Zh1, Matematika II, 2012.03.21. NEPTUN: Gyak.Vez.:

Név: Javítókulcs Aláírás:

1. (3 + 2 + 3 + 2 pont)

$y'(x) = x^2$, $y(1) = 2$. Mennyi $y(3)$?

$$y_{ált}(x) = \int x^2 dx = \frac{x^3}{3} + C. \ (1) \qquad y(1) = 2 \longleftrightarrow \frac{1^3}{3} + C = 2 \longrightarrow C = \frac{5}{3}$$

$$y_{part}(x) = \frac{x^3}{3} + \frac{5}{3} \ (1) \longrightarrow y(3) = \frac{3^3}{3} + \frac{5}{3} = \frac{32}{3} = 10\frac{2}{3} \ (1)$$

$\int \frac{1}{\sqrt[4]{7x}} + \sqrt[4]{(7x)} + \frac{9}{1+9x^2} + e^{-2x}\, dx =$

$$\int \frac{1}{\sqrt[4]{7}} x^{-\frac{1}{4}} + (7x)^{1/4} + 9 \cdot \frac{1}{1+(3x)^2} + e^{-2x}\, dx =$$

$$= \frac{1}{\sqrt[4]{7}} \frac{x^{3/4}}{3/4} + \frac{\frac{(7x)^{5/4}}{5/4}}{7} + 9 \cdot \frac{\operatorname{arctg}(3x)}{3} + \frac{e^{-2x}}{-2} + C$$

(1) (1)

$$\int x \sin(6x)\, dx = \left| \begin{matrix} f' = \sin 6x & g = x \\ f = \frac{-\cos 6x}{6} & g' = 1 \end{matrix} \right| = -\frac{\cos 6x}{6} \cdot x - \int -\frac{\cos 6x}{6} \cdot 1\, dx$$

(1) (1)

$$= -\frac{\cos 6x}{6} \cdot x + \frac{\sin 6x}{6 \cdot 6} + C$$

(1)

$\int x \sin(6x^2)\, dx =$

$$= \frac{1}{12} \int \sin(6x^2) \cdot 12x\, dx = \frac{1}{12} \int \sin(6x^2) \cdot (6x^2)'\, dx =$$

(1)

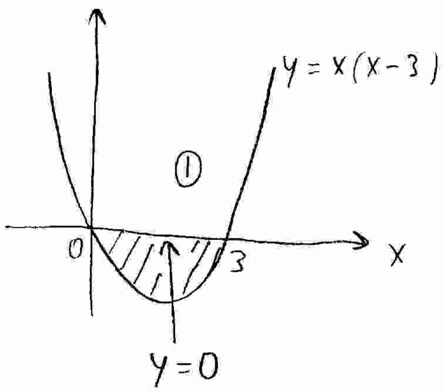
$$= \frac{1}{12} \left(-\cos(6x^2)\right) + C$$

(1)

2. (3+3+4 pont)
$\int_{-\infty}^{0} e^{2x-2}\,dx =$ $\lim_{R\to-\infty} \int_R^0 e^{2x-2}\,dx = \lim_{R\to-\infty} \left[\frac{e^{2x-2}}{2}\right]_R^0 =$ (1) (1)

$\lim_{R\to-\infty} \frac{e^{-2}}{2} - \frac{e^{2R-2}}{2} = \frac{e^{-2}}{2}$ (1), mivel $2R-2 \to -\infty$ ha $R \to -\infty$

Rajzold le az $y = 0$, illetve az $y = x(x-3)$ gorbeket! Szamitsd ki az altaluk kozrezart teruletet!



$T = \int_0^3 0 - x(x-3)\,dx = \int_0^3 -x^2+3x\,dx =$ (1)

$\left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3 = \left(-\frac{3^3}{3} + \frac{3\cdot 3^2}{2}\right) - (0) =$

$= \frac{9}{2}$ (1)

Mennyi $\int\int_D x - y^2\,dA$, ahol $D = \{(x,y);\ 0 \le x \le 3,\ 0 \le y \le 2\}$? (1)

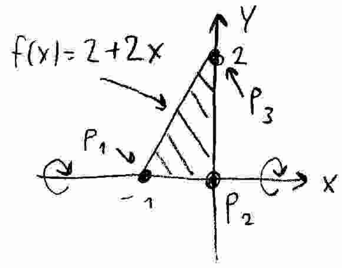
$\iint_D x - y^2\,dA = \int_{x=0}^{3} \left(\int_{y=0}^{2} x - y^2\,dy\right) dx = \int_{x=0}^{3} \left[xy - \frac{y^3}{3}\right]_{y=0}^{y=2} dx$ (1)

