Vizsga 05.31

~~Zh1,~~ Matematika II, 2013.~~05.23.~~ NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5 × 2 pont.

1. Ird fel a kovetkezo $f(x, y)$ fuggveny kozelito elsorendu $T_1(x, y)$ Taylor-polinomjat az $(x, y) = (0, 0)$ pont korul! $f(x, y = \cancel{e^{3x-5y+xy}}$. $f(x,y) = \frac{1}{1+x-2y}$ $\quad f(0,0) = 1$

$f'_x = -1\cdot(1+x-2y)^{-2}\cdot 1 \qquad f'_x(0,0) = -1$

$f'_y = -1\cdot(1+x-2y)^{-2}\cdot(-2) \qquad f'_y(0,0) = 2$

$f(x,y) \approx T_1(x,y) = 1 - 1\cdot x + 2y$

2. Kersed meg A ~~sajatvektorait~~ es sajatertekeit!

$A = \cancel{\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}} \quad \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$

$$0 = \begin{vmatrix} 2-\lambda & 0 \\ 1 & 5-\lambda \end{vmatrix} = (2-\lambda)(5-\lambda) - 0\cdot 1 = (2-\lambda)(5-\lambda) = 0$$

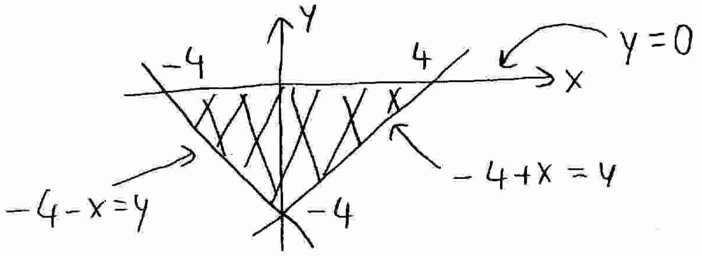
$\lambda_1 = 2 \qquad \lambda_2 = 5$

3. Legyen $f(x, y) = \frac{x+3}{2y+1}$

$f'_x = \frac{1}{2y+1}$ vagy $\frac{1\cdot(2y+1)-(x+3)\cdot 0}{(2y+1)^2}$

$f'_y = (x+3)\cdot(-1)\cdot(2y+1)^{-2}\cdot 2$ vagy $\frac{0\cdot(2y+1)-(x+3)\cdot 2}{(2y+1)^2}$

4. Rajzold le a kovetkezo tartomanyt! $D = \{(x, y);\ -4 + x \le y,\ -4 - x \le y,\ y \le 0\}$



5. Mennyi $\int \cancel{\frac{x}{y}}dx$ es $\int \cancel{\frac{y}{x}}dy$?

$\cancel{\int \frac{x}{y}dx} \neq \int \frac{x+1}{2y+3}dx = \frac{1}{2y+3}\cdot\left(\frac{x^2}{2}+x\right) + C$

$\cancel{\int \frac{y}{x}dy} \neq \int \frac{x+1}{2y+3}dy = (x+1)\cdot\frac{\ln|2y+3|}{2} + C$

2.((2+2+1)+5 pont)
Szamold ki a kovetkezoket!

- $\int \ln(2x)x\,dx =$ $\left| \begin{array}{ll} f' = x & g = \ln(2x) \\ f = \frac{x^2}{2} & g' = \frac{1}{2x}\cdot 2 = \frac{1}{x} \end{array} \right| = \frac{x^2}{2}\cdot\ln(2x) - \int \overbrace{\frac{x^2}{2}\cdot\frac{1}{x}}^{x/2}\,dx =$

$$= \frac{x^2}{2}\cdot\ln(2x) - \frac{x^2}{2\cdot 2} + C$$

- $\int \sin(2x^2)x\,dx = \frac{1}{4}\int \sin(2x^2)\cdot 4x\,dx = \frac{1}{4}\int \sin(2x^2)\cdot(2x^2)'\,dx$

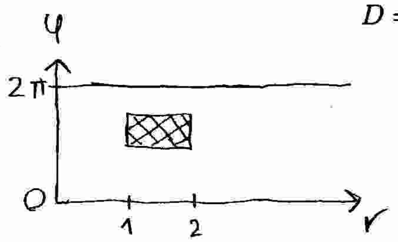
$$= \frac{1}{4}\cdot\left(-\cos(2x^2)\right) + C$$

