zh2 13.05.07

~~Zh1~~, Matematika II, ~~2012.03.21~~. NEPTUN: Gyak.Vez.:

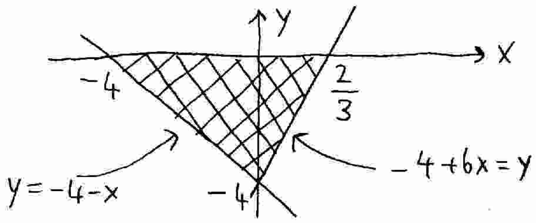
Név: Aláírás:

1.

1. Szamitsd ki a kovetkezo integralokat!
   $\int e^{-x+y}\, dx, \quad \int e^{-x+y}\, dy,$

   $$\int e^{-x+y} dx = \frac{e^{-x+y}}{-1} + c \qquad \int e^{-x+y} dy = e^{-x+y} + C$$

2. Rajzold le a kovetkezo tartomanyt! $D = \{(x, y);\ -4 + 6x \le y,\ -4 - x \le y,\ y \le 0\}$



3. Ird fel a kovetkezo $f(x, y)$ fuggveny kozelito elsorendu $T_1(x, y)$ Taylor-polinomjat az $(x, y) = (0, 0)$ pont

   $f(x,y) \approx T_1(x,y) = 2 - x + 4y$ ← az $f(x,y)$ polinom maximum elsőrendű tagjai

   Vagy: $f'_x = -1 + y + 12x$ $\quad f(0,0) = 2$
   $f'_y = 4 + x$ $\quad f'_x(0,0) = -1$ $\quad f'_y(0,0) = 4$

   $$T_1(x,y) = 2 + \begin{pmatrix} -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2 - x + 4y$$

   korul! $f(x, y) = 2 - x + 4y + xy + 6x^2$

4. Oldd meg a kovetkezo DE-t! $y'' = -y$. Harmonikus oszcillátor: $y = C_1 \cdot \cos x + C_2 \sin x$

   Vagy: $\lambda^2 = -1$, $\lambda_1 = 0 + 1 \cdot i$, $\lambda_2 = 0 - 1 \cdot i$

   $$y_{ált} = e^{0 \cdot x}(C_1 \cdot \cos(1 \cdot x) + C_2 \sin(1 \cdot x)) = C_1 \cos x + C_2 \sin x$$

5. Keresd meg a kovetkezo matrix sajatvektorait es sajatertekeit!

   $$A = \begin{pmatrix} -4 & 0 \\ 0 & 4 \end{pmatrix},$$

   Mivel A diagonális,

   $\lambda_1 = -4 \qquad \lambda_2 = 4$

   $\overline{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \overline{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

2. Legyen $y'' - 4y = 0$.

- Ird fel a DE karakterisztikus egyenletet es keresd meg a gyokeit!

$$\lambda^2 - 4 = 0 \qquad \lambda_1 = 2 \qquad \lambda_2 = -2$$

- Ird fel a DE altalanos megoldasat!

$$y_{ált} = C_1 e^{2x} + C_2 e^{-2x}$$

- Ird fel a DE partikularis megoldasat, ha $y(0) = 2$, $y'(0) = -4$!

$$y'_{ált} = C_1 \cdot 2 \cdot e^{2x} + C_2(-2)\, e^{-2x}$$

$$\left.\begin{array}{l} y(0) = 2 \leftrightarrow C_1 \cdot e^{2\cdot 0} + C_2 e^{-2\cdot 0} = C_1 + C_2 = 2 \\ y'(0) = -4 \leftrightarrow C_1 \cdot 2 \cdot e^{2\cdot 0} + C_2(-2) e^{-2\cdot 0} = 2C_1 - 2C_2 = -4 \end{array}\right\} \rightarrow \begin{array}{l} C_1 = 0 \\ C_2 = 2 \end{array}$$

$$y_{part} = 0 \cdot e^{2x} + 2 \cdot e^{-2x} = 2e^{-2x}$$

Ird fel f masodrendu kozelito Taylor-polinomjat az $(x, y) = (0, 0)$ pont korul, ha $f(x, y) = \cos(x - 7y^2 + 2xy)$!

