Feladatok.Megoldas (Diff.egy., Tobbvalt.fuggv.)

vp

April 15, 2012

(2) $f(x,y) = x^3 - 3x + 3y - y^3$

$f'_x = 3x^2 - 3$, $f'_y = 3 - 3y^2$

$\left.\begin{array}{l} 3x^2-3=0 \to x=\pm 1 \\ 3-3y^2=0 \to y=\pm 1 \end{array}\right\}$

Lehetséges szélsőértékhelyek: $P_1(1,1)$, $P_2(1,-1)$, $P_3(-1,1)$, $P_4(-1,-1)$

$f''_{xx} = 6x$

$f''_{xy} = 0 = f''_{yx} = 0$

$f''_{yy} = -6y$

$H(f) = \begin{pmatrix} 6x & 0 \\ 0 & -6x \end{pmatrix}$

$(H(f))(P_1) = \begin{pmatrix} 6\cdot 1 & 0 \\ 0 & 6\cdot(-1) \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & -6 \end{pmatrix}$

$(H(f))(P_2) = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

$(H(f))(P_3) = \begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix}$

$(H(f))(P_4) = \begin{pmatrix} -6 & 0 \\ 0 & +6 \end{pmatrix}$

Diagonális mátrix sajátértékei: a diagonális elemek

P_1	P_2	P_3	P_4
$\lambda_1=6, \lambda_2=-6$	$\lambda_1=6, \lambda_2=6$	$\lambda_1=-6, \lambda_2=-6$	$\lambda_1=-6, \lambda_2=6$
NYEREG	MIN	MAX	NYEREG
	Azonos előjelek		vegyes előjelek

$f(x,y) = x^2 - xy - y^2 - x - y$

$f'_x = 2x - y - 1$

$f'_y = -x - 2y - 1$

$\left.\begin{array}{l} 2x-y-1=0 \\ -x-2y-1=0 \end{array}\right\} \to x = \frac{1}{5}, \; y = -\frac{3}{5}$ $\quad P\left(\frac{1}{5}, -\frac{3}{5}\right)$

$f''_{xx} = 2$

$f''_{xy} = f''_{yx} = -1$

$f''_{yy} = -2$

$H(f) = \begin{pmatrix} 2 & -1 \\ -1 & -2 \end{pmatrix}$ $\quad (H(f))(P) = \begin{pmatrix} 2 & -1 \\ -1 & -2 \end{pmatrix}$

A mátrix sajátértékei: $\begin{vmatrix} 2-\lambda & -1 \\ -1 & -2-\lambda \end{vmatrix} =$

$= (2-\lambda)(-2-\lambda) - (-1)(-1) = \lambda^2 - 5 = 0$

$\underbrace{\lambda_1 = \sqrt{5}, \; \lambda_2 = -\sqrt{5}}_{\text{NYEREGPONT}}$

③

$$
\begin{array}{rl|l}
 & f(x,y) = e^{-2x+y} & f(0,0) = 1 \\
 & f'_x = e^{-2x+y}\cdot(-2) & f'_x(0,0) = -2 \\
 & f'_y = e^{-2x+y}\cdot 1 & f'_y(0,0) = 1 \\
 & f''_{xx} = e^{-2x+y}\cdot(-2)^2 & f''_{xx}(0,0) = 4 \\
 f''_{yx} = & f''_{xy} = e^{-2x+y}\cdot(-2) & f''_{xy}(0,0) = -2 \\
 & f''_{yy} = e^{-2x+y}\cdot 1^2 & f''_{yy}(0,0) = 1
\end{array}
$$

$$f(x,y) \approx T_2(x,y) = 1 + (-2 \;\; 1)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}\,(x \;\; y)\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 - 2x + y + 2x^2 - 2xy + \frac{1}{2}y^2$$

