Zh1, Matematika II, 2013.04.09. NEPTUN: Gyak.Vez.:

Név: Aláírás:

Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.
Szamold ki a kovetkezoket!

- $\int \sin(6x-6)\,dx = \dfrac{-\cos(6x-6)}{6} + C$

- $\int \frac{1}{\sqrt[7]{7x-77}} = \int (7x-77)^{-1/7} dx = \dfrac{\frac{(7x-77)^{6/7}}{6/7}}{7} + C$

- $\int_5^6 e^{2x-2}\,dx = \left[\dfrac{e^{2x-2}}{2} \right]_5^6 = \dfrac{e^{2\cdot 6-2}}{2} - \dfrac{e^{2\cdot 5-2}}{2} = \frac{1}{2}(e^{10} - e^{8})$

- $f(x,y) = (3x+2y)^{-2}$. $f'_x = -2(3x+2y)^{-3} \cdot 2$

- $f(x,y) = \sqrt{(3x+2y+6)}$. $f'_y = \left[(3x+2y+6)^{1/2} \right]'_y = \frac{1}{2}(3x+2y+6)^{-1/2} \cdot 2$

(5 × 2 pont)

$y'(x) = x^2 - 1,\ \ y(1) = 2$. Mennyi $y(3)$?

$y_{ált}(x) = \int x^2 - 1\,dx = \frac{x^3}{3} - x + C$. Ha $y(1) = 2$, akkor $\frac{1^3}{3} - 1 + C = 2$, tehát $C = 2\frac{2}{3}$.

$y_{part}(x) = \frac{x^3}{3} - x + 2\frac{2}{3}$, $y_{part}(3) = \frac{3^3}{3} - 3 + 2\frac{2}{3} = 8\frac{2}{3}$

$\int \frac{1}{\sqrt[3]{-2x}} + \sqrt[2]{(7x)} + \frac{3}{4+9x^2} + e^{-2x}\,dx = \int (-2x)^{-1/3} + (7x)^{1/2} + \frac{3}{4}\frac{1}{1+\left(\frac{3}{2}x\right)^2} + e^{-2\cdot x}\,dx =$

$$= \frac{\frac{(-2x)^{2/3}}{2/3}}{-2} + \frac{\frac{(7x)^{3/2}}{3/2}}{7} + \frac{3}{4}\frac{\operatorname{arctg}\left(\frac{3}{2}x\right)}{3/2} + \frac{e^{-2x}}{-2} + C$$

$\int \cos(6x)x\,dx = \begin{vmatrix} f' = \cos(6x) & g = x \\ f = \frac{\sin(6x)}{6} & g' = 1 \end{vmatrix} = \frac{\sin(6x)}{6}x - \int \frac{\sin(6x)}{6}\cdot 1\,dx =$

$$= \frac{\sin(6x)}{6}x - \frac{-\cos(6x)}{6\cdot 6} + C =$$

$$= \frac{\sin(6x)}{6}x + \frac{\cos(6x)}{36} + C$$

$\int \cos(9 - 5x^2)x\,dx =$

$$= -\frac{1}{10}\int \cos(9-5x^2)\cdot(-10x)\,dx = -\frac{1}{10}\int \cos(9-5x^2)\cdot(9-5x^2)'\,dx =$$

$$= -\frac{1}{10}\sin(9-5x^2) + C$$

$\int_0^\infty e^{-2x}\,dx =$

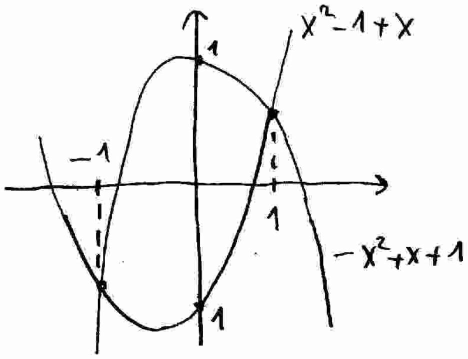
$$= \lim_{R\to\infty}\int_0^R e^{-2x}\,dx = \lim_{R\to\infty}\left[\frac{e^{-2x}}{-2}\right]_0^R = \lim_{R\to\infty}\frac{e^{-2R}}{-2} - \frac{e^{-2\cdot 0}}{-2} =$$

$$= 0 - \frac{1}{-2} = \frac{1}{2}$$

(4+3+3 pont)