$$f'_x = -\sin(x - 7y^2 + 2xy) \cdot (1 + 2y)$$

$$f'_y = -\sin(x - 7y^2 + 2xy) \cdot (-14y + 2x)$$

$$f''_{xx} = -\cos(x - 7y^2 + 2xy) \cdot (1 + 2y)^2$$

$$f''_{xy} = -\cos(x - 7y^2 + 2xy) \cdot (-14y + 2x)(1 + 2y) - \sin(x - 7y^2 + 2xy) \cdot 2$$

$$f''_{yy} = -\cos(x - 7y^2 + 2xy) \cdot (-14y + 2x)^2 - \sin(x - 7y^2 + 2xy) \cdot (-14)$$

Ezek értéke, ha $x = y = 0$:

$f \to 1$

$f'_x \to 0$

$f'_y \to 0$

$f''_{xx} \to -1$

$f''_{xy} \to 0$

$f''_{yy} \to 0$

$$f(x,y) \approx T_2(x,y) = 1 + (0\ 0)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x\ y)\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= 1 + 0x + 0y + \frac{1}{2}\left(-1 \cdot x^2 + 2 \cdot 0 \cdot xy + 0 \cdot y^2\right) =$$

$$= 1 - \frac{1}{2}x^2$$

3. Legyen $y' = f(y) = (y+1)(y-3)(y-5)$.

- Keresd meg a DE fixpontjait!

$$y' = f(y) = (y+1)(y-3)(y-5) = 0$$

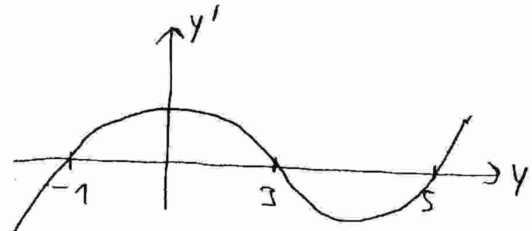
$$y_1 = -1 \qquad y_2 = 3 \qquad y_3 = 5$$

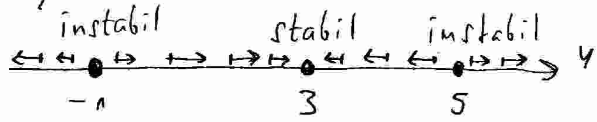
- Vizsgald meg azok stabilitasat! (indokold valaszodat!)

-1 : instabil

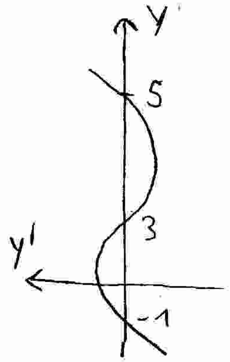
3 : stabil

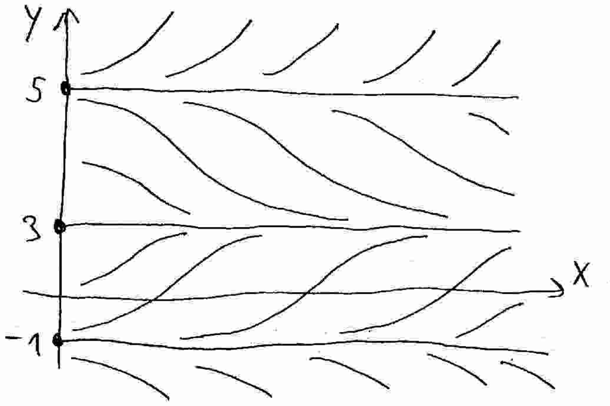
5 : instabil





- Rajzold le a DE megoldassereget!





Legyen $y' = -(y+1)$.

- Keresd meg a DE altalanos megoldasat!

$$\frac{dy}{dx} = -(y+1)$$

$$-\frac{dy}{y+1} = dx$$

$$-\int \frac{dy}{y+1} = \int dx$$

$$-\ln|y+1| = x + c$$

$$|y+1| = e^{-x-c}$$

$$y+1 = \pm e^{-x-c}$$

$$y = -1 \pm e^{-x-c}$$

általános megoldás

$$y(1) = 3 \longleftrightarrow 3 = -1 + e^{-1-c}$$

$$c = -1 - \ln 4$$

$$y = -1 + e^{-x-(-1-\ln 4)} =$$

$$= -1 + 4e^{-x+1}$$

partikuláris megoldás

- Keresd meg a DE partikularis megoldasat, ha $y(1) = 3$!

