Vizsga, Matematika II, 2013.06.12. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5 × 2 pont.

1. Ird fel a kovetkezo $f(x, y)$ fuggveny kozelito elsorendu $T_1(x, y)$ Taylor-polinomjat az $(x, y) = (0, 0)$ pont korul! $f(x, y = \sin(x - 5y + 2xy)$.

$f'_x = \cos(x-5y+2xy)\cdot(1+2y)$

$f'_y = \cos(x-5y+2xy)\cdot(-5+2x)$

$f(0,0)=0, \; f'_x(0,0)=1, \; f'_y(0,0)=-5$

$f(x,y) \approx T_1(x,y) =$

$= 0 + (1 \;\; -5)\begin{pmatrix} x \\ y \end{pmatrix} =$

$= x - 5y$

2. Kersed meg A sajatvektorait es sajatertekeit! $A = \begin{pmatrix} 0 & 0 \\ 0 & 9 \end{pmatrix}$

Mivel A diagonális,

$\lambda_1 = 0 \quad \bar{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

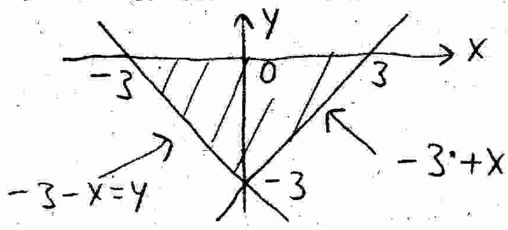
$\lambda_2 = 9 \quad \bar{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

3. Legyen $f(x, y) = \frac{x+3}{y+2}$.

$f'_x = \frac{1}{y+2}$

$f'_y = (x+3)\cdot(-1)\cdot(y+2)^{-2}$

4. Rajzold le a kovetkezo tartomanyt! $D = \{(x, y);\ -3 + x \le y,\ -3 - x \le y,\ y \le 0\}$



Mennyi $\iint_D 3\, dA$?

D területe $= \frac{6\cdot 3}{2} = 9$, így $\iint_D 3\,dA = 3\cdot 9 = 27$

5. $\int \frac{x+3}{y+2}\, dx =$

$= \frac{1}{y+2}\int x+3\, dx =$

$= \frac{1}{y+2}\left(\frac{x^2}{2} + 3x\right) + C$

C tetszőleges függvénye y-nak

$\int \frac{x+3}{y+2}\, dy = (x+3)\int \frac{1}{y+2}\, dy$

$= (x+3)\ln|y+2| + C$

(x-nek)

2.((2+2+1)+5 pont)

Szamold ki a kovetkezoket!

- $\int \ln(2x)x^5\,dx = \left| \begin{matrix} f' = x^5 & g = \ln(2x) \\ f = \frac{x^6}{6} & g' = \frac{1}{2x}\cdot 2 = \frac{1}{x} \end{matrix} \right| =$

$$= \frac{x^6}{6}\ln(2x) - \int \underbrace{\frac{x^6}{6}\cdot\frac{1}{x}}_{x^5/6}dx = \frac{x^6}{6}\ln(2x) - \frac{x^6}{6\cdot 6} + C$$

- $\int \frac{x}{3x^2+1} = \int x\cdot(3x^2+1)^{-1}dx = \frac{1}{6}\int(3x^2+1)^{-1}\cdot 6x\,dx =$

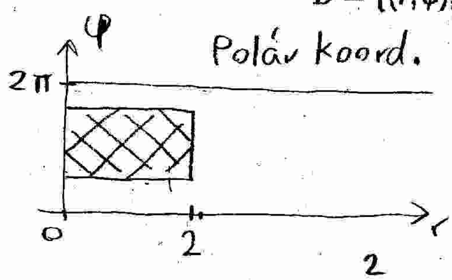
$$= \frac{1}{6}\int(3x^2+1)^{-1}\cdot(3x^2+1)'\,dx = \frac{1}{6}\ln|3x^2+1| + C$$