[Vagy: $e^x = 1 + x + \frac{x^2}{2} + \ldots$

$$e^{(-2x+y)} = 1 + (-2x+y) + \frac{(-2x+y)^2}{2} + \ldots \approx 1 - 2x + y + \frac{1}{2}(4x^2 - 4xy + y^2)$$

$$
\begin{array}{rl|l}
 & f(x,y) = x e^y & f(0,0) = 0 \\
 & f'_x(x,y) = e^y & f'_x(0,0) = 1 \\
 & f'_y = x e^y & f'_y(0,0) = 0 \\
 & f''_{xx} = 0 & f''_{xx}(0,0) = 0 \\
 f''_{yx} = & F''_{xy} = e^y & f''_{xy}(0,0) = 1 \\
 & f''_{yy} = e^y 0 & f''_{yy}(0,0) = 1
\end{array}
$$

$$f(x,y) \approx T_2(x,y) = 0 + (1 \;\; 0)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}\,(x \;\; y)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= x + xy$$

vagy $f(x,y) = x e^y \approx x\left(1 + y + \frac{y^2}{2} + \ldots\right) = x + xy + \ldots$

4) a) $y' = x \to y = \frac{x^2}{2} + C$

$y(0) = 1 \to 1 = \frac{0^2}{2} + C \to C = 1 \to y(x) = \frac{x^2}{2} + 1$

---

b) $y' = xe^{-x} \to y(x) = \int x e^{-x} dx = \left| \begin{matrix} f' = e^{-x} & g = x \\ f = -e^{-x} & g' = 1 \end{matrix} \right| = -e^{-x}x - \int -e^{-x} dx$

$= -e^{-x}x - e^{-x} + C$

$y(0) = 0 \to 0 = -e^{-0} \cdot 0 - e^{-0} + C \to C = 1 \to y(x) = -e^{-x}x - e^{-x} + 1$

---

c) $y' = xe^{x^2} \to y = \int x e^{x^2} dx = \frac{1}{2} \int e^{x^2} \cdot 2x \, dx = \frac{1}{2} e^{x^2} + C$

$y(0) = 1 \to 1 = \frac{1}{2} e^{0^2} + C \to C = \frac{1}{2} \to y(x) = \frac{1}{2} e^{x^2} + \frac{1}{2}$

---

d) $y'' = 1$, $y(0) = 1$, $y'(0) = 2$

$y' = \int 1 \, dx = x + C_1$

$y = \int x + C_1 \, dx = \frac{x^2}{2} + C_1 x + C_2$ (általános megoldás)

$\left. \begin{aligned} y(0) = 1 &\to \frac{0^2}{2} + C_1 \cdot 0 + C_2 = 1 \\ y'(0) = 2 &\to 0 + C_1 = 2 \end{aligned} \right\} C_1 = 2, \; C_2 = 1$

$y(x) = \frac{x^2}{2} + 2x + 1$ (partikuláris megoldás)

---

f) $y' = -3y$, $y(0) = 1$

$\frac{dy}{dx} = -3y$

$\frac{dx}{dy} = -\frac{1}{3y}$ $\left(\text{vagy} \int \frac{dy}{-3y} = \int dx \right)$

$x = -\frac{1}{3} \ln|y| + \tilde{c}$

$-3(x - \tilde{c}) = \ln|y|$

$e^{-3(x-\tilde{c})} = |y|$

$y = \pm e^{-3(x-\tilde{c})}$ mivel a DE időfüggetlen

$= \pm e^{-3\tilde{c}} \cdot e^{-3x}$

$= C \cdot e^{-3x}$

mivel a DE lineáris

vagy: $y' + 3y = 0$

állandó együttható lin. DE

$y = e^{\lambda x} \to y' = \lambda e^{\lambda x}$

$\lambda e^{\lambda x} + 3 e^{\lambda x} = 0$

$\lambda + 3 = 0 \to \lambda = -3$

tehát $y = e^{-3x}$ megoldás

A DE lin., elsőrendű $\to$

$y_{ált} = C \cdot e^{-3x}$

$y(0) = 1 \to C \cdot e^{-3 \cdot 0} = 1 \to C = 1$

$y_{part} = 1 \cdot e^{-3x}$

④ h) $y'+3y=e^{2x}$ (Inhomogén lin. DE)