4. Szamold ki a kovetkezo kettos integralt!

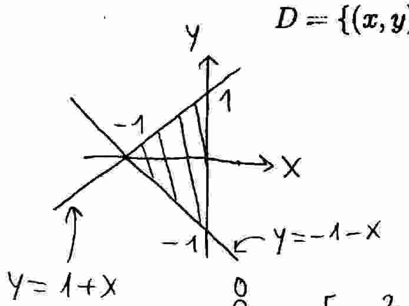
$$\int_{y=0}^{2}\int_{x=3}^{4} x^2 2y \; dxdy =$$

$$= \int_{y=0}^{2} \left[\frac{x^3}{3}\cdot 2y\right]_{x=3}^{4} dy = \int_{y=0}^{2} \left(\frac{4^3}{3}\cdot 2y\right) - \left(\frac{3^3}{3}\cdot 2y\right) dy = \int_{y=0}^{2} \frac{55}{3}\cdot 2y\, dy$$

$$= \left[\frac{55}{3}\, y^2\right]_{y=0}^{2} = \frac{55}{3}\cdot 2^2 - \frac{55}{3}\cdot 0^2 = \frac{220}{3}$$

Szamold ki a kovetkezo kettos integralokat es rajzold le a D integralasi tartomanyt!

$$D = \{(x,y);\; x \leq 0, y \leq 1+x, y \geq -1-x\}, \qquad \iint_D x^2 + y^2 \, dA$$



$$\iint_D x^2+y^2\, dA = \int_{x=-1}^{0} \left(\int_{y=-1-x}^{1+x} x^2+y^2 \, dy \right) dx =$$

$$= \int_{x=-1}^{0} \left[x^2 y + \frac{y^3}{3} \right]_{y=-1-x}^{1+x} dx = \int_{x=-1}^{0} \left(x^2(1+x) + \frac{(1+x)^3}{3}\right) - \left(x^2(-1-x) + \frac{(-1-x)^3}{3}\right) dx$$

$$= \int_{x=-1}^{0} 2\cdot\left(x^2(1+x) + \frac{(1+x)^3}{3}\right) dx = \int_{x=-1}^{0} 2\tfrac{2}{3}\cdot x^3 + 4x^2 + 2x + \frac{2}{3}\, dx =$$

$$= \left[2\tfrac{2}{3}\cdot\frac{x^4}{4} + \frac{4}{3}x^3 + x^2 + \frac{2}{3}x\right]_{x=-1}^{0} = 0 - \left(2\tfrac{2}{3}\cdot\frac{1}{4} + \frac{4}{3}\cdot(-1) + 1 + \frac{2}{3}(-1)\right)$$

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1.

1. Ird fel a kovetkezo $f(x,y)$ fuggveny kozelito elsorendu $T_1(x,y)$ Taylor-polinomjat az $(x,y)=(0,0)$ pont korul! $f(x,y) = \sin(3x) - 5y + xy$.

$f(0,0) = 0 - 0 + 0 = 0$

$f'_x = 3\cdot\cos(3x) + y$ $\quad f'_x(0,0) = 3\cdot 1 + 0 = 3$

$f'_y = -5 + x$ $\quad f'_y(0,0) = -5 + 0 = -5$

$f(x,y) \approx T_1(x,y) =$

$= 0 + (3 \;\; -5)\begin{pmatrix} x \\ y \end{pmatrix} =$

$= 3x - 5y$

2. Kersed meg A sajatvektorait es sajatertekeit!

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

Mivel A diagonális, így

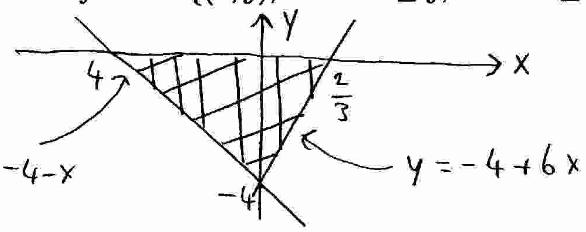
$\lambda_1 = 2$ $\quad \lambda_2 = 0$

$\bar{V}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \bar{V}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

