Zh2, Matematika II, 2012.05.02. NEPTUN: Gyak.Vez.: 56 d6

Név: Aláírás:

1. Ird fel az $f(x,y) = e^{x^2-3y}$ fuggveny masodrendu kozelito T_2 Taylor polinomat at $(0,0)$ pont korul! (4+3+3 pont)

f elso es masodrendu derivaltjai:

$$f'_x = e^{x^2-3y} \cdot 2x \quad 1$$

$$f'_y = e^{x^2-3y} \cdot (-3) \quad 1$$

$$f''_{xx} = e^{x^2-3y} \cdot (2x)^2 + e^{x^2-3y} \cdot 2 = e^{x^2-3y} \cdot (4x^2+2) \quad 1$$

$$\left.\begin{aligned} f''_{yx} = f''_{xy} &= e^{x^2-3y} \cdot 2x \cdot (-3) \\ f''_{yy} &= e^{x^2-3y} \cdot (-3)^2 = e^{x^2-3y} \cdot 9 \end{aligned}\right\} \quad 1$$

Ezek ertekei a $(0,0)$ pontban:

f: 1 $\quad f''_{xx}$: 2

f'_x: 0 $\quad f''_{yx} = f''_{xy}$: 0 $\quad$ 3.

f'_y: -3 $\quad f''_{yy}$: 9

T_2:

$$1 + \begin{pmatrix} 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \quad 2$$

$$= 1 - 3y + x^2 + \frac{9}{2} y^2 \quad 1$$

vagy: $f = 1 + (x^2-3y) + \frac{1}{2}(x^2-3y)^2 + \ldots \; 1 - 3y + x^2 + \cancel{\frac{1}{2}x^4} - \cancel{3x^2y} + \frac{9}{2}y^2 + \ldots$

$$? \approx 1 - 3y + x^2 + \frac{9}{2} y^2$$

vagy 3

2.((1+3+1)+(1+1+3) pont)

a) Oldd meg az $y' - 2y = 5,\ y(0) = 1$ inhomogen linearis DE-t az allando varialasanak a modszerevel!

A homogen egyenlet altalanos megoldasa:

$$y' = 2y \longrightarrow y = C \cdot e^{2x} \quad (1)$$

A inhomogen egyenlet altalanos megoldasa:

$$(1)\quad \left.\begin{array}{l} y = C \cdot e^{2x} \\ y' = C' e^{2x} + C \cdot 2 e^{2x} \end{array}\right\} \longrightarrow (C' e^{2x} + 2C e^{2x}) - 2C e^{2x} = 5 \longrightarrow C' e^{2x} = 5 \longrightarrow$$

$$\longrightarrow C' = 5 \cdot e^{-2x} \longrightarrow C = \int 5 e^{-2x} dx \overset{(1)}{=} 5 \cdot \frac{e^{-2x}}{-2} + k = -\frac{5}{2} e^{-2x} + k$$

$$y = C \cdot e^{2x} = \left(-\frac{5}{2} e^{-2x} + k\right) e^{2x} = -\frac{5}{2} + k e^{2x} \quad (1)$$

A inhomogen egyenlet partikularis megoldasa:

$$y(0) = 1 \longrightarrow -\frac{5}{2} + k e^{2 \cdot 0} = 1 \longrightarrow k = \frac{7}{2}$$

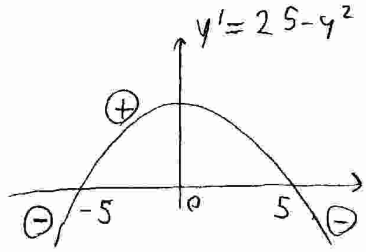
$$y_{part} = -\frac{5}{2} + \frac{7}{2} e^{2x} \quad (1)$$

b) Legyen $y' = 25 - y^2$.

Keresd meg a DE fixpontjait!

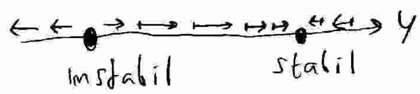
$$y' = 0 = 25 - y^2 \longrightarrow y_1 = -5 \quad , \quad y_2 = 5 \quad (1)$$

Vizsgald meg azok stabilitasat! (indokold valaszodat!)

