Pót Zh1 05.14

Zh1, Matematika II, 2013.~~04.09.~~ NEPTUN: Gyak.Vez.:

Név: Aláírás:

Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.
Szamold ki a kovetkezoket!

- $\int \sin(6x-6)\,dx = \dfrac{-\cos(6x-6)}{6} + C$

- $\int \frac{1}{\sqrt[7]{7x-77}}dx = \int (7x-77)^{-1/7}\,dx = \dfrac{\frac{(7x-77)^{6/7}}{6/7}}{7} + C$

- $\int_1^3 e^{2x-2}\,dx = \left[\dfrac{e^{2x-2}}{2}\right]_1^3 = \dfrac{e^{2\cdot 3-2}}{2} - \dfrac{e^{2\cdot 1-2}}{2} = \frac{1}{2}(e^4-1)$

- $f(x,y) = e^{(3x+2y)}$. $f'_x = e^{(3x+2y)} \cdot 3$ $\quad (3x+2y)'_x = 3$

- $f(x,y) = \sqrt{(3x+2y)}$. $f'_y = \left[(3x+2y)^{1/2}\right]'_y = \frac{1}{2}(3x+2y)^{-1/2} \cdot 2$ $\quad (3x+2y)'_y$

(5 × 2 pont)

$y'(x) = x^3$, $y(1) = 2$. Mennyi $y(3)$?

$$y(x) = \int x^3\,dx = \frac{x^4}{4} + C.$$

$$y(1) = 2 \longrightarrow \frac{1^4}{4} + C = 2 \longrightarrow C = 1\tfrac{3}{4} \longrightarrow y(x) = \frac{x^4}{4} + 1\tfrac{3}{4}$$

$$y(3) = \frac{3^4}{4} + 1\tfrac{3}{4} = 22$$

$\int \frac{1}{\sqrt[3]{-2x}} + \sqrt[2]{(7x)} + \frac{3}{4+9x^2} + e^{-2x}\,dx = \int (-2x)^{-1/3} + (7x)^{1/2} + 3 \cdot \frac{1}{4} \cdot \frac{1}{1+\left(\frac{3}{2}x\right)^2} + e^{-2x}\,dx$ $\quad \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$$= \frac{\frac{(-2x)^{2/3}}{2/3}}{-2} + \frac{\frac{(7x)^{3/2}}{3/2}}{7} + \frac{\frac{3}{4}\operatorname{arctg}\left(\frac{3}{2}x\right)}{3/2} + \frac{e^{-2x}}{-2} + C$$

$\int \cos(6x)x\,dx =$ $\quad \begin{vmatrix} f' = \cos(6x) & g = x \\ f = \frac{\sin(6x)}{6} & g' = 1 \end{vmatrix} = \underset{f \cdot g}{\frac{\sin(6x)}{6}x} - \int \underset{f \cdot g'}{\frac{\sin(6x)}{6} \cdot 1}\,dx =$

$$= \frac{\sin(6x)}{6} \cdot x - \frac{-\cos(6x)}{6 \cdot 6} + C$$

$\int \cos(6x^2)x\,dx = \frac{1}{12}\int \cos(6x^2)(12x)\,dx = \frac{1}{12}\int \cos(6x^2) \cdot (6x^2)'\,dx =$

$$\boxed{\begin{array}{l} \int f(g(x)) \cdot g'(x)\,dx = F(g(x)), \\ \text{ha } F'(x) = f(x), \text{ vagyis } F(x) = \int f(x)\,dx \end{array}} \quad = \frac{1}{12} \cdot \sin(6x^2) + C$$

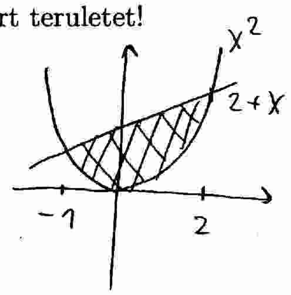
$\int_{-\infty}^{0} e^{2x}\,dx = \lim\limits_{R \to -\infty} \int_R^0 e^{2x}\,dx = \lim\limits_{R \to -\infty} \left[\frac{e^{2x}}{2}\right]_R^0 =$

$$= \lim_{R \to -\infty} \frac{e^{2 \cdot 0}}{2} - \frac{e^{2 \cdot R}}{2} = \frac{1}{2}$$

$$\frac{e^{2 \cdot R}}{2} \to \frac{1}{2} \cdot e^{-\infty} = 0$$

(4+3+3 pont)