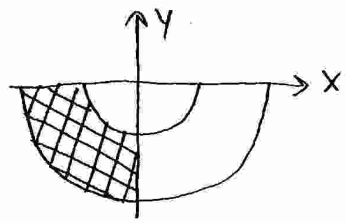
- $\int \frac{1}{\sqrt[5]{7x}} + \sqrt[5]{7x^2}\,dx = \int (7x)^{-1/5} + \sqrt[5]{7}\cdot x^{2/5}\,dx =$

$$= \frac{\frac{(7x)^{4/5}}{4/5}}{7} + \sqrt[5]{7}\,\frac{x^{7/5}}{7/5} + C$$

Szamold ki a kovetkezo kettos integralt es rajzold le a D integralasi tartomanyt!

$$D = \{(r,\phi);\ 1 \le r \le 2,\ \pi \le \phi \le 3\pi/2\}, \qquad \iint_D x+y\,dA$$





$$\iint_D x+y\,dA = \int_{r=1}^{2}\int_{\varphi=\pi}^{3\pi/2} (r\cos\varphi + r\cdot\sin\varphi)\cdot\overbrace{r\cdot d\varphi\,dr}^{dA} =$$

$$= \int_{r=1}^{2} r^2\cdot\left[\sin\varphi - \cos\varphi\right]_{\varphi=\pi}^{3\pi/2} dr =$$

$$= \int_{r=1}^{2} r^2\cdot\left\{\left(\underbrace{\sin\tfrac{3\pi}{2}}_{-1} - \underbrace{\cos\tfrac{3\pi}{2}}_{0}\right) - \left(\underbrace{\sin\pi}_{0} - \underbrace{\cos\pi}_{-1}\right)\right\} dr$$

$$= \int_{r=1}^{2} r^2\cdot(-2)\,dr = \left[-2\cdot\frac{r^3}{3}\right]_1^2 = \left(-2\cdot\frac{2^3}{3}\right) - \left(-2\cdot\frac{1^3}{3}\right) = -\frac{14}{3}$$

3. ((2+1+3)+(2+2) pont) $f(x,y) = x^2 + 2xy + 2y^2 - x$

Legyen $f(x,y) = \cancel{x^4 + y^2 + x^2 - y}$. Hatarozd meg f kritikus pontjanak a helyet es a tipusat!

f parcialis derivaltjai:

$$f'_x = 2x + 2y - 1 \qquad f''_{xx} = 2$$

$$f'_y = 2x + 4y \qquad f''_{xy} = f''_{yx} = 2$$

$$f''_{yy} = 4$$

A kritikus pont helye:

$$\left.\begin{array}{l} 2x+2y-1=0 \\ 2x+4y=0 \end{array}\right\} \longrightarrow \begin{array}{l} y = -\frac{1}{2} \\ x = 1 \end{array} \qquad P_{krit}\left(1, -\frac{1}{2}\right)$$

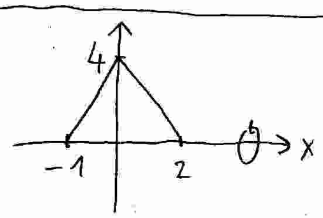
A kritikus pont tipusanak a meghatarozasa:

$$H(f) = \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}, \quad (H(f))(P_{krit}) = \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}$$

sajátértékek: $0 = \begin{vmatrix} 2-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = (2-\lambda)(4-\lambda) - 2\cdot 2$

$= \lambda^2 - 6\lambda + 4 \longrightarrow$

$\lambda_{1,2} = \frac{6 \pm \sqrt{36-24}}{2} = 3 \pm \sqrt{3} > 0$ MINIMUM



f(x)-t

~~Kosd ossze a $P_1(-3,3)$, $P_2(0,3)$, $P_3(3,0)$ pontokat~~. Forgasd meg ~~oket~~ az x-tengely korul! Szamitsd ki a kapott forgastest terfogatat es feluletet!

$$f(x) = \begin{cases} 4+4x, & \text{ha } x \in [-1,0] \\ 4-2x, & \text{ha } x \in [0,2] \end{cases}$$