① Homogén egyenlet:

$$y'+3y \longrightarrow y=C\cdot e^{-3x}$$

② Inhomogén egyenlet

$$y=C(x)\cdot e^{-3x}=C\cdot e^{-3x} \qquad \text{(C itt már függvény)}$$

$$y'=C'e^{-3x}+C\cdot(-3)\cdot e^{-3x}$$

$$\underbrace{(C'e^{-3x}-3Ce^{-3x})}_{y'}+3\underbrace{Ce^{-3x}}_{y}=e^{2x}$$

$$C'e^{-3x}=e^{2x} \longrightarrow C'=e^{5x} \rightarrow C=\int e^{5x}dx=\frac{e^{5x}}{5}+k$$

$$y=\left(\frac{e^{5x}}{5}+k\right)\cdot e^{-3x}=\underbrace{\frac{e^{2x}}{5}}_{\text{inhomogén DE egy megoldása}}+\underbrace{k\cdot e^{-3x}}_{\text{homogén DE ált. megold.}}$$

---

i) $y'+3y=x+1$

① $y'+3y=0 \longrightarrow y=C\cdot e^{-3x}$

② $y=Ce^{-3x}$, $y'=C'e^{-3x}+C\cdot(-3)e^{-3x}$

$$(C'e^{-3x}-3Ce^{-3x})+3Ce^{-3x}=x+1$$

$$C'e^{-3x}=x+1 \longrightarrow C'=(x+1)e^{+3x}$$

$$C=\int e^{+3x}(x+1)dx=\frac{e^{+3x}}{3}x+\frac{2}{9}e^{-3x}+k$$

$$\left(\text{Mivel } \int e^{+3x}x\,dx=\left|\begin{matrix} f'=e^{+3x} & g=x \\ f=\frac{e^{+3x}}{+3} & g'=1\end{matrix}\right|=\frac{e^{+3x}}{+3}-\int\frac{e^{+3x}}{+3}\cdot 1\,dx= =+\frac{e^{+3x}}{3}x-\frac{e^{+3x}}{9}+k\right)$$

$$y=C\cdot e^{-3x}=\left(\frac{1}{3}x+\frac{2}{9}\right)+k\,e^{-3x}$$

---

j) $y'+3y=e^{2x}+x+1$

Mivel $y_1'+3y_1=e^{2x}$ egy partikuláris megoldása $y_1=\frac{e^{2x}}{5}$

és $y_2'+3y_2=x+1$ —"— $y_2=\frac{1}{3}x+\frac{2}{9}$,

ezért a DE egy part. megoldása: $y_i=y_1+y_2=\frac{e^{2x}}{5}+\left(\frac{1}{3}x+\frac{2}{9}\right)$

Ált. megold: $y=y_{hom}+y_i=K\cdot e^{-3x}+\left(\frac{e^{2x}}{5}+\frac{1}{3}x+\frac{2}{9}\right)$

(4) k) $y''+y=0$, $y'(0)=2$, $y(0)=1$

Állandó együttható homogén lin. DE

$y=e^{\lambda x}$, $y'=\lambda e^{\lambda x}$, $y''=\lambda^2 e^{\lambda x}$

$\lambda^2 e^{\lambda x}+e^{\lambda x}=0 \longrightarrow \lambda^2+1=0 \longrightarrow \lambda_1=0+1\cdot i$, $\lambda_2=0-1\cdot i$

$$y_{ált}=\tilde{C}_1\cdot e^{(0+1\cdot i)x}+\tilde{C}_2\, e^{(0-1\cdot i)x}=e^{0\cdot x}\left(C_1\cos(1\cdot x)+C_2\sin(1\cdot x)\right)$$
$$=C_1\cos x+C_2\sin x$$

