PotZh1, Matematika II, 2012.05.09. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. (3 + 2 + 3 + 2 pont)

$y'(x) = (2x)^3,\ \ y(0) = 2$. Mennyi $y(3)$ ?

$$y = \int (2x)^3 dx = \frac{\frac{(2x)^4}{4}}{2} = 8 \cdot \frac{x^4}{4} + C = 2x^4 + C \quad ①$$

$$y(0) = 2 \longrightarrow 2 \cdot 0^4 + C = 2 \rightarrow C = 2 \ ① \longrightarrow y(3) = 2 \cdot 3^4 + 2 = 164 \ ①$$

$$\int \frac{1}{\sqrt[4]{7x^2}} + \sqrt[4]{(7x)} + \frac{1}{1+4x^2} + \cos(-2x)\, dx =$$

$$= \int \frac{1}{\sqrt[4]{7}} \cdot x^{-2/4} + (7x)^{1/4} + \frac{1}{1+(2x)^2} + \cos(-2x)\, dx =$$

$$= \frac{1}{\sqrt[4]{7}} \frac{x^{1/2}}{1/2} + \frac{\frac{(7x)^{5/4}}{5/4}}{7} + \frac{arctg(2x)}{2} + \frac{\sin(-2x)}{-2} + C \quad ① + ①$$

$$\int x \ln(7x)\, dx =$$

$$= \left| \begin{matrix} f' = x & g = \ln(7x) \\ f = \frac{x^2}{2} & g' = \frac{1}{7x} \cdot 7 = \frac{1}{x} \end{matrix} \right| ① = \frac{x^2}{2} \ln(7x) - \int \underbrace{\frac{x^2}{2} \cdot \frac{1}{x}}_{x/2} dx = ①$$

$$= \frac{x^2}{2} \ln(7x) - \frac{x^2}{4} + C \quad ①$$

$$\int x \ln(5x^2)\, dx = \frac{1}{10} \int \ln(5x^2) \cdot 10x \ ② = \frac{1}{10} \int \ln(5x^2) \cdot (5x^2)'\, dx =$$

$$= \frac{1}{10} \left[ (5x^2) \ln(5x^2) - (5x^2) \right] + C,$$

mivel $\int \ln x\, dx = \int 1 \cdot \ln x\, dx = \left| \begin{matrix} f' = 1 & g = \ln x \\ f = x & g' = \frac{1}{x} \end{matrix} \right| = x \ln x - \underbrace{\int x \cdot \frac{1}{x} dx}_{x + C}$.

Vagy $\int x \ln(5x^2)\, dx = \int x [\ln 5 + 2 \ln x]\, dx = \frac{x^2}{2} \ln 5 + 2 \int x \ln x\, dx =$

$$= \frac{x^2}{2} \ln 5 + 2 \left| \begin{matrix} f' = x & g = \ln x \\ f = \frac{x^2}{2} & g' = \frac{1}{x} \end{matrix} \right| = \frac{x^2}{2} \ln 5 + 2 \left( \frac{x^2}{2} \ln x - \underbrace{\int \frac{x^2}{2} \cdot \frac{1}{x} dx}_{x^2/4 + C} \right) \quad +2$$

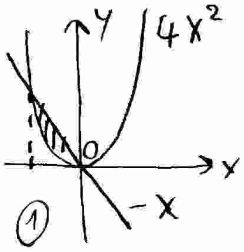
2. (3+3+4 pont)

$\int_\infty^0 (3x+3)^{-4}\,dx = \lim_{R\to\infty} \int_R^0 (3x+3)^{-4}dx = \lim_{R\to\infty}\left[\frac{(3x+3)^{-3}}{-3\cdot 3}\right]_R^0 =$ (1) (1)

$$= \frac{(3\cdot 0+3)^{-3}}{-3\cdot 3} - 0 = -\frac{1}{3^5} = -\frac{1}{243}$$ (1)

$$\int_0^\infty (3x+3)^{-4}dx = -\int_\infty^0 (3x+3)^{-4}dx = \frac{1}{243}$$

Rajzold le az $y = -x$, illetve az $y = 4x^2$ gorbeket! Szamitsd ki az altaluk kozrezart teruletet!