Rajzold le az $y = x^2 - 1 + x$, illetve az $y = -x^2 + x + 1$ gorbeket! Szamitsd ki az altaluk kozrezart teruletet!

$$x^2 - 1 + x = -x^2 + x + 1$$
$$2x^2 = 2$$
$$x_1 = -1 \quad x_2 = 1$$



$$T = \int_{-1}^{1} (-x^2+x+1) - (x^2-1+x)\,dx =$$
$$= \int_{-1}^{1} -2x^2+2\,dx = \left[-2\frac{x^3}{3} + 2x\right]_{-1}^{1} =$$
$$= \left(-2\frac{1^3}{3} + 2\cdot 1\right) - \left(-2\frac{(-1)^3}{3} + 2\cdot(-1)\right) = \frac{8}{3}$$

Legyen $f(x,y) = x/y + 3$, $x_0 = 2, y_0 = 3$. Ird fel az $f(x,y)$ fuggveny altal leirt felulet erintosikjanak a $z = z(x,y)$ egyenletet az (x_0, y_0) pontban! Ird fel az erintosik normalvektorat!

$$f(2,3) = \frac{2}{3} + 3 = 3\frac{2}{3} = \frac{11}{3}$$
$$f'_x = \frac{1}{y} \qquad f'_x(2,3) = \frac{1}{3}$$
$$f'_y = x\cdot(-1)\cdot y^{-2} \qquad f'_y(2,3) = -\frac{2}{3^2} = -\frac{2}{9}$$
$$f(2+\Delta x, 3+\Delta y) \approx z(2+\Delta x, 3+\Delta y) = \frac{11}{3} + \frac{1}{3}\Delta x + \left(-\frac{2}{9}\right)\Delta y$$
$$f(x,y) \approx z(x,y) = \frac{11}{3} + \frac{1}{3}(x-2) + \left(-\frac{2}{9}\right)(y-3)$$
$$\frac{11}{3} + \frac{1}{3}(x-2) + \left(-\frac{2}{9}\right)(y-3) - 1\cdot z = 0,$$
$$\text{tehát} \quad \bar{n} = \left(\frac{1}{3}, -\frac{2}{9}, -1\right)$$

Keresd meg a kovetkezo gorbe ivhosszat!

$\bar{r}(t) = (4 + 2\cos t, 3 + 2\sin t),\quad t \in [\pi, 3\pi/2]$

$$\bar{v}(t) = (-2\sin t, 2\cos t)$$
$$\text{ivhossz} = \int_{\pi}^{3\pi/2} \sqrt{(-2\sin t)^2 + (2\cos t)^2}\,dt = \int_{\pi}^{3\pi/2} 2\,dt = \left(\frac{3\pi}{2} - \pi\right)\cdot 2 = \pi$$

$f(x) = e^{(3x+y^2)}y$. (6+4 pont)

$f'_x = y \cdot e^{3x+y^2} \cdot 3$

$f'_y = \left(e^{3x+y^2} \cdot 2y\right) \cdot y + e^{3x+y^2} \cdot 1 = (2y^2+1) \cdot e^{3x+y^2}$

$f''_{xx} = 3y \cdot e^{3x+y^2} \cdot 3$

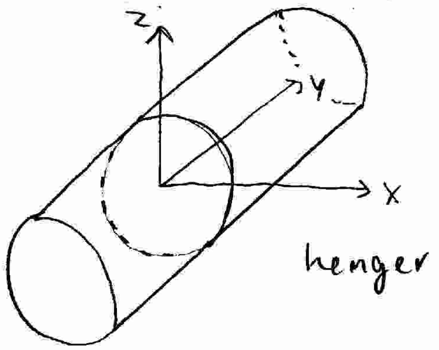
$f''_{xy} = 3 \cdot \left[e^{3x+y^2} \cdot 2y^2 + e^{3x+y^2} \right]$

$f''_{yx} = (2y^2+1)\, e^{3x+y^2} \cdot 3$

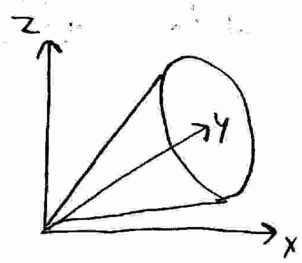
$f''_{yy} = 4y \cdot e^{3x+y^2} + (2y^2+1)\, e^{3x+y^2} \cdot 2y$

Rajzold le a kovetkezo feluleteteket!

