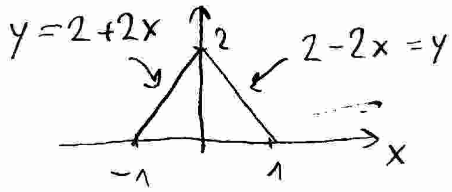
Viszga, Matematika II, 2012.05.18. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. (2+3+1+4 pont)
Egy T haromszog csucspontjai legyenek az $P_1(-1,0)$, $P_2((0,2)$, $P_3(1,0)$ pontok. Forgasd meg T-t az x-tengely korul! Szamitsd ki a kapott forgastest terfogatat es feluletet!



Terfogat=

$$\pi\int_{-1}^{1} f^2(x)\,dx = \pi\left(\int_{-1}^{0}(2+2x)^2\,dx+\int_0^1(2-2x)^2\,dx\right)= \quad (1)$$

$$=\pi\left(\left[\frac{(2+2x)^3}{3\cdot 2}\right]_{-1}^{0}+\left[\frac{(2-2x)^3}{3\cdot(-2)}\right]_{0}^{1}\right)=$$

$$=\pi\left(\left(\frac{2^3}{3\cdot 2}-0\right)+\left(0-\frac{2^3}{3\cdot(-2)}\right)\right)=\pi\cdot\frac{8}{3} \quad (1)$$

$$f(x)=\begin{cases}2+2x, & -1\le x\le 0\\ 2-2x, & 0\le x\le 1\end{cases}$$

Felulet=

$$2\pi\int_{-1}^{1} f(x)\sqrt{1+(f'(x))^2}\,dx = 2\pi\left(\int_{-1}^{0}(2+2x)\sqrt{1+2^2}\,dx+\int_0^1(2-2x)\sqrt{1+(-2)^2}\,dx\right) \quad (1)\ (1)$$

$$=2\pi\cdot\sqrt{5}\left(\left[\frac{(2+2x)^2}{2\cdot 2}\right]_{-1}^{0}+\left[\frac{(2-2x)^2}{2\cdot(-2)}\right]_{0}^{1}\right)=$$

$$=2\pi\cdot\sqrt{5}\left(\left(\frac{2^2}{4}-0\right)+\left(0-\frac{2^2}{-4}\right)\right)=4\pi\sqrt{5} \quad (1)$$

Add meg a T tartomanyt egyenlotlensegek segitsegevel! Szamitsd ki, hogy mennyi $\int\int_T x+y\,dA$!

$$T=\{(x,y);\ y\ge 0,\ y\le 2+2x,\ y\le 2-2x\}=$$

$$=\left\{(x,y);\ 0\le y\le 2,\ \frac{y-2}{2}\le x\le\frac{2-y}{2}\right\} \quad (1)$$

$$\iint_T x+y\,dA=\int_{y=0}^{2}\left(\int_{x=\frac{y-2}{2}}^{\frac{2-y}{2}} x+y\,dx\right)dy=\int_{y=0}^{2}\left[\frac{x^2}{2}+xy\right]_{x=\frac{y-2}{2}}^{\frac{2-y}{2}}dy= \quad (1)\ (1)$$

$$=\int_{y=0}^{2}\left(\frac{\left(\frac{2-y}{2}\right)^3}{3}+\left(\frac{2-y}{2}\right)y\right)-\left(\frac{\left(\frac{y-2}{2}\right)^3}{3}+\left(\frac{y-2}{2}\right)y\right)dy= \quad (1)$$

$$=\int_{y=0}^{2}-y^2+2y\,dy=\left[-\frac{y^3}{3}+y^2\right]_0^2=-\frac{8}{3}+4=\frac{4}{3} \quad (1)$$

2.((1+3+1)+(1+1+3) pont)

a) Oldd meg az $y' + y = 4,\ y(0) = 1$ inhomogen linearis DE-t az allando varialasanak a modszerevel!

A homogen egyenlet altalanos megoldasa:

$$y' + y = 0 \rightarrow y' = -y \longrightarrow y = C \cdot e^{-x} \quad 1$$

A inhomogen egyenlet altalanos megoldasa:

$$y = C \cdot e^{-x}$$
$$y' = C' e^{-x} - C e^{-x}$$
$$1 \quad (C' e^{-x} - C e^{-x}) + C e^{-x} = 4$$
$$C' e^{-x} = 4$$
$$C' = 4e^{x} \quad 1$$

$$C = \int 4e^{x} dx = 4e^{x} + k$$
$$y_{ált} = (4e^{x} + k) e^{-x} = 4 + k e^{-x}$$
1

A inhomogen egyenlet partikularis megoldasa:

$$y(0) = 1 \longrightarrow 1 = 4 + k e^{-0} \longrightarrow k = -3$$
$$\longrightarrow y_{part} = 4 - 3e^{-x} \quad 1$$

b) Legyen $y' = -y + y^2$.