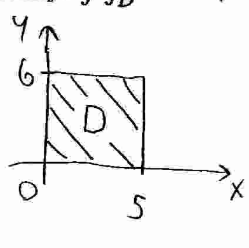
3. Ird fel a kovetkezo DE karakterisztikus egyenletet es keresd meg a gyokeit! $y' + 8y = 0$.

$\lambda + 8 = 0 \longrightarrow \lambda_1 = -8$

4. Rajzold le a kovetkezo tartomanyt! $D = \{(x,y);\ -4 + 6x \le y,\ -4 - x \le y,\ y \le 0\}$



5. Mennyi $\int\int_D -3\,dA$, ha $D = \{(x,y);\ 0 \le x \le 5,\ 0 \le y \le 6\}$?



D területe $= 6\cdot 5 = 30$, így

$\iint_D -3\,dA = -3\cdot 30 = -90$

2. Szamold ki a kovetkezo kettos integralt!

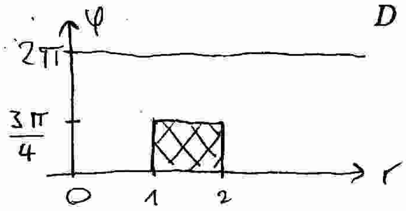
$$\int_{y=1}^{2}\int_{x=3}^{4} x\cos(3y)\ dxdy = \qquad \frac{5}{2}$$

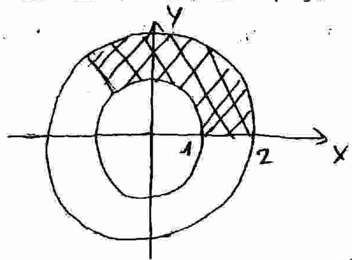
$$= \int_{y=1}^{2} \left[\frac{x^2}{2}\cos(3y)\right]_{x=3}^{4} dy = \int_{y=1}^{2} \cos(3y)\cdot\left(\frac{4^2}{2}-\frac{3^2}{2}\right) dy =$$

$$= \frac{5}{2}\left[\frac{\sin(3y)}{3}\right]_{y=1}^{2} = \frac{5}{6}\cdot(\sin 6 - \sin 3)$$

Szamold ki a kovetkezo kettos integralokat es rajzold le a D integralasi tartomanyt!

$D = \{(r,\phi);\ 1 \le r \le 2,\ 0 \le \phi \le 3\pi/4\}$, $\qquad \iint_D x - y\, dA$





$$\iint_D x-y\, dA = \int_{r=1}^{2}\int_{\varphi=0}^{3\pi/4} (r\cdot\cos\varphi - r\cdot\sin\varphi)\cdot \overbrace{r\, d\varphi\, dr}^{dA} =$$

$$= \int_{r=1}^{2} r^2\left(\left[\sin\varphi - (-\cos\varphi)\right]_{\varphi=0}^{3\pi/4}\right) dr = \int_{r=1}^{2} r^2\cdot\Big(\left(\sin 135^\circ + \cos 135^\circ\right) - \left(\sin 0^\circ + \cos 0^\circ\right)\Big) dr$$

$$= \int_{r=1}^{2} r^2\cdot(-1)\, dr = \left[(-1)\cdot\frac{r^3}{3}\right]_{r=1}^{2} = \left(-1\cdot\frac{2^3}{3}\right) - \left(-1\cdot\frac{1^3}{3}\right) = -\frac{7}{3}$$

3. Legyen

$$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

- Keresd meg A sajatertekeit!

$$0 = \begin{vmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 3^2 = \lambda^2 - 4\lambda - 5 = 0$$

$$\lambda_1 = 5 \qquad \lambda_2 = -1$$

- Keresd meg A sajatvektorait!

