Vizsga, Matematika II, 2013.06.06. NEPTUN: Gyak.Vez.: 15.30

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

1. $\int \frac{1}{\sqrt[7]{7x-1}} = \int (7x-1)^{-1/7} dx = \dfrac{\dfrac{(7x-1)^{6/7}}{6/7}}{7} + C$

2. $f(x,y) = \sqrt{3x+2y}$.

$f'_y = \left[(3x+2y)^{1/2} \right]'_y = \frac{1}{2}(3x+2y)^{-1/2} \cdot 2$

$(3x+2y)'_y$

3. Szamitsd ki a kovetkezo integralokat!

$\int e^{-x+y}\, dx = \dfrac{e^{-x+y}}{-1} + C$

$\int e^{-x+y}\, dy = e^{-x+y} + C$

4. Keresd meg a kovetkezo matrix sajatertekeit es sajatvektorait!

$\begin{pmatrix} -6 & 0 \\ 0 & 8 \end{pmatrix}$,

$\lambda_1 = -6$, $\bar{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\lambda_2 = 8$, $\bar{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

5. Oldd meg a kovetkezo DE-t! $y' = 99y$

$y = C \cdot e^{99x}$

2. $f(x) = \sin(3x + y^2)$. (6+4 pont)

$f'_x = \cos(3x+y^2)\cdot 3$

$f'_y = \cos(3x+y^2)\cdot 2y$

$f''_{xx} = -\sin(3x+y^2)\cdot 3\cdot 3$

$f''_{xy} = \left[\cos(3x+y^2)\cdot 3\right]'_y =$

$= -\sin(3x+y^2)\cdot 2y\cdot 3$

//

$f''_{yx} = \left[\cos(3x+y^2)\cdot 2y\right]'_x =$

$= 2y\cdot(-\sin(3x+y^2))\cdot 3$

$f''_{yy} = \left[\cos(3x+y^2)\cdot(2y)\right]'_y =$

$= \left[\cos(3x+y^2)\right]'_y (2y) +$

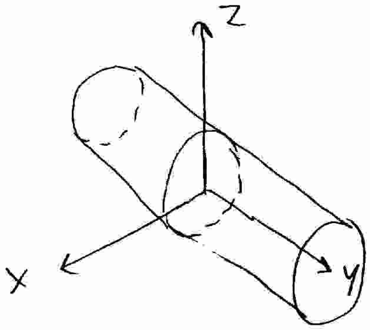
$+ \cos(3x+y^2)\cdot\left[2y\right]'_y =$

$= -\sin(3x+y^2)\cdot(2y)\cdot(2y) +$

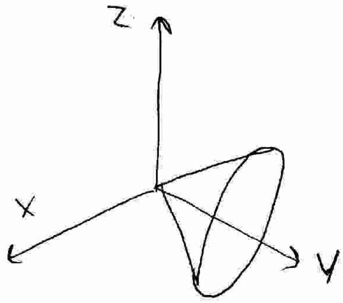
$+ \cos(3x+y^2)\cdot 2$

Rajzold le a kovetkezo feluleteteket!

$x^2 + z^2 = 16$

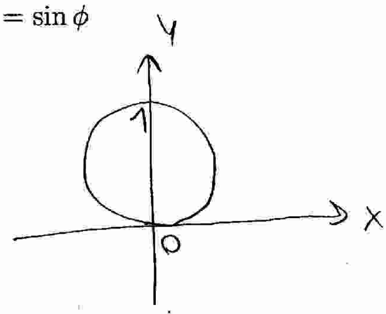


$y = \sqrt{x^2 + z^2}$

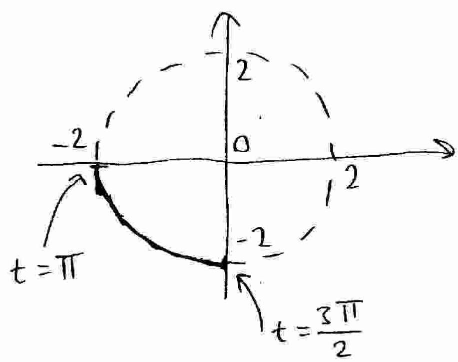


Rajzold le a kovetkezo gorbeket!