$x^2 + z^2 = 16$

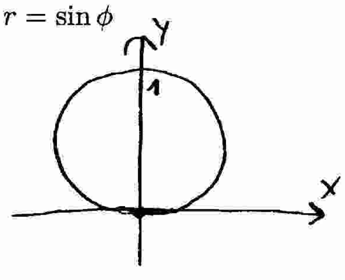


$y = \sqrt{x^2 + z^2}$

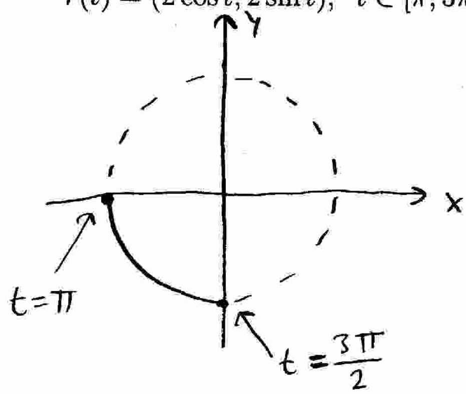


Rajzold le a kovetkezo gorbeket!

$r = \sin \phi$



$\bar{r}(t) = (2\cos t, 2\sin t), \quad t \in [\pi, 3\pi/2]$



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Név: Aláírás:

Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5 × 2 pont.
Szamold ki a kovetkezoket!

- $\int \cos(-6x+6)\,dx = \dfrac{\sin(-6x+6)}{-6} + C$

- $\int \frac{1}{\sqrt[3]{2x-22}} = \int (2x-22)^{-1/3}\,dx = \dfrac{\frac{(2x-22)^{2/3}}{2/3}}{2} + C$

- $\int_1^3 (2x-2)^2\,dx = \left[\dfrac{\frac{(2x-2)^3}{3}}{2}\right]_1^3 = \dfrac{(2\cdot3-2)^3}{6} - \dfrac{(2\cdot1-2)^3}{6} = \dfrac{64}{6} - 0 = 10\frac{2}{3}$

- $f(x,y) = \ln(3x-2y)$. $f'_x = \dfrac{1}{(3x-2y)}\cdot 3$

- $f(x,y) = \sqrt[3]{(3x-2y)}$. $f'_y = \frac{1}{3}(3x-2y)^{-2/3}\cdot(-2)$

(5 × 2 pont)

$y'(x) = x - 1,\ \ y(1) = 3$. Mennyi $y(3)$?

$y_{\acute{a}lt}(x) = \int x-1\,dx = \frac{x^2}{2} - x + C$

Ha $y(1)=3$, akkor $\frac{1^2}{2} - 1 + C = 3 \Rightarrow C = 3\frac{1}{2}$

$y_{part}(x) = \frac{x^2}{2} - x + 3\frac{1}{2}$ $\qquad y_{part}(3) = \frac{3^2}{2} - 3 + 3\frac{1}{2} = 5$

$\int \frac{1}{\sqrt[3]{-2x}} + \sqrt[4]{(-x)} + \frac{5}{16+9x^2} + e^{-x}\,dx =$

$\frac{1}{\sqrt[3]{-2x}} = (-2x)^{-1/3}$ $\qquad \frac{5}{16+9x^2} = \frac{5}{16}\cdot\frac{1}{1+\left(\frac{3}{4}x\right)^2}$

$\sqrt[4]{(-x)} = (-1\cdot x)^{1/4}$

$= \frac{\frac{(-2x)^{2/3}}{2/3}}{-2} + \frac{\frac{(-1\cdot x)^{5/4}}{5/4}}{-1} + \frac{5}{16}\cdot\frac{\operatorname{arctg}\left(\frac{3}{4}x\right)}{3/4} + \frac{e^{-x}}{-1} + C$

$\int \sin(6x)x\,dx = \left| \begin{matrix} f' = \sin(6x) & g = x \\ f = \frac{-\cos(6x)}{6} & g' = 1 \end{matrix} \right| = -\frac{\cos(6x)}{6}\cdot x - \int \frac{-\cos(6x)}{6}\cdot 1\,dx$