$= \int_{x=0}^{3} \left(x\cdot 2 - \frac{2^3}{3}\right) - \left(x\cdot 0 - \frac{0^3}{3}\right) dx = \int_{x=0}^{3} 2x - \frac{8}{3}\,dx =$ (1)

$= \left[x^2 - \frac{8}{3}x\right]_{x=0}^{3} = \left(3^2 - \frac{8}{3}\cdot 3\right) - \left(0^2 - \frac{8}{3}\cdot 0\right) = 1$ (1)

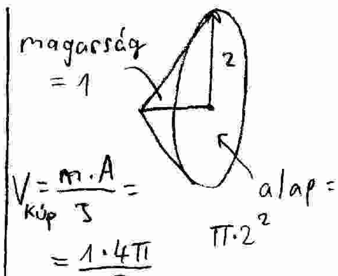
3. (2+3+1+4 pont)

Egy T haromszog csucspontjai legyenek az $P_1(-1,0)$, $P_2((0,0)$, $P_3(0,2)$ pontok. Forgasd meg T-t az x-tengely korul! Szamitsd ki a kapott forgastest terfogatat es feluletet!



Terfogat= $\pi \int_a^b f^2(x)\,dx = \pi \int_{-1}^{0} (2+2x)^2\,dx =$ ①

$= \pi \left[\frac{\frac{(2+2x)^3}{3}}{2} \right]_{-1}^{0} = \pi\left(\frac{4}{3} - 0\right) = \frac{4\pi}{3}$ ①



$V_{kúp} = \frac{m \cdot A}{3} = \frac{1 \cdot 4\pi}{3}$

Felulet= $2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx = 2\pi \int_{-1}^{0} (2+2x)\sqrt{1+2^2}\,dx =$ ① ①

$= 2\pi \cdot \sqrt{5} \left[\frac{\frac{(2+2x)^2}{2}}{2} \right]_{-1}^{0} = \pi \frac{\sqrt{5}}{2} \left[(2+2x)^2 \right]_{-1}^{0} =$

$= \pi \frac{\sqrt{5}}{2} \cdot \left(2^2 - 0^2\right) = 2\sqrt{5} \cdot \pi$ ①

Add meg a T tartomanyt egyenlotlensegek segitsegevel! Szamitsd ki, hogy mennyi $\int\int_T x + y\, dA$!

$T = \{ (x,y);\ -1 \le x \le 0,\ 0 \le y \le 2+2x \}$ ①

$\iint_T x+y\,dA = \int_{x=-1}^{0} \left(\int_{y=0}^{2+2x} x+y\,dy \right) dx = \int_{x=-1}^{0} \left[xy + \frac{y^2}{2} \right]_{y=0}^{2+2x} dx =$ ① ①

$= \int_{x=-1}^{0} \left(x(2+2x) + \frac{(2+2x)^2}{2} \right) - \left(x \cdot 0 + \frac{0^2}{2} \right) dx =$

$= \int_{x=-1}^{0} 4x^2 + 6x + 2\,dx = \left[\frac{4x^3}{3} + \frac{6x^2}{2} + 2x \right]_{x=-1}^{0} = 0 - \left(-\frac{4}{3} + \frac{6}{2} - 2 \right) = \frac{1}{3}$ ① ①

f(x,y)=

4. ~~f(x)~~ $= \sin(x + y^2)y$. (6+4 pont)

$f'_x = \cos(x+y^2)\cdot y$ (1)

$f'_y = \cos(x+y^2)\cdot 2y\cdot y + \sin(x+y^2)$ (1)

$f''_{xx} = -\sin(x+y^2)\cdot y$ (1)

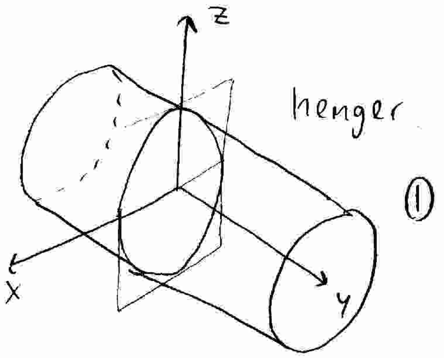
$f''_{xy} = -\sin(x+y^2)\cdot 2y\cdot y + \cos(x+y^2)$ (1)

$f''_{yx} = -\sin(x+y^2)\cdot 2y\cdot y + \cos(x+y^2)$ (1)