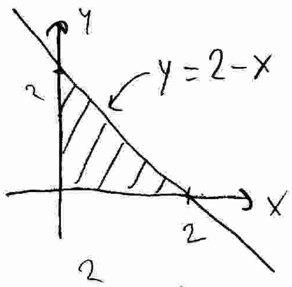

(1)

$$-x = 4x^2 \longrightarrow x_1 = -\frac{1}{4},\ x_2 = 0$$ (1)

$$T = \int_{-\frac{1}{4}}^{0} -x-4x^2\,dx = \left[-\frac{x^2}{2} - \frac{4x^3}{3}\right]_{-\frac{1}{4}}^{0} =$$ (1)

$$= 0 - \left[-\frac{(-1/4)^2}{2} - \frac{4\cdot(-1/4)^3}{3}\right]$$

Mennyi $\int\int_D 1+x-y\,dA$, ahol $D = \{(x,y);\ 0\le x\le 2,\ 0\le y\le 2,\ y\le 2-x\}$?




$$\iint_D 1+x-y\,dA = \int_{x=0}^{2}\left(\int_{y=0}^{2-x} 1+x-y\,dy\right)dx =$$ (1)

$$= \int_{x=0}^{2}\left[y+xy-\frac{y^2}{2}\right]_{y=0}^{2-x}dx = \int_{x=0}^{2}\left(2-x+x(2-x)-\frac{(2-x)^2}{2}\right)-0\,dx$$ (1)

$$= \int_{x=0}^{2} -\frac{3}{2}x^2+3x\,dx = \left[-\frac{3}{2}\frac{x^3}{3}+3\frac{x^2}{2}\right]_{x=0}^{2} =$$ (1) (1)

$$= \left(-\frac{3}{2}\frac{2^3}{3}+3\frac{2^2}{2}\right) - 0 = -4+6 = 2$$

PotZh2, Matematika II, 2012.05.09. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Keresd meg az $f(x,y) = x^2 - 2x + y^2 + 4y$ fuggveny szelsoertekeit! (4+3+3 pont)

$f$ elso es masodrendu derivaltjai:

$$f'_x = 2x-2 \qquad f''_{xx} = 2$$

$$f'_y = 2y+4 \quad (2) \qquad f''_{yy} = 2 \quad (2)$$

$$f''_{xy} = f''_{yx} = 0$$

A szelsoertekek lehetseges helyei:

$$\left.\begin{array}{l} 2x-2=0 \\ 2y+4=0 \end{array}\right\} \longrightarrow \begin{array}{l} x=1 \\ y=-2 \end{array} \qquad P_{\text{kritikus p.}} = (1,-2)$$

(1) (2)

A szelsoertekek tipusai:

$$H(f) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad (1)$$

$$(H(f))(P_{\text{kritikus p.}}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ sajátértékei: $\lambda_1 = \lambda_2 = 2$ (1), mivel a mátrix diagonális

vagy $0 = \begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 0^2 \rightarrow \lambda_{1,2} = 2$

$\lambda_1 = 2 > 0$, $\lambda_2 = 2 > 0$ MINIMUM (1)

$+$ $+$

2.((1+3+1)+(1+1+3) pont)

a) Oldd meg az $y'' - 16y = 5,\ y(0) = 1,\ y(0) = 2$ linearis DE-t!