$= \frac{-\cos(6x)\cdot x}{6} + \frac{\sin(6x)}{6\cdot 6} + C$

$\int \cos(6x^2)x\,dx = \frac{1}{12}\int \cos(6x^2)\cdot 12x\,dx = \frac{1}{12}\int \cos(6x^2)\cdot(6x^2)'\,dx =$

$= \frac{1}{12}\sin(6x^2) + C$

$\int_{-\infty}^{-1} \frac{1}{2x}\,dx = \lim_{R\to-\infty} \int_R^{-1} \frac{1}{2x}\,dx = \lim_{R\to-\infty} \frac{1}{2}\left[\ln|x|\right]_R^{-1} =$

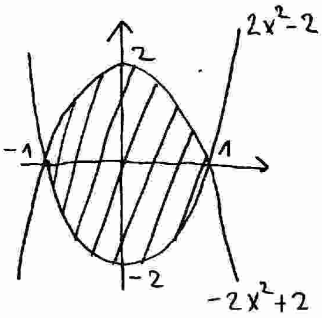
$= \lim_{R\to-\infty} \frac{1}{2}\left(\ln|-1| - \ln|R|\right) = -\infty,$

mivel $|R| \to \infty$, ha $R \to -\infty$, így

ekkor $\ln|R| \to \infty$

(4+3+3 pont)

Rajzold le az $y = 2x^2 - 2$, illetve az $y = -2x^2 + 2$ gorbeket! Szamitsd ki az altaluk kozrezart teruletet!



$$2x^2-2=-2x^2+2 \Longrightarrow x_1=-1,\ x_2=-2$$

$$T=\int_{-1}^{1}(-2x^2+2)-(2x^2-2)\,dx=$$

$$=\int_{-1}^{1}-4x^2+4\,dx=\left[-4\cdot\frac{x^3}{3}+4x\right]_{-1}^{1}=$$

$$=\left(-4\cdot\frac{1^3}{3}+4\cdot1\right)-\left(-4\cdot\frac{(-1)^3}{3}+4\cdot(-1)\right)=8\cdot\frac{2}{3}=5\frac{1}{3}$$

Legyen $f(x,y) = xy + x - y - 3$, $x_0 = 2, y_0 = 3$. Ird fel az $f(x,y)$ fuggveny altal leirt felulet erintosikjanak a $z = z(x,y)$ egyenletet az (x_0, y_0) pontban! Ird fel az erintosik normalvektorat!

$$f(2,3)=2\cdot3+2-3-3=2$$

$$f'_x=y+1\ ,\quad f'_x(2,3)=3+1=4$$

$$f'_y=x-1\ ,\quad f'_y(2,3)=2-1=1$$

$$f(2+\Delta x,3+\Delta y)\approx z(2+\Delta x,3+\Delta x)=2+4\Delta x+1\cdot\Delta y$$

$$f(x,y)\approx z(x,y)=2+4(x-2)+1\cdot(y-3)$$

$$2+4(x-2)+1\cdot(y-3)-z=0$$

$$\bar{n}=(4,1,-1)$$

Keresd meg a kovetkezo gorbe ivhosszat!

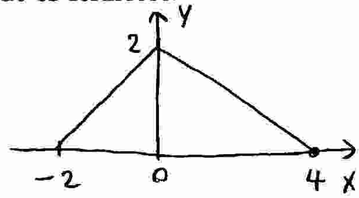
$\bar{r}(t) = (4 - 3\cos t, 3 - 3\sin t),\quad t \in [\pi, 2\pi]$

$$\bar{v}(t)=\frac{d\bar{r}}{dt}=\left(-3\cdot(-\sin t)\ ,\ 3\cdot\cos t\right)$$

$$\text{Ívhossz}=\int_{\pi}^{2\pi}\sqrt{[-3(-\sin t)]^2+[3\cdot\cos t]^2}\,dt=\int_{\pi}^{2\pi}3\,dt=(2\pi-\pi)\cdot3=3\pi$$

((2+3)+(2+1+2) pont) $P_3(4,0)$

Egy T haromszog csucspontjai legyenek az $P_1(-2,0)$, $P_2(0,2)$, ~~$P_3(0,4)$~~ pontok. Rajzold le a haromszoget! Forgasd meg T-t az x-tengely korul! Szamitsd ki a kapott forgastest terfogatat es feluletet!



