Vizsga, Matematika II, 2013.06.~~06.~~ 04 NEPTUN: Gyak.Vez.: 35

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

1. $\int \frac{1}{7x-1} dx = \int (7x-1)^{-1} dx = \frac{\ln|7x-1|}{7} + C$

2. $f(x,y) = \ln(3x+2y)$.

$f'_y = \frac{1}{3x+2y} \cdot 2$

3. Szamitsd ki a kovetkezo integralokat!

$\int \sin(-x+y)\, dx = \frac{-\cos(-x+y)}{-1} + C = \cos(-x+y)$

$\int \cos(-x+y)\, dy = \sin(-x+y) + C$

4. Keresd meg a kovetkezo matrix sajatertekeit!

$\begin{pmatrix} -6 & -6 \\ 0 & 8 \end{pmatrix}$,

Mivel a mátrix triangulárís, így $\lambda_1 = -6$, $\lambda_2 = 8$

Vagy: $\begin{vmatrix} -6-\lambda & -6 \\ 0 & 8-\lambda \end{vmatrix} =$

$= (-6-\lambda)(8-\lambda) - (-6)\cdot 0 = 0$

$\Rightarrow \lambda_1 = -6, \lambda_2 = 8$

5. Oldd meg a kovetkezo DE-t! $y' = 1 - x$, $y(0) = 8$

$y = \int 1 - x\, dx = x - \frac{x^2}{2} + C \leftarrow$ általános megoldás

$0 - \frac{0^2}{2} + C = 8 \rightarrow C = 8$

$y = x - \frac{x^2}{2} + 8 \leftarrow$ partikuláris megoldás

2. $f(x) = \frac{1}{3x+y^2}$. (6+4 pont) $= (3x+y^2)^{-1}$

$f'_x = -1 \cdot (3x+y^2)^{-2} \cdot 3 = -\frac{3}{(3x+y^2)^2}$

$f'_y = -1 \cdot (3x+y^2)^{-2} \cdot 2y = \frac{-2y}{(3x+y^2)^2}$

$f''_{xx} = -1 \cdot (-2)(3x+y^2)^{-3} \cdot 3 \cdot 3 = + \frac{18}{(3x+y^2)^3}$

$f''_{xy} = 12 \frac{y}{(3x+y^2)^3}$

$f''_{yx} = -3 \cdot (-2) \cdot (3x+y^2)^{-3} \cdot 2y$

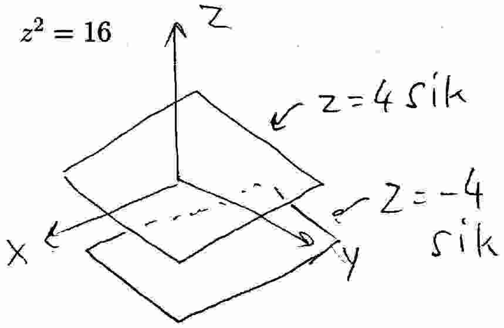
$f''_{yy} = -1 \cdot (-2) \cdot (3x+y^2)^{-3} \cdot (2y)^2 + {}$

$+ (-1) \cdot (3x+y^2)^{-2} \cdot 2$

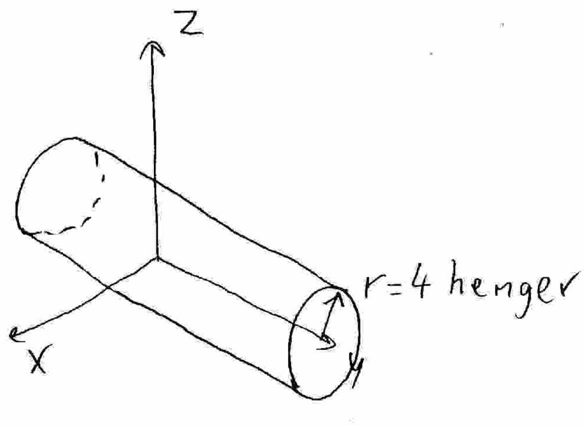
$= \frac{-2 \cdot (3x+y^2)^2 - (-2y) \cdot 2 \cdot (3x+y^2) \cdot 2y}{(3x+y^2)^4}$

Rajzold le a kovetkezo feluleteteket!

$z^2 = 16$

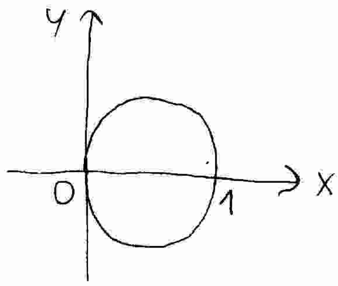


$x^2 + z^2 = 16$

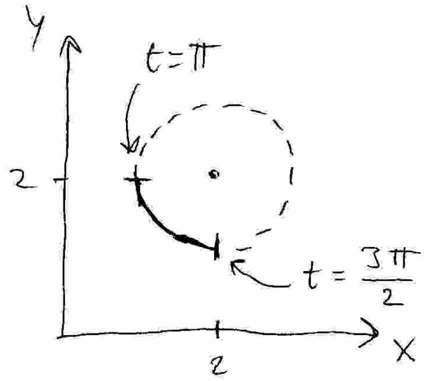


Rajzold le a kovetkezo gorbeket!

