Viszga, Matematika II, 2012.05.25. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1a. ((2+3)+(1+4) pont)
$f(x) = \begin{cases} 1, & \text{ha} -1 \le x \le 0 \\ 1-x, & \text{ha } 0 \le x \le 1 \end{cases}$ Forgasd meg f-t az x-tengely korul! Szamitsd ki a kapott forgastest terfogatat es feluletet!

1

Terfogat= $\pi \int_a^b f^2(x)\,dx = \pi\left(\int_{-1}^{0} 1^2\,dx + \int_0^1 (1-x)^2\,dx\right) = \pi\left([x]_{-1}^{0} + \left[\frac{(1-x)^3}{3\cdot(-1)}\right]_0^1\right) =$

$= \pi\left((0-(-1)) + \left(0 - \frac{1^3}{-3}\right)\right) = \pi\left(1+\frac{1}{3}\right) = \frac{4}{3}\pi$ 1

1 1

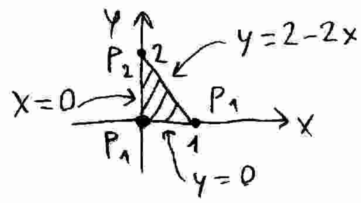
Felulet= $2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx = 2\pi\left(\int_{-1}^{0} 1\cdot\sqrt{1+0^2}\,dx + \int_0^1 (1-x)\sqrt{1+(-1)^2}\,dx\right) =$

$= 2\pi\left([x]_{-1}^{0} + \left[\frac{(1-x)^2}{2\cdot(-1)}\cdot\sqrt{2}\right]_0^1\right) = 2\pi\left((0-(-1)) + \left(0 - \frac{\sqrt{2}\cdot 1^2}{-2}\right)\right) =$

$= 2\pi\left(1+\frac{\sqrt{2}}{2}\right) = \pi(2+\sqrt{2})$ 1

1b. Egy T haromszog csucspontjai legyenek az $P_1(0,0)$, $P_2((0,2)$, $P_3(1,0)$ pontok. Add meg a T tartomanyt egyenlotlensegek segitsegevel! Szamitsd ki, hogy mennyi $\int\int_T x - y\,dA$!

$T = \{(x,y);\ x \ge 0,\ y \ge 0,\ y \le 2-2x\}$ 1



$\iint_T x-y\,dA = \int_{x=0}^{1}\left(\int_{y=0}^{2-2x} x-y\,dy\right)dx =$ 1

$= \int_{x=0}^{1}\left[xy - \frac{y^2}{2}\right]_{y=0}^{2-2x} dx = \int_{x=0}^{1}\left(x(2-2x) - \frac{(2-2x)^2}{2}\right) - 0\,dx =$

1

$= \int_{x=0}^{1} -4x^2+6x-2\,dx = \left[-4\cdot\frac{x^3}{3} + 6\cdot\frac{x^2}{2} - 2x\right]_0^1 =$

1

$= -\frac{4}{3}+3-2 = -\frac{1}{3}$ 1

2.((3+2)+(1+1+3) pont)

a) Oldd meg az $y'' - 4y = 4,\ y(0) = 6,\ y'(0) = 4$ DE-t!

Az egyenlet altalanos megoldasa:

1 $\lambda^2 - 4 = 0 \longrightarrow \lambda_1 = 2,\ \lambda_2 = -2$ 1

$y_{ált} = C_1 e^{2x} + C_2 e^{-2x}$ 1

$y'_{ált} = 2C_1 e^{2x} - 2C_2 e^{-2x}$

Az egyenlet partikularis megoldasa: 1

$$\left.\begin{array}{l} y(0)=6 \longrightarrow C_1 + C_2 = 6 \\ y'(0)=4 \longrightarrow 2C_1 - 2C_2 = 4 \end{array}\right\} \longrightarrow C_1 = 4,\ C_2 = 2$$

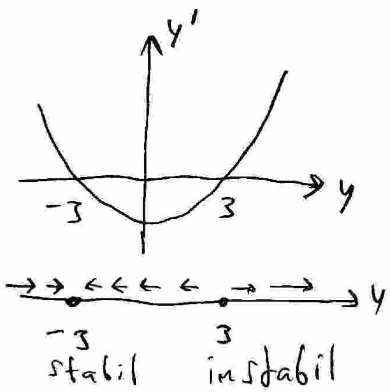
$y_{part} = 4e^{2x} + 2e^{-2x}$

1

b) Legyen $y' = -9 + y^2$.

Keresd meg a DE fixpontjait!