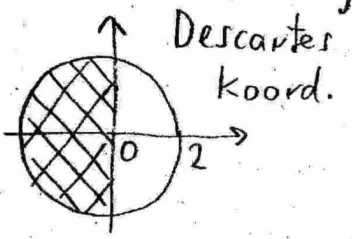
- $\int \frac{1}{\sqrt[3]{(5x)^2}} + \sqrt[5]{3x^2}\,dx = \int (5x)^{-2/3} + \sqrt[5]{3}\,x^{2/5}\,dx =$

$$= \frac{\frac{(5x)^{1/3}}{1/3}}{5} + \sqrt[5]{3}\,\frac{x^{7/5}}{7/5} + C$$

Szamold ki a kovetkezo kettos integralt es rajzold le a D integralasi tartomanyt!

$$D = \{(r,\phi);\ 0 \le r \le 2,\ \pi/2 \le \phi \le 3\pi/2\}, \qquad \iint_D x - y\,dA$$





$$\iint_D x - y\,dA = \int_{r=0}^{2}\int_{\varphi=\frac{\pi}{2}}^{3\pi/2}\underbrace{(r\cos\varphi - r\sin\varphi)\cdot r\,d\varphi\,dr}_{dA} =$$

$$= \int_{r=0}^{2} r^2\cdot\left[\sin\varphi - (-\cos\varphi)\right]_{\varphi=\pi/2}^{3\pi/2}dr =$$

$$= \int_{r=0}^{2} r^2\cdot\left[((-1)-0)-(1-0)\right]dr = \int_{r=0}^{2} r^2\cdot(-2)\,dr = \left[-\frac{2}{3}r^3\right]_0^2$$

$$= -\frac{8}{3}$$

3. ((2+1+3)+(2+2) pont)

Legyen $f(x,y) = -2x^3 + y^2 + 3x^2 - 2y$. Hatarozd meg f kritikus pontjanak a helyet es a tipusat!

f parcialis derivaltjai:

$$f'_x = -6x^2+6x \qquad f''_{xx} = -12x+6$$

$$f'_y = 2y-2 \qquad f''_{yy} = 2 \qquad f''_{xy} = f''_{yx} = 0$$

A kritikus pont helye:

$$\left.\begin{aligned} 0 &= -6x^2+6x = 6x(-x+1) \\ 0 &= 2y-2 \end{aligned}\right\} \quad \begin{aligned} &x_1=0,\ x_2=1 \\ &y=1 \end{aligned} \qquad \begin{aligned} &P_1(0,1) \\ &P_2(1,1) \end{aligned}$$

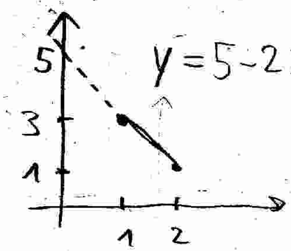
A kritikus pont tipusanak a meghatarozasa:

$$H(f) = \begin{pmatrix} -12x+6 & 0 \\ 0 & 2 \end{pmatrix}$$

$$(H(f))(P_1) = \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix} \qquad \lambda_1 = 6 > 0,\ \lambda_2 = 2 > 0 \text{ MINIMUM}$$

$$(H(f))(P_2) = \begin{pmatrix} -6 & 0 \\ 0 & 2 \end{pmatrix} \qquad \lambda_1 = -6 < 0,\ \lambda_2 = 2 > 0 \text{ NYEREGPONT}$$

Kosd ossze a $P_1(1,3)$, $P_2(2,1)$ pontokat. Forgasd meg a $\overline{P_1P_2}$ szakaszt az x-tengely korul! Szamitsd ki a kapott forgastest terfogatat es feluletet!



$$\frac{y-3}{x-1} = \frac{1-3}{2-1}$$

Terfogat= $\pi\int_1^2 f^2(x)\,dx = \pi\int_1^2 (5-2x)^2\,dx =$

$$= \pi\left[\frac{\frac{(5-2x)^3}{3}}{2}\right]_1^2 = \pi\left[\frac{(5-2\cdot2)^3}{6} - \frac{(5-2\cdot1)^3}{6}\right]$$