$y'_{ált}=-C_1\sin x+C_2\cos x$

$y(0)=1 \longrightarrow C_1\cdot\cos 0+C_2\sin 0=1 \qquad C_1=1$

$y'(0)=2 \longrightarrow -C_1\sin 0+C_2\cos 0=2 \qquad C_2=2$

$$y_{part}=1\cdot\cos x+2\sin x$$

---

l) $y''-y=0$, $y'(0)=2$, $y(0)=1$

karakterisztikus egyenlet: $\overset{y''}{\lambda^2}-\overset{y}{1}=0 \rightarrow \lambda_1=1,\ \lambda_2=-1$

$y_{ált}=C_1 e^{1\cdot x}+C_2 e^{(-1)\cdot x}$

$y'_{ált}=C_1 e^{x}-C_2 e^{-x}$

$y(0)=1 \longrightarrow C_1\cdot e^{1\cdot 0}+C_2 e^{-1\cdot 0}=1 \rightarrow C_1+C_2=1$

$y'(0)=2 \longrightarrow C_1\cdot e^{1\cdot 0}-C_2 e^{-1\cdot 0}=2 \longrightarrow C_1-C_2=2$

$\longrightarrow C_1=3/2$, $C_2=-1/2$

$$y_{part}=\frac{3}{2}e^{x}-\frac{1}{2}e^{-x}$$

---

m) $y''+2y'+5y=0$

Kar. egy.: $\lambda^2+2\lambda+5=0 \longrightarrow \lambda_1=-1+2i$, $\lambda_2=-1-2i$

$$y_{ált}=\tilde{C}_1\, e^{(-1+2i)x}+\tilde{C}_2\, e^{(-1-2i)x}=$$
$$=e^{-x}\left(C_1\cos(2x)+C_2\sin(2x)\right)$$

$e^{ix}=\cos x+i\sin x$

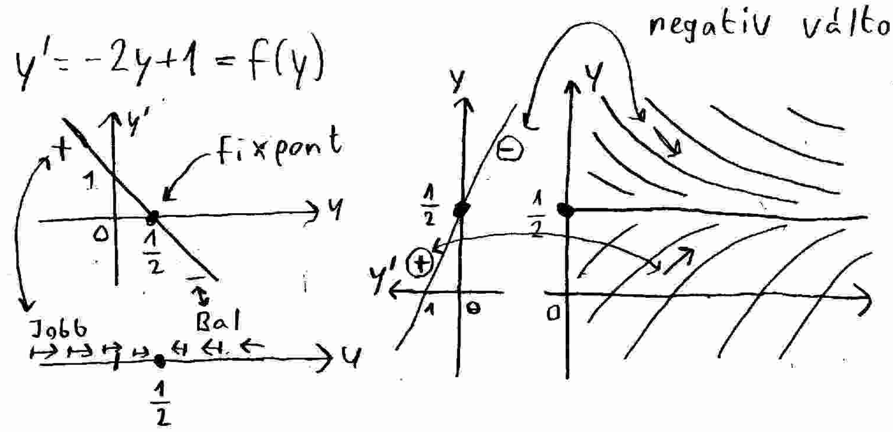
$e^{-ix}=\cos(-x)+i\sin(-x)$

$=\cos x-i\sin x$

$\longrightarrow \cos x=\dfrac{e^{ix}+e^{-ix}}{2}$

$\sin x=\dfrac{e^{ix}-e^{-ix}}{2i}$

5. $y' = -2y + 1 = f(y)$



negatív változási sebesség ↔
↔ csökkenő
megoldás görbe

pozitív vált. seb ↔
↔ növekvő meg. g.

Fixpont: $0 = y' = -2y + 1 \longrightarrow y = \frac{1}{2}$

Linearizált egyenlet a fixpont körül:

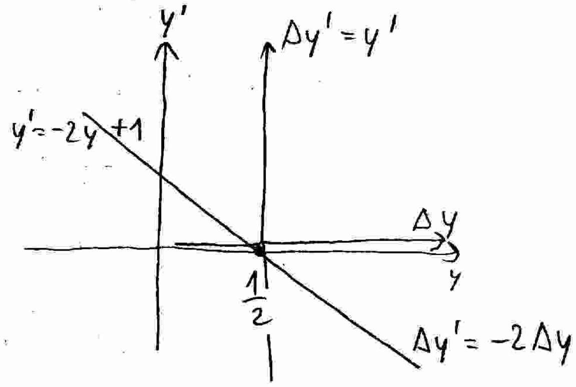
$\Delta y = y - \frac{1}{2} \longrightarrow \Delta y' = y'$