$r = \cos\phi$



$\bar{r}(t) = (2+\cos t, 2+\sin t), \quad t \in [\pi, 3\pi/2]$



3. a) Legyen $y'' + 4y = 0$. (2+1+2 pont)

1. Ird fel a DE karakterisztikus egyenletet es keresd meg a gyokeit!

$$\lambda^2 + 4 = 0 \qquad \lambda_1 = 0 + 2i$$
$$\lambda = \sqrt{-4} \qquad \lambda_2 = 0 - 2i$$

2. Ird fel a DE altalanos megoldasat!

$$y_{ált} = e^{0\cdot x}\left(C_1 \cos(2x) + C_2 \sin(2x)\right) = C_1 \cos(2x) + C_2 \sin(2x)$$

3. Ird fel a DE partikularis megoldasat, ha $y(0) = 2$, $y'(0) = -4$!

$$y'_{ált} = -2C_1 \sin(2x) + 2C_2 \cos(2x)$$
$$y(0) = 2 \longrightarrow C_1 \cos(2\cdot 0) + C_2 \sin(2\cdot 0) = C_1 = 2$$
$$y'(0) = -4 \rightarrow -2C_1 \sin(2\cdot 0) + 2C_2 \cos(2\cdot 0) = 2C_2 = -4 \rightarrow C_2 = -2$$
$$y = 2\cos(2x) - 2\sin(2x)$$

b) Ird fel f masodrendu kozelito Taylor-polinomjat az $(x, y) = (0, 0)$ pont korul, ha $f(x, y) = \ln(1 + 5x + 3y)$! (2+2+1 pont)

1. f-nek a megoldashoz szukseges parcialis derivaltjai:

$$f'_x = \frac{1}{1+5x+3y}\cdot 5 \qquad f''_{xx} = 5\cdot(-1)(1+5x+3y)^{-2}\cdot 5$$
$$f'_y = \frac{1}{1+5x+3y}\cdot 3 \qquad f''_{yy} = 3\cdot(-1)(1+5x+3y)^{-2}\cdot 3$$
$$f''_{xy} = f''_{yx} = 5\cdot(-1)\cdot(1+5x+3y)^{-2}\cdot 3$$

2. Ezek erteke az $(x, y) = (0, 0)$ pontban:

$$f(0,0) = 0 \qquad f''_{xx}(0,0) = -25 \qquad f''_{xy}(0,0) = -15$$
$$f'_x(0,0) = 5 \qquad f''_{yy}(0,0) = -9$$
$$f'_y(0,0) = 3$$

3. f masodrendu kozelito Taylor-polinomja:

$$f(x,y) \approx T_2(x,y) = 0 + \begin{pmatrix} 5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} -25 & -15 \\ -15 & -9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= 0 + 5x + 3y - \frac{25}{2}x^2 - 15xy - \frac{9}{2}y^2$$

4. a) Szamold ki a kovetkezo integralokat! (2+2+1 pont)

1. $\int \ln(2x)x^3\,dx = \left| \begin{array}{ll} f' = x^3 & g = \ln(2x) \\ f = \frac{x^4}{4} & g' = \frac{1}{2x}\cdot 2 = \frac{1}{x} \end{array} \right| = \frac{x^4}{4}\cdot \ln(2x) - \int \overbrace{\frac{x^4}{4}\cdot\frac{1}{x}}^{x^3/4}\,dx$

$= \frac{x^4}{4}\ln(2x) - \frac{x^4}{4\cdot 4} + C$

2. $\int x\cos(2x^2) = \frac{1}{4}\int \cos(2x^2)\cdot 4x\,dx = \frac{1}{4}\int \cos(2x^2)\cdot(2x^2)'\,dx =$

$= \frac{1}{4}\sin(2x^2) + C$

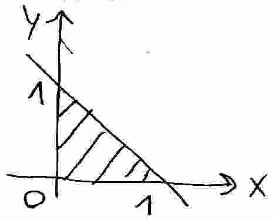
3. $\int \frac{1}{1+4x^2} + \frac{4}{1+x^2}\,dx = \int \frac{1}{1+(2x)^2} + 4\cdot\frac{1}{1+x^2}\,dx =$

$= \frac{\operatorname{arctg}(2x)}{2} + 4\cdot \operatorname{arctg}(2x) + C$

4. b) (1+1+3 pont)

1. Szamold ki! $\int x - y\,dy = \quad xy - \frac{y^2}{2} + C$

2. Rajzold le a D integralasi tartomanyt, ahol $D = \{(x,y);\ x \geq 0, y \leq 1 - x, y \geq 0\}$!



3. Szamold ki a kovetkezo kettos integralt! $\iint_D (x-y)\,dA =$

$= \int_{x=0}^{1}\left(\int_{y=0}^{1-x} x - y\,dy\right)dx = \int_{x=0}^{1}\left[xy - \frac{y^2}{2}\right]_{y=0}^{1-x} dx =$

$= \int_{x=0}^{1} x(1-x) - \frac{(1-x)^2}{2}\,dx = \int_{x=0}^{1} -3\cdot\frac{x^2}{2} + 2x - \frac{1}{2}\,dx =$

$= \left[-\frac{x^3}{2} + x^2 - \frac{1}{2}x\right]_0^1 = 0$

vagy: Mivel D szimmetrikus az $x \leftrightarrow y$ cseré nézve, viszont $(x-y) = -(y-x)$, így az integrál automatikusan nulla.