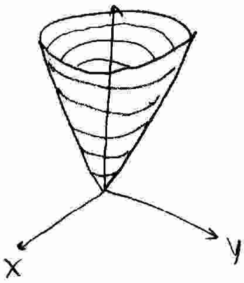
$f(x,y) = -x^2 - y^2$



$f(x,y) = x^2 - y^2$

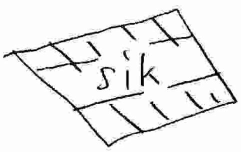


$f(x,y) = \sqrt{x^2 + y^2}$

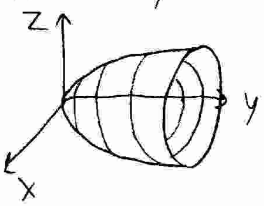


12) Rajzold le a kövzetkező felületeket!

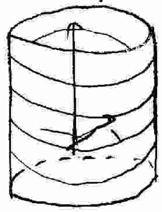
a) $x - y - z - 1 = 0$



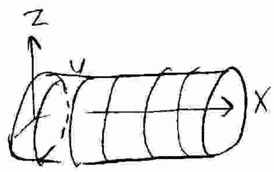
$x^2 + z^2 - y = 0$



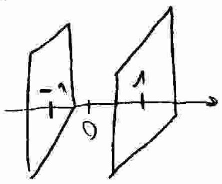
$x^2 + y^2 = 1$



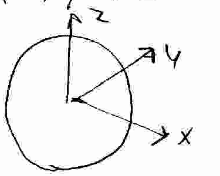
$y^2 + z^2 - 1 = 0$



$x^2 - 1 = 0$

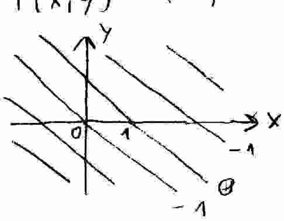


$x^2 + y^2 + z^2 = 1$

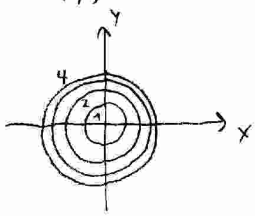


13) Rajzolj szintvonal térképeket a következő függvényekhez!

$f(x,y) = x + y - 1$

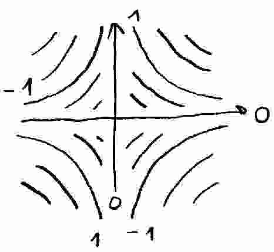


$f(x,y) = x^2 + y^2$

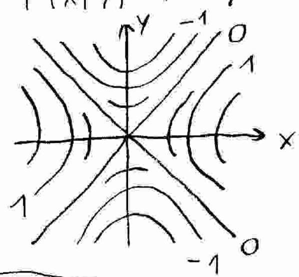


Minimum

$f(x,y) = xy$



$f(x,y) = x^2 - y^2$



Nyeregpont

12 b) Add meg a felületek paraméteres alakját!

$x-y-z-1=0$: $(x,y,z)=(s,t,s-t-1)$ $\quad s,t\in\mathbb{R}^2$

$x^2+z^2-y=0$: $(x,y,z)=(s,s^2+t^2,t)$ vagy $s,t\in\mathbb{R}^2$

$(x,y,z)=(r\cos\varphi, r^2, r\sin\varphi)$ $\quad r\in[0,\infty), \varphi\in[0,2\pi]$

$x^2+y^2=1$: $(x,y,z)=(\cos\varphi, \sin\varphi, t)$ $\quad t\in\mathbb{R}, \varphi\in[0,2\pi]$

$y^2+z^2-1=0$: $(x,y,z)=(t,\cos\varphi,\sin\varphi)$ $\quad t\in\mathbb{R}, \varphi\in[0,2\pi]$

$x^2+y^2+z^2=1$: $(x,y,z)=(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$, $\vartheta\in[0,\pi], \varphi\in[0,2\pi]$

(14) Számold ki a következő deriváltakat!

$\frac{d}{dx}(c^2 \cdot x^3) = c^2 \cdot 3x^2$, $\frac{d}{dc}(c^2 \cdot x^3) = 2c \cdot x^3$

$f(x,y) = y^2 x^3$

$\frac{\partial}{\partial x} f = f'_x = y^2 3x^2$ $\frac{\partial}{\partial y} f = f'_y = 2y \cdot x^3$

(15) Számold ki a következő deriváltakad!

a) $f(x,y) = \frac{x^2}{y}$

$f'_x = \frac{2x}{y}$, $f'_y = x^2 \cdot \left(-\frac{1}{y^2}\right)$

$f''_{xx} = \frac{2}{y}$, $f''_{yy} = x^2\left(\frac{2}{y^3}\right)$ $f''_{xy} = f''_{yx} = -\frac{2x}{y^2}$

b) $f(x,y) = \sin(x+y^2)$

$f'_x = \cos(x+y^2) \cdot 1$, $f'_y = \cos(x+y^2) \cdot 2y$

$f''_{xx} = -\sin(x+y^2)$, $f''_{yy} = -\sin(x+y^2) \cdot 2y \cdot 2y + \cos(x+y^2) \cdot 2$

$f''_{xy} = f''_{yx} = -\sin(x+y^2) \cdot 2y$

Itt $\left(h(g(x,y))\right)'_x = h'(g(x,y)) \cdot g'_x$

$\left(h(g(x,y))\right)_y = h'(g(x,y)) \cdot g'_y$

d) $f(x,y) = x^3 - xy + y^2$

$f'_x = 3x^2 - y$ $f'_y = -x + 2y$

$f''_{xx} = 6x$ $f''_{yy} = 2$ $f''_{xy} = f''_{yx} = -1$

(16) $f(x,y) = x^3 y^2$. írd fel f érintősíkjának az egyenletét $(x,y) = (5,7)$-ben!

$f'_x = 3x^2y^2$ → $f'_x(5,7) = 3 \cdot 5^2 \cdot 7^2 = 3675$

$f'_y = x^3 2y$ → $f'_y(5,7) = 5^3 \cdot 2 \cdot 7 = 1750$

$f(5,7) = 5^3 \cdot 7^2 = 6125$

Érintősík: $z(x,y) = f(x_0,y_0) + f'_x(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)(y-y_0)$

$z(x,y) = 6125 + 3675(x-5) + 1750(y-7)$

a sík érintkezési pontja: $(5, 7, 6125)$

normálvektora: $(3675, 1750, -1)$

mivel

$3675(x-5) + 1750(y-7) + (-1) \cdot (z - 6125) = 0$