$-9 + y^2 = 0 \longrightarrow y_1 = 3,\ y_2 = -3$ 1

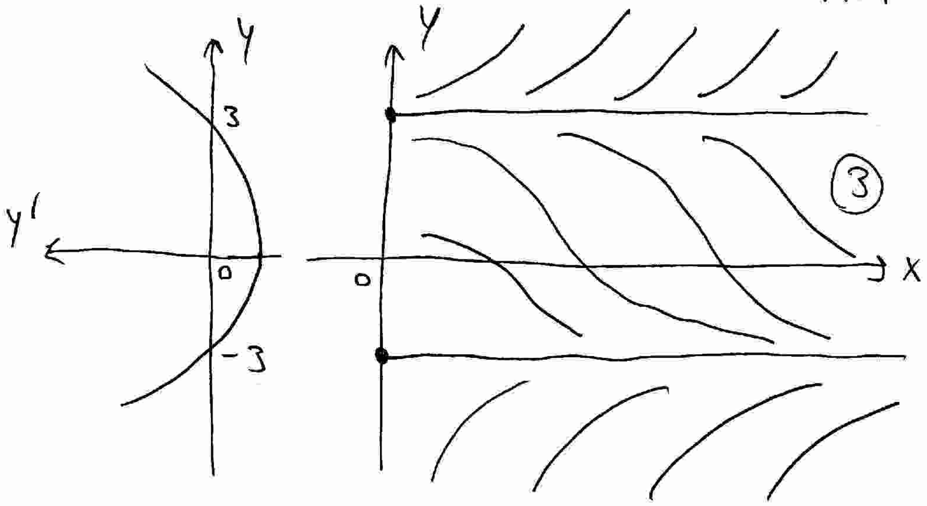


Vizsgald meg azok stabilitasat!

$3 = y_1$: insabil

$-3 = y_2$: sdabil 1

Rajzold le a DE megoldassereget!



3. (2+3+2+3 pont)

$$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad A = \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}.$$

A sajatertekei:

$$0 = \begin{vmatrix} 4-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix} = (4-\lambda)(2-\lambda) - 0 \cdot 1 \longrightarrow \lambda_1 = 4, \ \lambda_2 = 2$$

1 1

A sajatvektorai:

$\lambda_1 = 4$ $\quad \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 4 \begin{pmatrix} u \\ v \end{pmatrix}$ 1

$$\left.\begin{array}{l} 4u = 4u \\ u + 2v = 4v \end{array}\right\} u = 2v$$

$$\overline{V}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\lambda_2 = 2$ $\quad \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 2 \begin{pmatrix} u \\ v \end{pmatrix}$

$$\left.\begin{array}{l} 4u = 2u \\ u + 2v = 2v \end{array}\right\} u = 0$$

$$\overline{V}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2

A DE altalanos megoldasa:

$$\overline{y}_{ált} = C_1 e^{4x} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{2x} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2

Legyen $\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Ekkor a DE partikularis megoldasa:

$$C_1 \cdot \underset{e^{4\cdot 0}}{1} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 \cdot \overset{e^{2\cdot 0}}{1} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad 1$$

$$\begin{pmatrix} 2C_1 \\ C_1 + C_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \rightarrow \begin{array}{l} C_1 = 1 \\ C_2 = 2 \end{array} \quad 1$$

$$y_{part} = e^{4x}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + 2 \cdot e^{2x}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad 1$$

4a. Ird fel az $f(x,y) = e^{x+y+y^2}$ masodrendu T_2 Taylor polinomjat a $(0,0)$ pont korul! ((2+2+2)+(2+2) pont)

f elso es masodrendu derivaltjai:

$f'_x = e^{x+y+y^2}$ 1

$f'_y = e^{x+y+y^2} \cdot (1+2y)$

$(x+y+y^2)'_y$

$f''_{xx} = e^{x+y+y^2}$

$f''_{xy} = e^{x+y+y^2} \cdot (1+2y)$

$f''_{yy} = e^{x+y+y^2}(1+2y)^2 + e^{x+y+y^2} \cdot 2$

1

$(1+2y)'_y$

Ezek ertekei a $(0,0)$ pontban:

$f'_x(0,0) = 1$ $\quad f''_{xx}(0,0) = 1$ $\quad f''_{yy}(0,0) = 3$ $\quad f(0,0) = 1$

$f'_y(0,0) = 1$ $\quad f''_{xy}(0,0) = 1$ 2

T_2: $1 + (1, 1)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x \;\; y)\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$

$= 1 + x + y + \frac{1}{2}x^2 + xy + \frac{3}{2}y^2$ 2

4b. Szamitsd ki a kovetkezo integralokat:

$\int x\cos(3x)\,dx = \left| \begin{array}{ll} f' = \cos(3x) & g = x \\ f = \frac{\sin(3x)}{3} & g' = 1 \end{array} \right| = \frac{\sin(3x)}{3}x - \int \frac{\sin(3x)}{3} \cdot 1\,dx =$ 1

$= \frac{\sin(3x)}{3}x - \left(\frac{-\cos(3x)}{3 \cdot 3}\right) + C = \frac{\sin(3x)}{3}x + \frac{\cos(3x)}{9} + C$ 1

$\int x^2\cos(3x^3)\,dx = \frac{1}{9}\int \cos(3x^3) \cdot \underbrace{9x^2}_{(3x^3)'}\,dx = \frac{1}{9}\sin(3x^2) + C$ 2

mivel $\int \cos x\,dx = \sin x + C$