$$f(x)=\begin{cases} 2+x & -2\le x\le 0 \\ 2-\frac{1}{2}x & 0\le x\le 4 \end{cases}$$

Terfogat= $\pi\int_{-2}^{4} f^2(x)\,dx = \pi\left(\int_{-2}^{0}(2+x)^2dx + \int_{0}^{4}\left(2-\frac{1}{2}x\right)^2dx\right) =$

$$=\pi\left(\left[\frac{(2+x)^3}{3}\right]_{-2}^{0} + \left[\frac{(2-\frac{1}{2}x)^3}{3\cdot(-\frac{1}{2})}\right]_{0}^{4}\right) =$$

$$\pi\left(\left(\frac{2^3}{3}-\frac{0^3}{3}\right)+\left(\frac{0}{-3/2}-\frac{2^3}{-3/2}\right)\right) = \pi\cdot\frac{24}{3} = 8\pi$$

Felulet= $2\pi\cdot\int_{-2}^{4} f(x)\cdot\sqrt{1+[f'(x)]^2}\,dx = 2\pi\left(\int_{-2}^{0}(2+x)\cdot\sqrt{1+1^2}\,dx + \int_{0}^{4}\left(2-\frac{1}{2}x\right)\cdot\sqrt{1+\left(-\frac{1}{2}\right)^2}\,dx\right)$

$$=2\pi\left(\sqrt{2}\left[\frac{(2+x)^2}{2}\right]_{-2}^{0} + \frac{\sqrt{5}}{2}\left[\frac{(2-\frac{1}{2}x)^2}{2\cdot(-\frac{1}{2})}\right]_{0}^{4}\right) = 2\pi\left(\sqrt{2}\left(\frac{2^2}{4}-\frac{0^2}{4}\right)+\frac{\sqrt{5}}{2}\left(\frac{0^2}{-1}-\frac{2^2}{-1}\right)\right)$$

$$=\pi\left(2\sqrt{2}+4\sqrt{5}\right)$$

Legyen $f(x,y) = -x^3 + x - y^2$. Hatarozd meg f kritikus pontjainak a helyet es a tipusat!

f parcialis derivaltjai:

$f'_x = -3x^2+1$ $\qquad f''_{xx} = -6x$

$f'_y = -2y$ $\qquad f''_{xy} = f''_{yx} = 0$

$\qquad f''_{yy} = -2$

A kritikus pont helye:

$$\left.\begin{aligned} -3x^2+1 &= 0 \\ -2y &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= -\frac{1}{\sqrt{3}}, +\frac{1}{\sqrt{3}} \\ y &= 0 \end{aligned} \qquad P_1\left(-\frac{1}{\sqrt{3}},0\right),\ P_2\left(+\frac{1}{\sqrt{3}},0\right)$$

A kritikus pont tipusanak a meghatarozasa::

$$H(f) = \begin{pmatrix} -6x & 0 \\ 0 & -2 \end{pmatrix} \qquad \big(H(f)\big)(P_1) = \begin{pmatrix} -6\cdot(-\frac{1}{\sqrt{3}}) & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 2\sqrt{3} & 0 \\ 0 & -2 \end{pmatrix}$$

$$\big(H(f)\big)(P_2) = \begin{pmatrix} -6\cdot(\frac{1}{\sqrt{3}}) & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} -2\sqrt{3} & 0 \\ 0 & -2 \end{pmatrix}$$

Mivel a mátrixok diagonálisak, így a sajátértékek a diagonális elemek

P_1: $\lambda_1 = 2\sqrt{3} > 0$, $\lambda_2 = -2 < 0$ $\qquad$ P_2: $\lambda_1 = -2\sqrt{3} < 0$, $\lambda_2 = -2 < 0$

vegyes sajátértékek $\qquad$ negativ sajátértékek

NYEREGPONT $\qquad$ MAXIMUM

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- $\int (6x-6)^4\,dx = \dfrac{\dfrac{(6x-6)^5}{5}}{6} + C$

- $\int \frac{1}{(7x-77)^7}dx = \int (7x-77)^{-7}dx = \dfrac{\dfrac{(7x-77)^{-6}}{-6}}{7} + C$