$f'(y) = \dfrac{d\,f(y)}{dy} = \dfrac{d(-2y+1)}{2} = -2$ ez a függvény kiszámolva az $y_{fix} = \frac{1}{2}$ helyen,

$\Delta y' \cong -2 \cdot \Delta y$

negatív, tehát a fixpon stabil

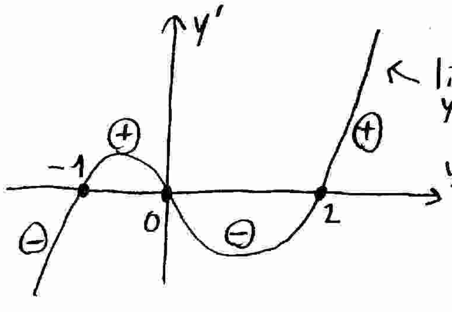
esetünkben az egyenlőség is fennáll.



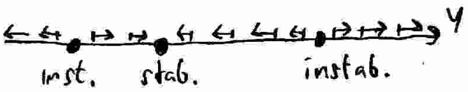
$\lim_{x\to\infty} y(x) = \frac{1}{2}$, ha $y(0) = 1{,}5$.

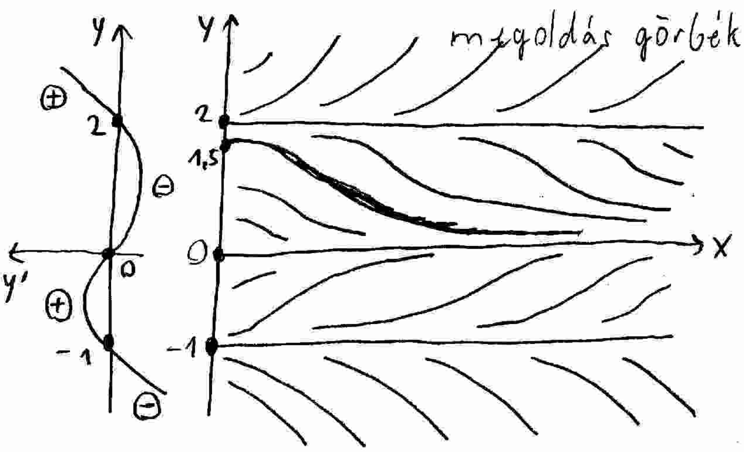
---

6. $y' = (y+1)y(y-2) = y^3 - y^2 - 2y = f(y)$



$\leftarrow \lim_{y\to\infty} y^3 - y^2 - 2y = \lim_{y\to\infty} y^3 = \infty$





megoldás görbék

$\dfrac{df(y)}{dy} = 3y^2 - 2y - 2 = f'(y)$

Fixpontok: $(y+1)\,y\,(y-2) = 0$

$y_1 = -1,$	$y_2 = 0,$	$y_3 = 2$
$f'(-1) = 3$	$f'(0) = -2$	$f'(2) = 6$ $3 \cdot 2^2 - 2 \cdot 2 - 2$
Instabil	Stabil	Instabil

Linearizált DE-k:

$\Delta y = y + 1$	$\Delta y = y - 0$	$\Delta y = y - 2$
$\Delta y' = 3\Delta y$	$\Delta y' = -2\Delta y$	$\Delta y' = 6\Delta y$

ha $y(0) = 1.5$

$\lim_{x\to\infty} y(x) = 0$

⑦ a) $$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4y_1 + 0y_2 \\ 0y_1 + 3y_3 \end{pmatrix} = \begin{pmatrix} 4y_1 \\ 3y_3 \end{pmatrix}$$