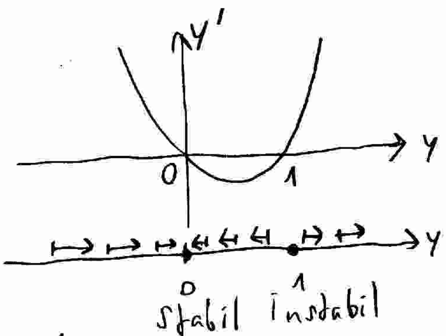
Keresd meg a DE fixpontjait!

$$-y + y^2 = y(-1 + y) = 0 \longrightarrow y_1 = 0,\ y_2 = 1 \quad 1$$

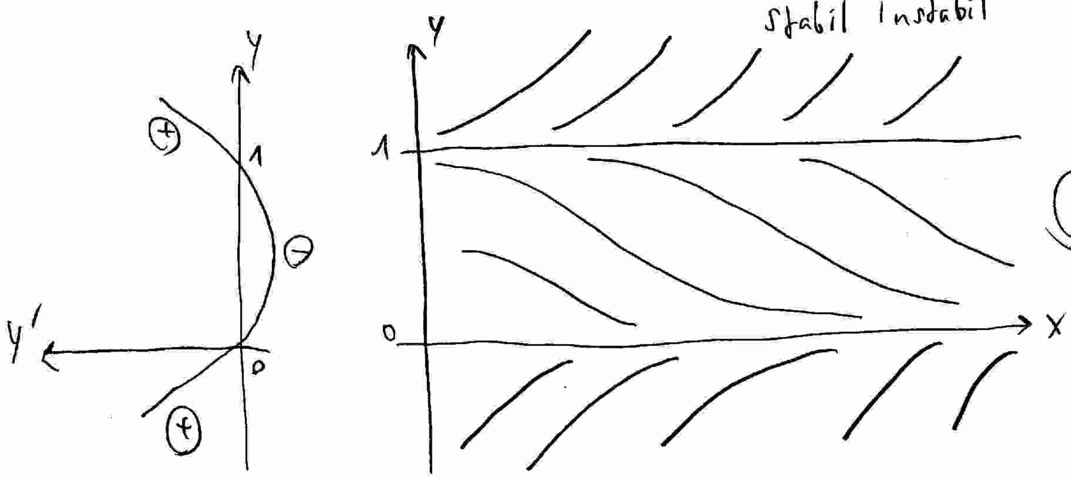
Vizsgald meg azok stabilitasat!

$y_1 = 0$ stabil

$y_2 = 1$ instabil 1



Rajzold le a DE megoldassereget!



3

3. (2+3+2+3 pont)

$$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad A = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}.$$

A sajatertekei:

$$0 = \begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 - 1^2 = \lambda^2 - 8\lambda + 15 \rightarrow \lambda_1 = 5, \quad \lambda_2 = 3 \qquad 1$$

1

A sajatvektorai:

$$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = 5\begin{pmatrix} U \\ V \end{pmatrix} \qquad 1 \qquad \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = 3\begin{pmatrix} U \\ V \end{pmatrix}$$

$$\begin{matrix} 4U + V = 5U \\ V + 4U = 5V \end{matrix} \rightarrow U = V \qquad\qquad \begin{matrix} 4U + V = 3U \\ U + 4V = 3V \end{matrix} \rightarrow U = -V$$

$$\lambda_1 = 5 \quad \overline{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad 2 \qquad \lambda_2 = 3 \quad \overline{V}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

A DE altalanos megoldasa:

$$\overline{y}_{ált} = c_1 e^{5x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3x}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

2

Legyen $\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Ekkor a DE partikularis megoldasa:

$$c_1 e^{5\cdot 0}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3\cdot 0}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad 1$$

$$\longrightarrow \begin{matrix} c_1 + c_2 = 2 \\ c_1 - c_2 = 3 \end{matrix} \longrightarrow c_1 = \frac{5}{2}, \quad c_2 = -\frac{1}{2} \qquad 1$$

$$y_{part} = \frac{5}{2} e^{5x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{1}{2} e^{3x}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad 1$$

4a. Keresd meg ay $f(x,y) = x^4 - 2x^2 + y^2 - 2y$ fuggveny szelsoertekeit! ((2+2+2)+(2+2) pont)

f elso es masodrendu derivaltjai:

$f'_x = 4x^3 - 4x \qquad f''_{xx} = 12x^2 - 4$

$f'_y = 2y - 2 \qquad f''_{yy} = 2$

1

$f''_{yx} = f''_{xy} = 0$ 1

A szelsoertekek lehetseges helyei:

1 $4x^3 - 4x = 0 \longrightarrow x_1 = -1, \quad x_2 = 0, \quad x_3 = 1$

$2y - 2 = 0 \longrightarrow y_1 = 1$

$P_1(-1, 1) \quad P_2(0, 1)$ 1 $P_3(1, 1)$

A szelsoertekek tipusai:

$H(f) = \begin{pmatrix} 12x^2-4 & 0 \\ 0 & 2 \end{pmatrix}$

1

$(H(f))(P_1) = \begin{pmatrix} 8 & 0 \\ 0 & 2 \end{pmatrix}, \quad P_2: \begin{pmatrix} -4 & 0 \\ 0 & 2 \end{pmatrix}, \quad P_3: \begin{pmatrix} 8 & 0 \\ 0 & 2 \end{pmatrix}$

sajátértékek: 8, 2   + +   MIN   -4, 2   − +   1 NYEREGP.   8, 2   + +   MIN

4b. Szamitsd ki a kovetkezo integralokat:

$\int xe^{3x}\,dx = \left| \begin{matrix} f' = e^{3x} & g = x \\ f = \frac{e^{3x}}{3} & g' = 1 \end{matrix} \right| = x\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} \cdot 1\,dx = x\frac{e^{3x}}{3} - \frac{e^{3x}}{3 \cdot 3} + C$

1   1

$\int x^2 e^{3x^3}\,dx = \frac{1}{9}\int e^{3x^3} \cdot \underbrace{9x^2}_{(3x^3)'}\,dx = \frac{1}{9} e^{3x^3} + C$

2

Jegy		Aláírás:	
	1–14: 1		1–13 NEM
	15–19: 2		14–40 IGEN
	20–24: 3	+jegy:	16–20: 2
	25–29: 4		21–25: 3
	30–40: 5		26–30: 4
			31–40: 5