Terfogat$= \pi \int_{-1}^{2} f^2(x)\,dx = \pi \int_{-1}^{0} (4+4x)^2\,dx + \pi \int_{0}^{2} (4-2x)^2\,dx$

$$= \pi \left[\frac{(4+4x)^3}{3\cdot 4} \right]_{-1}^{0} + \pi \left[\frac{(4-2x)^3}{3\cdot(-2)} \right]_{0}^{2} = 16\pi$$

Felulet$= 2\pi \int_{-1}^{2} f(x)\sqrt{1+[f'(x)]^2}\,dx = 2\pi \int_{-1}^{0} (4+4x)\cdot\sqrt{1+4^2}\,dx +$

$$+ 2\pi \int_{0}^{2} (4-2x)\cdot\sqrt{1+(-2)^2}\,dx =$$

$$= 2\pi\sqrt{17}\left[\frac{(4+4x)^2}{2\cdot 4} \right]_{-1}^{0} + 2\pi\sqrt{5}\left[\frac{(4-2x)^2}{2\cdot(-2)} \right]_{0}^{2} = 4\pi\cdot(\sqrt{17} + 2\cdot\sqrt{5})$$

4.((1+1+3))+(1+2+2) pont)

Legyen $y' = f(y) = \cancel{-y^2+16}$ $-y^2+y$

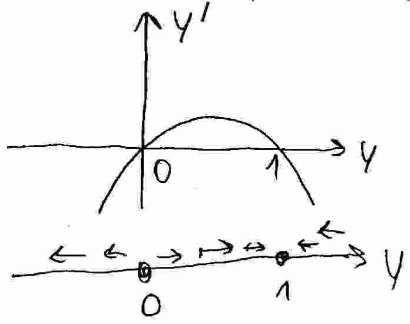
- Keresd meg a DE fixpontjait!

$$-y^2+y = y(-1+y) = 0 \longrightarrow y_1 = 0 \;,\; y_2 = 1$$

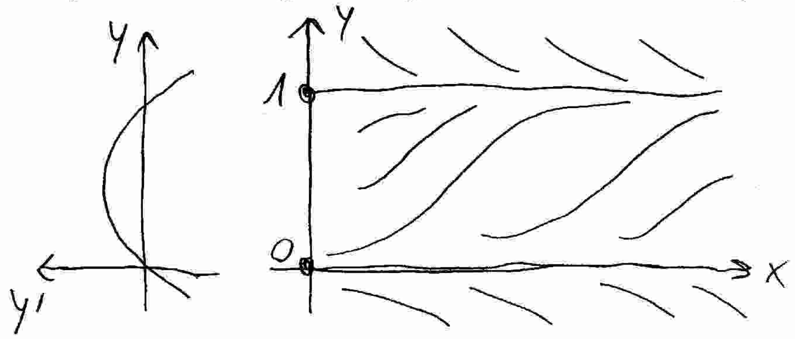
- Vizsgald meg azok stabilitasat!

$y_1 = 0$: instabil

$y_2 = 1$: stabil



- Rajzold le a DE megoldassereget!



Legyen $y'' = +16y$.

- Ird fel a DE karakterisztikus egyenletet es keresd meg a gyokeit!

$$\lambda^2 = 16 \longrightarrow \lambda_1 = 4 \quad \lambda_2 = -4$$

- Keresd meg a DE altalanos megoldasat!

$$y_{\acute{a}lt} = C_1 \cdot e^{4x} + C_2 e^{-4x}$$

$$y'_{\acute{a}lt} = 4C_1 e^{4x} - 4C_2 e^{-4x}$$

- Keresd meg a DE partikularis megoldasat, ha $y(0) = 3$, $y'(0) = 4$!

$$C_1 \cdot e^{4\cdot 0} + C_2 e^{-4\cdot 0} = C_1 + C_2 = 3$$

$$4C_1 e^{4\cdot 0} - 4C_2 e^{-4\cdot 0} = 4C_1 - 4C_2 = 4$$

$$C_1 = 2 \qquad C_2 = 1$$

$$y_{part} = 2 \cdot e^{4x} + 1 \cdot e^{-4x}$$