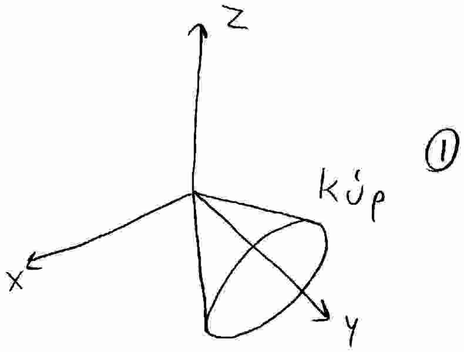
$f''_{yy} = -\sin(x+y^2)\cdot 4y^3 + \cos(x+y^2)\cdot 4y + \cos(x+y^2)\cdot 2y$ (1)

Rajzold le a kovetkezo feluleteteket!

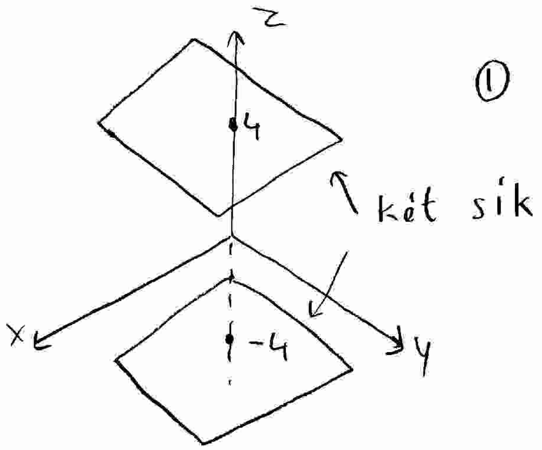
$x^2 + z^2 = 16$



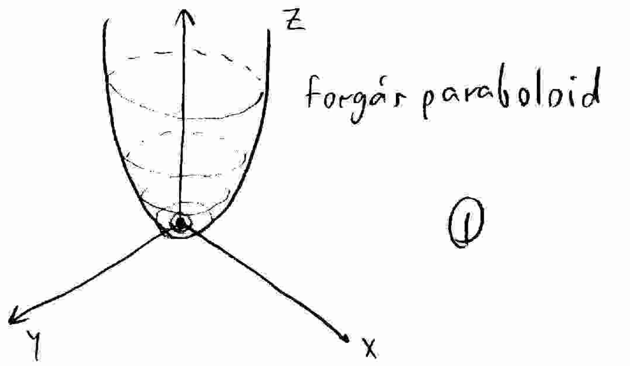
$y = \sqrt{x^2 + z^2}$



$z^2 = 16 \Rightarrow z = 4$ vagy $z = -4$



$z = x^2 + y^2$



(4a) $f(x,y) = \sin(x+y^2)\cdot y$

$$f'_x = \sin'(x+y^2)\cdot(x+y^2)'_x \cdot y = \cos(x+y^2)\cdot 1\cdot y$$

$$f'_y = (\sin(x+y^2))'_y \cdot y + \sin(x+y^2)\cdot(y)'_y =$$

$$= \cos(x+y^2)\cdot \underbrace{2y}_{(x+y^2)'_y} \cdot y + \sin(x+y^2)\cdot 1$$

$$f''_{xx} = (f'_x)'_x = (\cos(x+y^2)\,y)'_x = \cos'(x+y^2)\cdot(x+y^2)'_x\cdot y$$

$$= -\sin(x+y^2)\cdot 1 \cdot y$$

$$f''_{xy} = f''_{yx} = (f'_x)'_y = (\cos(x+y^2)\,y)'_y =$$

$$= (\cos(x+y^2))'_y\cdot y + \cos(x+y^2)\cdot(y)'_y =$$

$$= -\sin(x+y^2)\cdot(x+y^2)'_y\cdot y + \cos(x+y^2)\cdot 1 =$$

$$= -\sin(x+y^2)\cdot 2y\cdot y + \cos(x+y^2)$$

$$f''_{yy} = (f'_y)'_y = (\cos(x+y^2)\cdot 2y^2)'_y + (\sin(x+y^2))'_y =$$

$$= (\cos(x+y^2))'_y\cdot 2y^2 + \cos(x+y^2)\cdot(2y^2)'_y + (\sin(x+y^2))'_y =$$

$$= -\sin(x+y^2)\cdot \underbrace{2y}_{(x+y^2)'_y}\cdot 2y^2 + \cos(x+y^2)\cdot 4y + \underbrace{\cos}_{\sin'}(x+y^2)\cdot \underbrace{2y}_{(x+y^2)'_y}$$