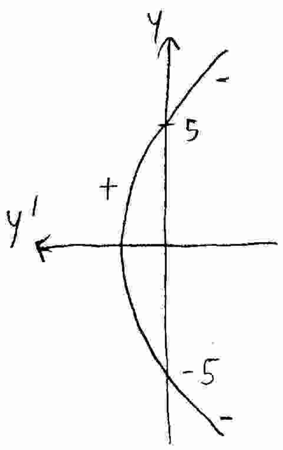


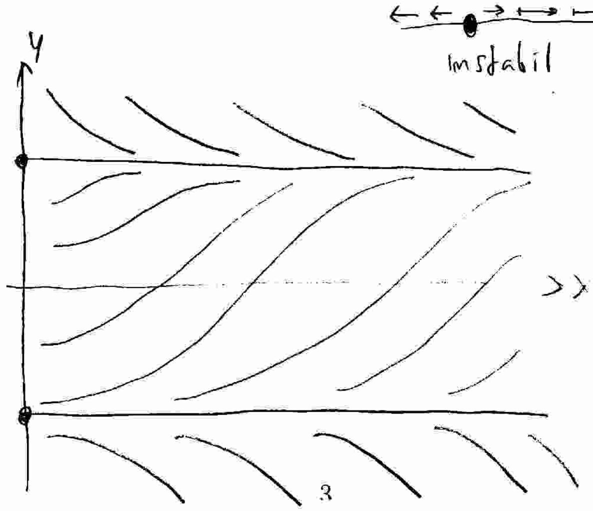
−5 instabil

5 stabil (1)



Rajzold le a DE megoldassereget!





(3)

3. (2+3+2+3 pont)

$$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \qquad A = \begin{pmatrix} 4 & 0 \\ 1 & 3 \end{pmatrix}.$$

A sajatertekei:

$$0 = \begin{vmatrix} 4-\lambda & 0 \\ 1 & 3-\lambda \end{vmatrix} = (4-\lambda)(3-\lambda) - 0\cdot 1 \longrightarrow \lambda_1 = 4 \ , \ \lambda_2 = 3$$

1 1

A sajatvektorai:

$$\begin{pmatrix} 4 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 4\begin{pmatrix} u \\ v \end{pmatrix} \qquad 1 \qquad \begin{pmatrix} 4 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 3\begin{pmatrix} u \\ v \end{pmatrix}$$

$4u = 4u \rightarrow u$ tetszőleges $\qquad 4u = 3u \rightarrow u = 0$

$1u + 3v = 4v \rightarrow u = v \qquad u + 3v = 3v \rightarrow v$ tetszőleges

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u \\ u \end{pmatrix} \quad \overline{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \overline{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

1 1

A DE altalanos megoldasa:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \cdot e^{4x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3x}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad 2$$

Legyen $\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Ekkor a DE partikularis megoldasa:

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = c_1 \cdot e^{4\cdot 0}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3\cdot 0}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad 1$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_1 + c_2 \end{pmatrix} \longrightarrow c_1 = 2 \ , \ c_2 = 1 \qquad 1$$

$$\overline{y}_{part} = 2\cdot e^{4x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1 \cdot e^{3x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

1

$$= \begin{pmatrix} 2e^{4x} \\ 2e^{4x} + e^{3x} \end{pmatrix}$$

4. Keresd meg ay $f(x,y) = x^3 - 3x + \overbrace{(y-1)^2}^{y^2-2y+1}$ fuggveny szelsoertekeit! (4+3+3 pont)

f elso es masodrendu derivaltjai:

$$f'_x = 3x^2 - 3$$
$$f'_y = 2y - 2 \quad 2$$
$$f''_{xx} = 6x$$
$$f''_{xy} = f''_{yx} = 0 \quad 2$$
$$f''_{yy} = 2$$

A szelsoertekek lehetseges helyei:

$$f'_x = 3x^2 - 3 = 0 \longrightarrow x_1 = 1, \quad x_2 = -1$$
$$1 \quad f'_y = 2y - 2 = 0 \longrightarrow y_0 = 1 \quad 1$$

$$P_1(1, 1), \qquad P_2(-1, 1)$$

1

A szelsoertekek tipusai:

$$H(f) = \begin{pmatrix} 6x & 0 \\ 0 & 2 \end{pmatrix} \quad 1$$

	sajátértékek	kritikus pont
$(H(f))(P_1) = \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$	$6, 2$ $+ \quad +$ 1	MIN 1
$(H(f))(P_2) = \begin{pmatrix} -6 & 0 \\ 0 & 2 \end{pmatrix}$	$-6, 2$ $- \quad +$	NYEREGPONT

[Mivel $\begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$ diagonális, sajátértékei 6 és 2,

vagy $\begin{vmatrix} 6-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = 0 = (6-\lambda)(2-\lambda) - 0 \cdot 0 \longrightarrow \lambda_1 = 6, \ \lambda_2 = 2$