$A = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}$. sajátértékek: $\underbrace{\text{mivel A diagonális}}$, $\lambda_1 = 4$, $\lambda_2 = 3$

saját vektorok: —||— , $\bar{v}_1 = \bar{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\bar{v}_2 = \bar{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\bar{y}_{ált} = c_1 \cdot e^{4x}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \cdot e^{3x}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 e^{4x} \\ c_2 e^{3x} \end{pmatrix}$$

$$\bar{y}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} c_1 e^{4\cdot 0} \\ c_2 e^{3\cdot 0} \end{pmatrix} \rightarrow c_1 = 2, c_3 = 3$$

$$\bar{y}_{part}(x) = 2 \cdot e^{4x}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3 \cdot e^{3x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

---

b) $$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4y_1 + 2y_2 \\ 2y_1 + 4y_2 \end{pmatrix}$$

sajátértékek: $0 = \det\left(\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) = \begin{vmatrix} 4-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 - 2 \cdot 2 = \lambda^2 - 8\lambda + 12$

tehát $\lambda_1 = 6$, $\lambda_2 = 2$

saját vektorok:

$\lambda_1 = 6$: $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 6\begin{pmatrix} u \\ v \end{pmatrix} \Leftrightarrow \begin{pmatrix} \overset{4-6}{-2} & 2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow u = v$, $\bar{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 2$: $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 2\begin{pmatrix} u \\ v \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 & 2 \\ 2 & \underset{4-2}{2} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow u = -v$, $\bar{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$y_{ált} = c_1 e^{6x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{2x}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} c_1 e^{6x} + c_2 e^{2x} \\ c_1 e^{6x} - c_2 e^{2x} \end{pmatrix}$$

$$\bar{y}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} c_1 e^{6\cdot 0} + c_2 e^{2\cdot 0} \\ c_1 e^{6\cdot 0} - c_2 e^{2\cdot 0} \end{pmatrix} = \begin{pmatrix} c_1 + c_2 \\ c_1 - c_2 \end{pmatrix} \longrightarrow \begin{matrix} c_1 = 5/2 \\ c_2 = -\frac{1}{2} \end{matrix}$$

$$\bar{y}_{part} = \frac{5}{2} e^{6x}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{1}{2} e^{2x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(8) $f(x,y,z)=x+xy+xyz$

$$\nabla f=\operatorname{grad} f=\left(\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\right)=(1+y+yz,\ x+xz,\ xy)$$

$$\Delta f=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)f=\operatorname{div}(\operatorname{grad} f)=\frac{\partial}{\partial x}(1+y+yz)+\frac{\partial}{\partial y}(x+xz)+\frac{\partial}{\partial z}(xy)$$

$$=0+0+0$$

$\operatorname{rot}\operatorname{grad} f=0$ (ez automatikusan teljesül)

vagy

$$\operatorname{rot}\operatorname{grad} f=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 1+y+yz & x+xz & xy\end{vmatrix}=\bar{i}\begin{vmatrix}\frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x+xz & xy\end{vmatrix}-\bar{j}\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial z}\\ 1+y+yz & xy\end{vmatrix}$$

$$+\bar{k}\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\\ 1+y+yz & x+xz\end{vmatrix}=$$

$\frac{\partial}{\partial y}(xy)$ $\quad$ $\frac{\partial}{\partial z}(x+xz)$

$$=\bar{i}(x-x)-\bar{j}(y-y)+\bar{k}(z-z)=\bar{0}$$

---

(9) $\bar{V}(x,y,z)=(x,z,y)$.

$$\operatorname{rot}\bar{V}=\nabla\times\bar{V}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x & z & y\end{vmatrix}=\bar{i}\left(\frac{\partial}{\partial y}y-\frac{\partial}{\partial z}z\right)-\bar{j}\left(\frac{\partial}{\partial x}y-\frac{\partial}{\partial z}x\right)+\bar{k}\left(\frac{\partial}{\partial x}z-\frac{\partial}{\partial y}x\right)$$

$$=0\bar{i}+0\bar{j}+0\bar{k}=\bar{0}$$

$$\operatorname{div}\bar{V}=\frac{\partial}{\partial x}x+\frac{\partial}{\partial y}z+\frac{\partial}{\partial z}y=1+0+0=1$$

vektorok skalárszorzata

$$\int_\Gamma \bar{V}(\bar{r}(t))\,d\bar{r}(t)=\int_\Gamma \left(\bar{V}(1+t,t,2t)\right)\cdot d\bar{r}=$$