Felulet= $2\pi\int_1^2 f(x)\sqrt{1+[f'(x)]^2}\,dx =$

$$= 2\pi\int_1^2 (5-2x)\sqrt{1+(-2)^2}\,dx =$$

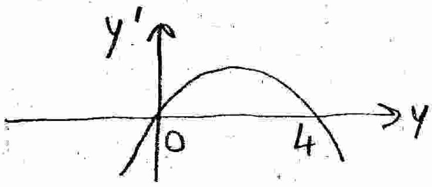
$$= 2\pi\sqrt{5}\left[\frac{(5-2x)^2}{2}\right]_1^2 = 2\pi\sqrt{5}\left(\frac{(5-2\cdot2)^2}{2} - \frac{(5-2\cdot1)^2}{2}\right)$$

4. ((1+1+3))+(3+2) pont)

Legyen $y' = f(y) = -y^2 + 4y$.

- Keresd meg a DE fixpontjait!

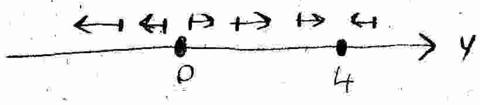
$$-y^2 + 4y = 0 = y(-y+4) \longrightarrow y_1 = 0 \qquad y_2 = 4$$



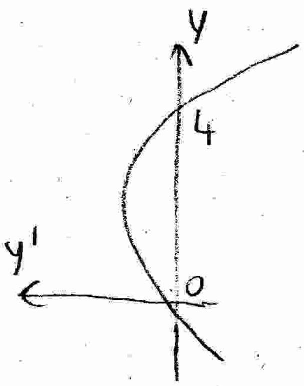
- Vizsgald meg azok stabilitasat!

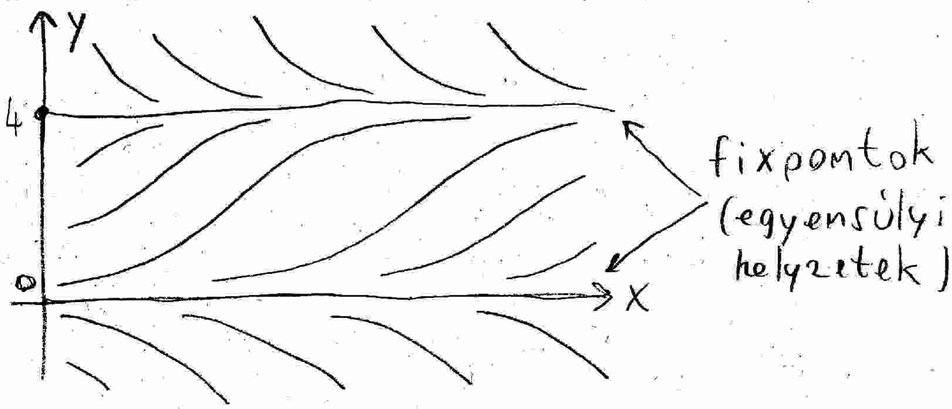
$y=0$ instabil

$y=4$ stabil



- Rajzold le a DE megoldassereget!





Legyen $y' = 6y + 6$.

- Keresd meg a DE altalanos megoldasat!

$$\frac{dy}{dx} = 6y+6$$

$$\frac{dy}{6y+6} = dx$$

$$\int \frac{dy}{6y+6} = \int dx$$

$$\frac{\ln|6y+6|}{6} = x + C$$

$$\ln|6y+6| = 6(x+C)$$

$$e^{\ln|6y+6|} = |6y+6| = e^{6(x+C)}$$

$$6y+6 = \pm e^{6(x+C)}$$

$$6y = \pm e^{6(x+C)} - 6$$

$$\boxed{y = \pm \frac{1}{6} e^{6(x+C)} - 1}$$

- Keresd meg a DE partikularis megoldasat, ha $y(0) = 3$!

$$y(0)=3 \longrightarrow +\frac{1}{6} e^{6(0+C)} - 1 = 3 \longrightarrow e^{6C} = 24 \longrightarrow 6C = \ln 24$$

$$C = \frac{\ln 24}{6} \longrightarrow y = \frac{1}{6} e^{6\left(x + \frac{\ln 24}{6}\right)} - 1 = 4 \cdot e^{6x} - 1$$

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