$r = \sin\phi$



$\bar{r}(t) = (2\cos t, 2\sin t),\ \ t \in [\pi, 3\pi/2]$



3. a) Legyen $y'' - 4y = 0$. (2+1+2 pont)

1. Ird fel a DE karakterisztikus egyenletet es keresd meg a gyokeit!

$$\lambda^2 - 4 = 0 \qquad \lambda_1 = 2 \qquad \lambda_2 = -2$$

2. Ird fel a DE altalanos megoldasat!

$$y_{ált}(x) = C_1 e^{2x} + C_2 e^{-2x}$$

3. Ird fel a DE partikularis megoldasat, ha $y(0) = 2$, $y'(0) = -4$!

$$y'_{ált}(x) = C_1 \cdot 2 e^{2x} + C_2 \cdot (-2) e^{-2x}$$

$$\left.\begin{array}{l} y(0)=2 \longrightarrow C_1 \cdot e^{2\cdot 0} + C_2 \cdot e^{-2\cdot 0} = C_1 + C_2 = 2 \\ y'(0)=4 \longrightarrow C_1 \cdot 2 e^{2\cdot 0} + C_2 (-2) e^{-2\cdot 0} = 2C_1 - 2C_2 = -4 \end{array}\right\} \quad \begin{array}{l} C_1 = 0 \\ C_2 = 2 \end{array}$$

$$y_{part}(x) = 0 \cdot e^{2x} + 2 \cdot e^{-2x} = 2 \cdot e^{-2x}$$

b) Ird fel f masodrendu kozelito Taylor-polinomjat az $(x, y) = (0, 0)$ pont korul, ha $f(x, y) = \sin(5x + 3y)$! (2+2+1 pont)

1. f-nek a megoldashoz szukseges parcialis derivaltjai:

$$f'_x = \cos(5x+3y) \cdot 5 \qquad f''_{xx} = -\sin(5x+3y) \cdot 5 \cdot 5$$

$$f'_y = \cos(5x+3y) \cdot 3 \qquad f''_{xy} = f''_{yx} = -\sin(5x+3y) \cdot 5 \cdot 3$$

$$f''_{yy} = -\sin(5x+3y) \cdot 3 \cdot 3$$

2. Ezek erteke az $(x, y) = (0, 0)$ pontban:

$$f(0,0) = \sin(5 \cdot 0 + 3 \cdot 0) = 0 \qquad f''_{xx}(0,0) = 0$$

$$f'_x(0,0) = 5 \qquad f''_{xy}(0,0) = 0$$

$$f'_y(0,0) = 3 \qquad f''_{yy}(0,0) = 0$$

3. f masodrendu kozelito Taylor-polinomja:

$$f(x,y) \approx 0 + (5 \;\; 3)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x \;\; y)\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= 5x + 3y$$

4. a) Szamold ki a kovetkezo integralokat! (2+2+1 pont)

1. $\int e^{(2x)}x\,dx = \left| \begin{matrix} f' = e^{2x} & g = x \\ f = \frac{e^{2x}}{2} & g' = 1 \end{matrix} \right| = \frac{e^{2x}}{2} \cdot x - \int \frac{e^{2x}}{2} \cdot 1\,dx =$

$$= \frac{e^{2x}}{2} \cdot x - \frac{e^{2x}}{2 \cdot 2} + C$$

2. $\int x \sin(2x^2) = \frac{1}{4} \int \sin(2x^2) \cdot 4x\,dx = \frac{1}{4} \int \sin(2x^2) \cdot (2x^2)'\,dx =$

$$= \frac{1}{4} \cdot (-\cos(2x^2)) + C$$

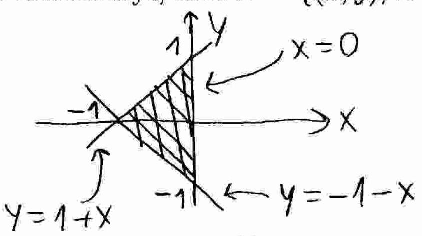
3. $\int \frac{4}{1+4x^2} + \sqrt[3]{-x^2}\,dx = \int 4 \cdot \frac{1}{1+(2x)^2} + (-x^{2/3})\,dx =$

$$= 4 \cdot \frac{\operatorname{arctg}(2x)}{2} - \frac{x^{5/3}}{5/3} + C$$

4. b) (1+1+3 pont)

1. Szamold ki! $\int x - 2y\,dy = xy - y^2 + C$

2. Rajzold le a D integralasi tartomanyt, ahol $D = \{(x, y);\ x \leq 0, y \leq 1 + x, y \geq -1 - x\}$!



3. Szamold ki a kovetkezo kettos integralt! $\iint_D (x - 2y)\,dA =$

$$\int_{x=-1}^{0} \left(\int_{y=-1-x}^{1+x} x - 2y\,dy \right) dx = \int_{x=-1}^{0} \left[xy - y^2 \right]_{y=-1-x}^{1+x} dx =$$

$$= \int_{x=-1}^{0} \left(x(1+x) - (1+x)^2 \right) - \left(x(-1-x) - (-1-x)^2 \right) dx =$$

$$= \int_{x=-1}^{0} 2x - 2x^2\,dx = \left[x^2 - 2\frac{x^3}{3} \right]_{x=-1}^{0} = -\frac{1}{3}$$