$\bar{r}(t)=(1+t,t,2t)$

$\frac{d\bar{r}}{dt}=(1,1,2)$

$d\bar{r}=(1,1,2)\,dt$

$0\le t\le 1$

$$=\int_0^1(1+t,2t,t)\cdot\overbrace{(1,1,2)\,dt}^{d\bar{r}}=\int_0^1(1+t)\cdot 1+2t\cdot 1+t\cdot 2\,dt=$$

$$=\int_0^1 1+5t\,dt=\left[t+5\frac{t^2}{2}\right]_0^1=3.5$$

$$\operatorname{rot}\bar{V}=0\longleftrightarrow\bar{V}=\operatorname{grad}\varphi$$

$$\operatorname{grad}\varphi=(\varphi'_x,\varphi'_y,\varphi'_z)=(x,z,y)\longrightarrow\quad \begin{aligned}\varphi'_x&=x\\ \varphi'_y&=z\\ \varphi'_z&=y\end{aligned}$$

(folyt.) $\rightarrow$

(9) folytatás:

$$\varphi'_x = x \longrightarrow \varphi = \frac{x^2}{2} + f(y,z) \quad \left.\begin{matrix} \\ \varphi'_y = z \end{matrix}\right\} \to \varphi = \frac{x^2}{2} + zy + f(z) \quad \left.\begin{matrix} \\ \varphi'_z = y \end{matrix}\right\} \to \begin{matrix} f(z) = \text{konst} = C \\ \varphi = \frac{x^2}{2} + yz + C \end{matrix}$$

Tehát $\overline{V} = (x, z, y) = \operatorname{grad}\left(\frac{x^2}{2} + yz + C\right)$

$$\int_\Gamma (\operatorname{grad}\varphi)(\overline{r})\,d\overline{r} = \varphi(\Gamma \text{ végpontja}) - \varphi(\Gamma \text{ kezdőpontja}),$$

tehát $\int_\Gamma \overline{V}(\overline{r})\,d\overline{r} = \varphi(1+1, 1, 2\cdot 1) - \varphi(1+0, 0, 2\cdot 0) =$

$$= \left(\frac{(1+1)^2}{2} + 1\cdot(2\cdot 1) + C\right) - \left(\frac{(1+0)^2}{2} + 0\cdot(2\cdot 0) + C\right) =$$

$$= 4 - \frac{1}{2} = 3.5$$

---

(10) Legyen $\varphi(x,t) = f(x+at)$. Ha $\varphi''_{xx} - 4\varphi''_{tt} = 0$, Mennyi a?

$$\varphi''_{xx} = \left(\left(f(x+at)\right)'_x\right)'_x = \left(f'(x+at)\right)'_x = f''(x+at)$$

$$\varphi''_{tt} = \left(\left(f(x+at)\right)'_t\right)'_t = \left(f'(x+at)\cdot \underset{(x+at)'_t}{a}\right)'_t = f''(x+at)\cdot a^2$$

tehát $f''(x+at) - 4f''(x+at)\cdot a^2 = 0 \longrightarrow a^2 = \frac{1}{4}, \; a = \pm\frac{1}{2}$

$\varphi''_{xx} - 4\varphi''_{tt} = 0$-t megoldják az $f_1(x + \frac{1}{2}t)$, $f_2(x - \frac{1}{2}t)$ függvények.

Mivel az egyenlet lineáris, így az általános megoldás:

$$\varphi(x,t) = f_1(x + \tfrac{1}{2}t) + f_2(x - \tfrac{1}{2}t)$$

— — — — — — — — — —