① $\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 5\begin{pmatrix} x \\ y \end{pmatrix}$

$$\left.\begin{aligned} 2x+3y &= 5x \\ 3x+2y &= 5y \end{aligned}\right\} x = y$$

$$\overline{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

② $\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -1 \cdot \begin{pmatrix} x \\ y \end{pmatrix}$

$$\left.\begin{aligned} 2x+3y &= -x \\ 3x+2y &= -y \end{aligned}\right\} x = -y$$

$$\overline{V}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- Ird fel a DE altalanos megoldasat!

$$\overline{y}_{ált} = C_1 \cdot e^{5x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \cdot e^{-1 \cdot x}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- Ird fel a DE partikularis megoldasat, ha

$$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$C_1 \cdot e^{5 \cdot 0}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \cdot e^{-1 \cdot 0}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} C_1 + C_2 \\ C_1 - C_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

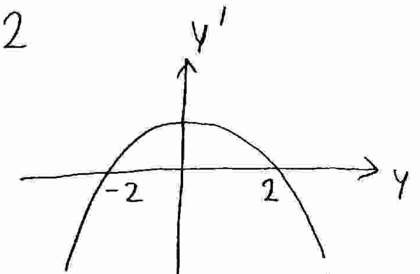
$$C_1 = \frac{13}{2} \qquad C_2 = -\frac{1}{2}$$

$$\overline{y}_{part} = \frac{13}{2} e^{5x}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{1}{2} e^{-x}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

4. Legyen $y' = f(y) = -y^2 + 4$.

- Keresd meg a DE fixpontjait!

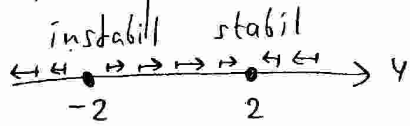
$$-y^2+4=0 \longrightarrow y_1 = 2 \qquad y_2 = -2$$



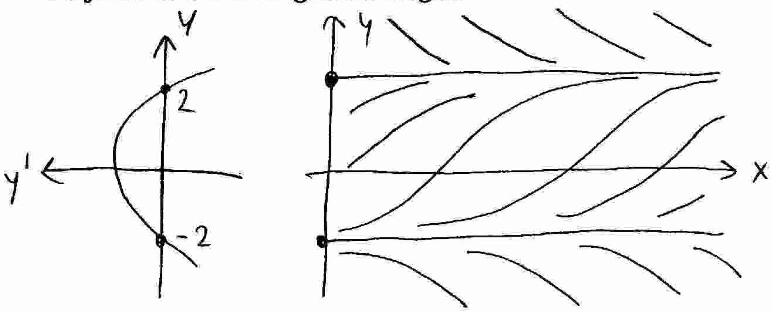
- Vizsgald meg azok stabilitasat! (indokold valaszodat!)

$y_1 = 2$ stabil

$y_2 = -2$ instabil



- Rajzold le a DE megoldassereget!



Legyen $y' = -4y + 1$.

- Keresd meg a DE altalanos megoldasat!

$$\frac{dy}{dx} = -4y+1 \qquad \frac{dy}{-4y+1} = dx$$

$$\int \frac{dy}{-4y+1} = \int dx \qquad \frac{\ln|-4y+1|}{-4} = x + C$$

$$|-4y+1| = e^{-4(x+C)}$$

$$y = -\frac{1}{4}\left(\pm e^{-4(x+C)}\right) + \frac{1}{4}$$

$$\left(= \frac{1}{4}\tilde{C} e^{-4x}\right)$$

- Keresd meg a DE partikularis megoldasat, ha $y(1) = 3$!

$$y = \frac{1}{4} + e^{-4(x+c)}$$

$$3 = \frac{1}{4} + e^{-4(1+c)}$$

$$2\tfrac{3}{4} = e^{-4(1+c)}$$

$$\ln\left(2\tfrac{3}{4}\right) = -4 - 4C$$

$$C = \frac{1}{4}\left(-4 - \ln\left(2\tfrac{3}{4}\right)\right)$$

$$y_{part} = \frac{1}{4} + e^{-4\left(x + \frac{1}{4}\left(-4 - \ln\left(2\frac{3}{4}\right)\right)\right)}$$