2012. Május 30.

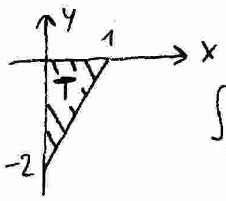
(1) Számítsd ki!

(2) $\int \frac{1}{\sqrt{4x}} + \frac{1}{\sqrt{4x^3}} + \frac{2}{3-x}\,dx =$

(3) $\int_1^\infty \frac{1}{x^7}\,dx =$

(2) $\int x^7 \ln(7x)\,dx =$

(5)



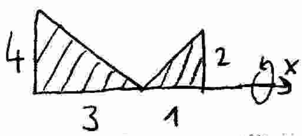
írd fel a T tartományt!

$\iint_T y-x\,dA =$

(2) Keresd meg az $f(x,y) = x^4 - 2x^2 + y^3 - 3y$ függvény

(8) szélsőértékeit, és határozd meg azok típusait!

(3) a)



Forgasd meg az ábrán látható alakzatot az x-tengely körül!

(5) ~~Számítsd ki a kapott forgástest térfogatát és felületét!~~

(5) b) $f(x,y) = \cos(x + x^2 - y)$. Mennyi $f'_x, f'_y, f''_{xx}, f''_{xy}, f''_{yy}$?

(4) $\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}}_{A}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$. Keresd meg A sajátértékeit és

(10) sajátvektorait! Írd fel a DE általános megoldását!

Írd fel a DE partikuláris megoldását, ha $\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$!

(4*) Írd fel az $f(x) = \begin{cases} 1, & \text{ha } -\pi \le x \le 0 \\ 0, & \text{ha } 0 < x < \pi \end{cases}$ függvény

(10) Fourier sorát!

Megoldókulcs, Mat. II. 12 Május 30

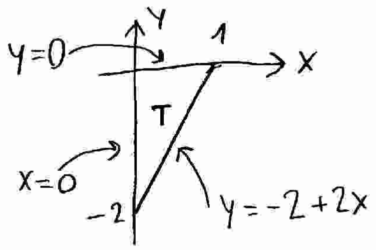
(1) $$\int \frac{1}{\sqrt{4x}} + \frac{1}{\sqrt{4x^3}} + \frac{2}{3-x}\,dx = \int (4x)^{-\frac{1}{2}} + \frac{1}{\sqrt{4}}\,x^{-\frac{3}{2}} + \frac{2}{3-x}\,dx =$$

$$= \frac{(4x)^{1/2}}{1/2\cdot 4} + \frac{1}{\sqrt{4}}\,\frac{x^{-1/2}}{1/2} + 2\,\frac{\ln|3-x|}{-1} + C \quad (2)$$

$$\int_1^\infty \frac{1}{x^7}\,dx = \lim_{R\to\infty} \int_1^R x^{-7}\,dx \;(1) = \lim_{R\to\infty}\left[\frac{x^{-6}}{-6}\right]_{x=1}^{R} (1) = \lim_{R\to\infty} \frac{R^{-6}}{-6} - \frac{1^{-6}}{-6} = \frac{1}{6} \;(1)$$

$$\int x^7 \ln(7x)\,dx = \left| \begin{array}{ll} f' = x^7 & g = \ln(7x) \\ f = \frac{x^8}{8}\;(1) & g' = \frac{1}{7x}\cdot 7 = \frac{1}{x} \end{array} \right| = \frac{x^8}{8}\ln(7x) - \int \underbrace{\frac{x^8}{8}\cdot\frac{1}{x}}_{x^7/8}\,dx =$$

$$= \frac{x^8}{8}\ln(7x) - \frac{x^8}{8\cdot 8} + C \;(1)$$



$$T = \{(x,y);\; x \geq 0,\; y \leq 0,\; y \geq -2+2x\} \;(1)$$

$$\iint_T y - x\,dA = \int_{x=0}^{1} \left(\int_{y=-2+2x}^{0} y - x\,dy \right) dx \;(1) =$$

$$= \int_{x=0}^{1} \left[\frac{y^2}{2} - xy\right]_{y=-2+2x}^{0} dx \;(1) = \int_{x=0}^{1} 0 - \left(\frac{(-2+2x)^2}{2} - x(-2+2x)\right) dx$$

$$= \int_{x=0}^{1} -2+2x\,dx \;(1) = \left[-2x + x^2\right]_0^1 = -1 - 0 = -1 \;(1)$$

(2) $f(x,y) = x^4 - 2x^2 + y^3 - 3y$

(1) $f'_x = 4x^3 - 4x$; $f'_y = 3y^2 - 3$ (1) $\left\{ \begin{array}{l} 4x^3 - 4x = 0 = 4x(x^2-1) \longrightarrow x_1 = -1,\ x_2 = 0,\ x_3 = 1 \\ 3(y^2-1) = 0 \longrightarrow y_1 = -1,\ y_2 = 1 \end{array} \right.$

$f''_{xx} = 12x^2 - 4$

$f''_{xy} = 0$ (1)

$f''_{yx} = 6y$

Lehetséges helyek:

$P_1(-1,-1)$, $P_2(0,-1)$, $P_3(1,-1)$

$P_4(-1,1)$, $P_5(0,1)$, $P_6(1,1)$ (2)

$$H(f) = \begin{pmatrix} 12x^2 - 4 & 0 \\ 0 & 6y \end{pmatrix} \quad (1)$$

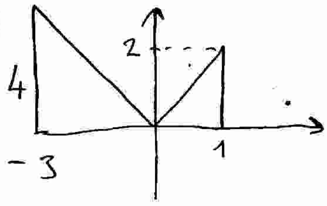
H(f) értékei:

$P_1(-1,-1)$	$P_2(0,-1)$	$P_3(1,-1)$	$P_4(-1,1)$	$P_5(0,1)$	$P_6(1,1)$
$\begin{pmatrix} 8 & 0 \\ 0 & -6 \end{pmatrix}$	$\begin{pmatrix} -4 & 0 \\ 0 & -6 \end{pmatrix}$	$\begin{pmatrix} 8 & 0 \\ 0 & -6 \end{pmatrix}$	$\begin{pmatrix} 8 & 0 \\ 0 & 6 \end{pmatrix}$	$\begin{pmatrix} -4 & 0 \\ 0 & 6 \end{pmatrix}$	$\begin{pmatrix} 8 & 0 \\ 0 & 6 \end{pmatrix}$ (2)
sajátértékek:					
$8,\ -6$	$-4,\ -6$	$8,\ -6$	$8,\ 6$	$-4,\ 6$	$8,\ 6$
$+\ -$	$-\ -$	$+\ -$	$+\ +$	$-\ +$	$+\ +$
NYEREG	MAX	NYEREG	MIN	NYEREG	MIN (2)

$$\begin{vmatrix} 8-\lambda & 0 \\ 0 & -9-\lambda \end{vmatrix} = (8-\lambda)(-9-\lambda) - 0 \cdot 0 = 0 \longrightarrow \lambda_1 = 8,\ \lambda_2 = -9$$

vagy: mivel a mátrix diagonális, így a sajátértékek a diagonális elemek.

③



$$f(x)=\begin{cases}-\frac{4}{3}x, & \text{ha } -3\le x\le 0\\ 2x, & \text{ha } 0\le x\le 1\end{cases}$$

$$V=\pi\left(\int_{-3}^{0}\left(\frac{4}{3}x\right)^2dx+\int_0^1(2x)^2dx\right)=\pi\left(\left[\frac{\left(\frac{4}{3}x\right)^3}{3\cdot 4/3}\right]_{-3}^{0}+\left[\frac{(2x)^3}{3\cdot 2}\right]_0^1\right) \quad ①$$

$$=\pi\left(16+\frac{4}{3}\right) \quad ①$$

$$F=2\pi\left(\int_{-3}^{0}-\frac{4}{3}x\cdot\sqrt{1+\left(\frac{4}{3}\right)^2}\,dx+\int_0^1 2x\cdot\sqrt{1+2^2}\,dx\right)= \quad ②$$

$$=2\pi\left(-\frac{4}{3}\cdot\sqrt{\frac{25}{9}}\left[\frac{x^2}{2}\right]_{-3}^{0}+2\sqrt{5}\cdot\left[\frac{x^2}{2}\right]_0^1\right)=$$

$$=20\pi+2\sqrt{5}\pi \quad ①$$

$$f(x,y)=\cos(x+x^2-y)$$

$$f'_x=-\sin(x+x^2-y)\cdot(1+2x) \quad ①$$

$$f'_y=-\sin(x+x^2-y)\cdot(-1) \quad ①$$

$$f''_{xx}=\left(-\cos(x+x^2-y)\cdot(1+2x)\right)\cdot(1+2x)-\sin(x+x^2-y)\cdot 2 \quad ①$$

$$f''_{xy}=-\cos(x+x^2-y)\cdot(1+2x) \quad ①$$

$$f''_{yy}=-\cos(x+x^2-y)\cdot\underbrace{(-1)^2}_{=1} \quad ①$$

4 $\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

1

sajátértékek: $0 = \begin{vmatrix} 3-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda) - 0\cdot 1$

$\longrightarrow \lambda_1 = 3, \; \lambda_2 = 2$ (mivel a mátrix trianguláris, ez automatikus)

1

sajátvektorok:

$\lambda_1 = 3 \quad \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = 3\begin{pmatrix} U \\ V \end{pmatrix}$ 1 $\quad \lambda_2 = 2 \quad \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = 2\begin{pmatrix} U \\ V \end{pmatrix}$

$\left.\begin{matrix} 3U = 3U \\ 1U + 2V = 3V \end{matrix}\right\} \to U = V \qquad \left.\begin{matrix} 3U = 2U \\ 1U + 2V = 2V \end{matrix}\right\} U = 0$

$\bar{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ 1 $\qquad$ 1 $\bar{V}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\bar{y}_{\acute{a}lt} = C_1 e^{3x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{2x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ 2

$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 \\ 2 \end{pmatrix} = C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2\begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{1}{=} \begin{pmatrix} C_1 \\ C_1 + C_2 \end{pmatrix}$

$\longrightarrow C_1 = 2, \; C_2 = 0$ 1

$\bar{y}_{part} = 2 \cdot e^{3x}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ 1