A DE karakterisztikus egyenlete es annak gyokei:

Mivel $y(0) = 1 = y(0) = 2$, ami nem igaz, így a DE-nek nincs megoldása! (5)

A DE altalanos megoldasa:

$$y'' - 16y = 0,\quad y(0) = 1,\ y'(0) = 2$$

$$\lambda^2 - 16 = 0,\quad \lambda_1 = 4,\ \lambda_2 = -4$$

$$y_{ált} = C_1 e^{4x} + C_2 e^{-4x}$$

$$y'_{ált} = 4C_1 e^{4x} - 4C_2 e^{-4x}$$

$$\left.\begin{array}{l} y(0) = 1 \rightarrow C_1 + C_2 = 1 \\ y'(0) = 2 \rightarrow 4C_1 - 4C_2 = 2 \end{array}\right\} \quad C_1 = \frac{3}{4},\ C_2 = \frac{1}{4}$$

$$y_{párt} = \frac{3}{4} e^{4x} + \frac{1}{4} e^{-4x}$$

(5)

A DE partikularis megoldasa:

$y'' - 16y = 5$ egy megoldása: $y_p = -\frac{5}{16}$

$$y_{ált} = y_{hom} + y_p = C_1 e^{4x} + C_2 e^{-4x} - \frac{5}{16}$$

$$\left.\begin{array}{l} y(0) = 1 \rightarrow C_1 + C_2 - \frac{5}{16} = 1 \\ y'(0) = 2 \rightarrow 4C_1 - 4C_2 = 2 \end{array}\right\} \quad C_1 = \frac{29}{32},\ C_2 = \frac{13}{32} \rightarrow y_{part} = \frac{29}{32} e^{4x} + \frac{13}{32} e^{-4x} - \frac{5}{16}$$

(+8)

b) Legyen $y' = -(y+1)(y-3)$.

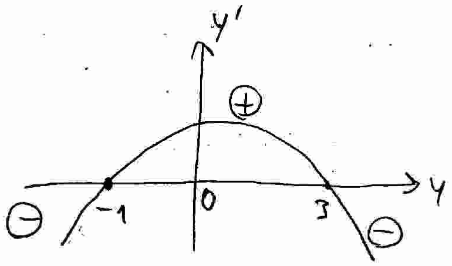
Keresd meg a DE fixpontjait!

$$-(y+1)(y-3) = 0 \rightarrow y_1 = -1,\ y_2 = 3$$

(1) (1)

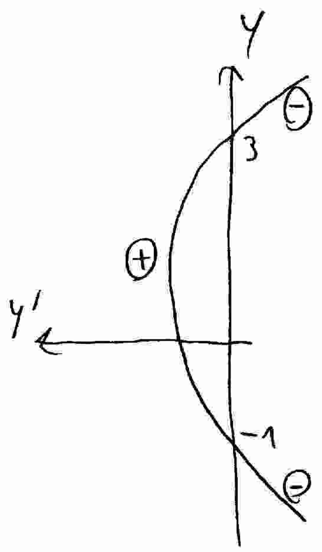
Vizsgald meg azok stabilitasat! (indokold valaszodat!)

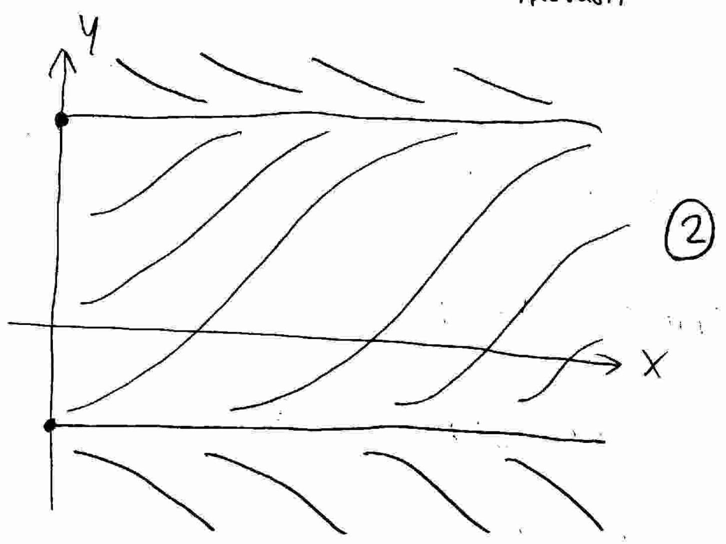
$y_1 = -1$ instabil, $y_2 = 3$ stabil (1)





Rajzold le a DE megoldassereget!




(2)