- $\int_2^4 \sin(2x-2)\,dx = \left[\dfrac{-\cos(2x-2)}{2}\right]_2^4 = -\dfrac{1}{2}\left(\cos \overset{2\cdot 4-2}{\overset{\downarrow}{6}} - \cos \overset{2\cdot 2-2}{\overset{\downarrow}{2}}\right) + C$

- $f(x,y) = \sin(-3x+2y)$. $f'_x = \cos(-3x+2y)\cdot(-3)$

- $f(x,y) = \sqrt[4]{(-3x-2y)}$. $f'_y = \left[(-3x-2y)^{1/4}\right]'_y = \dfrac{1}{4}(-3x-2y)^{-3/4}\cdot(-2)$

(5 × 2 pont)

$y'(x) = x^{-3}$, $y(1) = 2$. Mennyi $y(3)$?

$y_{ált}(x) = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C$. Ha $y(1)=2$, akkor $\frac{1^{-2}}{-2} + C = 2$,

tehát $C = 2\frac{1}{2}$, vagyis $y_{part}(x) = \frac{x^{-2}}{-2} + 2\frac{1}{2}$

$$y_{part}(3) = \frac{3^{-2}}{-2} + 2\frac{1}{2} = 2\frac{8}{18} = 2\frac{4}{9} = \frac{22}{9}$$

$\int \frac{1}{\sqrt[5]{2x}} + \sqrt[3]{(4x)} + \frac{4}{1+4x^2} + e^{-x}\,dx =$

$$= \int (2x)^{-1/5} + (4x)^{1/3} + 4\cdot\frac{1}{1+(2x)^2} + e^{-1\cdot x}\,dx =$$

$$= \frac{\frac{(2x)^{4/5}}{4/5}}{2} + \frac{\frac{(4x)^{4/3}}{4/3}}{4} + \frac{4\cdot \operatorname{arctg}(2x)}{-2} + \frac{e^{-x}}{-1} + C$$

$\int \sin(6x)x\,dx =$

$$= \left| \begin{matrix} f' = \sin(6x) & g = x \\ f = \frac{-\cos(6x)}{6} & g' = 1 \end{matrix} \right| = -\frac{\cos(6x)}{6}x - \int -\frac{\cos(6x)}{6}\cdot 1\,dx =$$

$$= -\frac{\cos(6x)}{6}\cdot x + \frac{\sin(6x)}{6\cdot 6} + C$$

$\int \sin(6x^2+6)x\,dx = \frac{1}{12}\int \sin(6x^2+6)\cdot 12x\,dx = \frac{1}{12}\int \sin(6x^2+6)\cdot(6x^2+6)'\,dx =$

$$= \frac{1}{12}\cdot\left(-\cos(6x^2+6)\right) + C$$

$\int_1^\infty \frac{1}{3x^2}\,dx = \lim\limits_{R\to\infty} \int_1^R \frac{1}{3x^2}\,dx = \lim\limits_{R\to\infty}\left[\frac{1}{3}\,\frac{x^{-1}}{-1}\right]_1^R =$

$$= \lim_{R\to\infty} -\frac{1}{3}\left(\underbrace{\frac{1}{R}}_{\to 0} - \frac{1}{1}\right) = \frac{1}{3}$$

(4+3+3 pont)
Rajzold le az $y = x^2$, illetve az $y = 2 - x$ gorbeket! Szamitsd ki az altaluk kozrezart teruletet!

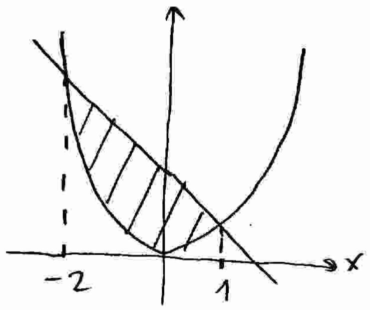
$x^2 = 2 - x$

$x^2 + x - 2 = 0$

$x_1 = -2 \quad x_1 = 1$

$$T = \int_{-2}^{1} (2-x) - x^2 dx = \left[-\frac{x^3}{3} + 2x - \frac{x^2}{2} \right]_{-2}^{1} =$$

$$= \left(-\frac{1^3}{3} + 2 \cdot 1 - \frac{1^2}{2} \right) - \left(-\frac{(-2)^3}{3} + 2 \cdot (-2) + \frac{(-2)^2}{2} \right)$$



Legyen $f(x, y) = x^2y^2 - 1$, $x_0 = 1, y_0 = 3$. Ird fel az $f(x, y)$ fuggveny altal leirt felulet erintosikjanak a $z = z(x, y)$ egyenletet az (x_0, y_0) pontban! Ird fel az erintosik normalvektorat!

$f(1,3) = 1^2 \cdot 3^2 - 1 = 8$

$f'_x = 2x \cdot y^2 \qquad f'_x(1,3) = 2 \cdot 1 \cdot 3^2 = 18$

$f'_y = x^2 \cdot 2y \qquad f'_y(1,3) = 1^2 \cdot 2 \cdot 3 = 6$

$f(1+\Delta x, 3+\Delta y) \approx z(1+\Delta x, 3+\Delta y) = 8 + 18\Delta x + 6\Delta y$

$z(x,y) = 8 + 18 \cdot (x-1) + 6 \cdot (y-3)$

$0 = 8 + 18(x-1) + 6 \cdot (y-3) - z$

$\bar{n} = (18, +6, -1)$

Keresd meg a kovetkezo gorbe ivhosszat!
$\bar{r}(t) = (4 - \cos t, 3 + \sin t), \quad t \in [\pi, 2\pi]$

$\bar{v}(t) = (\sin t, \cos t)$

$$\text{ivhossz} = \int_{\pi}^{2\pi} \sqrt{(\sin t)^2 + (\cos t)^2}\, dt = \int_{\pi}^{2\pi} 1\, dt = (2\pi - \pi) \cdot 1 = \pi$$

$f(x) = \sin(3x + y^2)y$. (6+4 pont)

$f'_x = \cos(3x+y^2)\cdot 3\cdot y$

$f'_y = \cos(3x+y^2)\cdot 2y\cdot y + \sin(3x+y^2)\cdot 1$

$f''_{xx} = -\sin(3x+y^2)\cdot 3\cdot 3\cdot y$

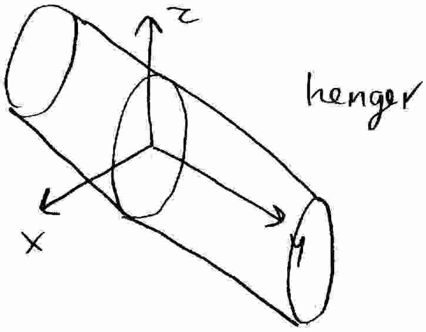
$f''_{xy} = -\sin(3x+y^2)\cdot 2y\cdot 3y + \cos(3x+y^2)\cdot 3\cdot 1$

$f''_{yx} = -\sin(3x+y^2)\cdot 3\cdot 2y^2 + \cos(3x+y^2)\cdot 1\cdot 3$

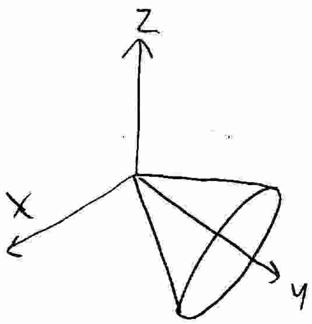
$f''_{yy} = -\sin(3x+y^2)\cdot 2y\cdot 2y^2 + \cos(3x+y^2)\cdot 2\cdot 2y^2 + \cos(3x+y^2)\cdot 2y$

Rajzold le a kovetkezo feluleteteket!

$x^2 + z^2 = 16$

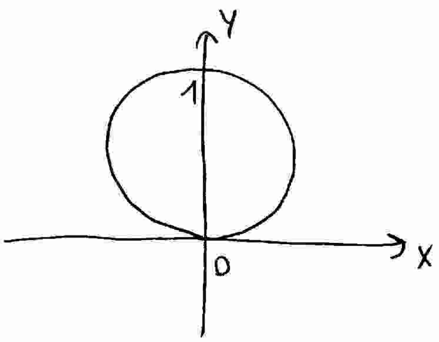


$y = \sqrt{x^2 + z^2}$



Rajzold le a kovetkezo gorbeket!

$r = \sin\phi$



$\bar{r}(t) = (2\cos t, 2\sin t),\quad t \in [\pi, 3\pi/2]$