Rajzold le az $y = x^2$, illetve az $y = 2 + x$ gorbeket! Szamitsd ki az altaluk kozrezart teruletet!



$$x^2 = 2+x \rightarrow x^2 - x - 2 = 0 \longrightarrow x_1 = -1, \quad x_2 = 2$$

$$T = \int_{-1}^{2} (2+x) - (x^2)\, dx = \left[2x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-1}^{2} =$$

$$= \left(2\cdot 2 + \frac{2^2}{2} - \frac{2^3}{3}\right) - \left(2\cdot(-1) + \frac{(-1)^2}{2} - \frac{(-1)^3}{3}\right)$$

Legyen $f(x,y) = x^2y + 3$, $x_0 = 2, y_0 = 3$. Ird fel az $f(x,y)$ fuggveny altal leirt felulet erintosikjanak a $z = z(x,y)$ egyenletet az (x_0, y_0) pontban! Ird fel az erintosik normalvektorat!

$$f'_x = 2xy \quad f'_y = x^2$$

$$\left.\begin{array}{l} f(2,3) = 2^2\cdot 3 + 3 = 15 \\ f'_x(2,3) = 2\cdot 2\cdot 3 = 12 \\ f'_y(2,3) = 2^2 = 4 \end{array}\right\} f(x,y) \approx T_1(x,y) = 15 + 12\cdot(x-2) + 4\cdot(y-3) = z$$

érintő sík egyenlete:

$$15 + 12\cdot(x-2) + 4(y-3) - z = 0$$

normálvektor:

$$\overline{n} = (12, 4, -1)$$

Keresd meg a kovetkezo gorbe ivhosszat!

$\bar{r}(t) = (4 + 2\cos t, 3 + 2\sin t)$, $t \in [\pi, 3\pi/2]$

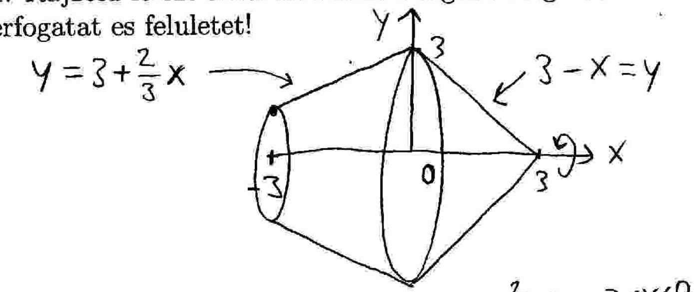
$$\overline{v}(t) = \frac{d\,\overline{r}(t)}{dt} = (-2\sin t, 2\cos t) = (\dot{x}, \dot{y})$$

$$\text{ívhossz} = \int_{\pi}^{3\pi/2} |\overline{v}(t)|\, dt = \int_{\pi}^{3\pi/2} \sqrt{(-2\sin t)^2 + (2\cos t)^2}\, dt =$$

mivel $\sin^2 t + \cos^2 t = 1$

$$= \int_{\pi}^{3\pi/2} 2\, dt = 2\cdot \underbrace{\frac{\pi}{2}}_{\frac{3\pi}{2} - \pi} = \pi$$

((2+3)+(2+1+2) pont)
Kosd ossze a $P_1(-3,1)$, $P_2(0,3)$, $P_3(3,0)$ pontokat. Rajzold le ezt a ket szakaszt! Forgasd meg oket az x-tengely korul! Szamitsd ki a kapott forgastest terfogatat es feluletet!



Terfogat$=\pi\int_a^b f^2(x)\,dx =$

$$=\pi\int_{-3}^{0}\left(3+\frac{2}{3}x\right)^2 dx+\pi\int_{0}^{3}(3-x)^2\,dx =$$

$$=\pi\left[\frac{\left(3+\frac{2}{3}x\right)^3}{3\cdot\frac{2}{3}}\right]_{-3}^{0}+\pi\left[\frac{(3-x)^3}{3\cdot(-1)}\right]_0^3 = \ldots\ldots$$

$$f(x)=\begin{cases}3+\frac{2}{3}x, & -3\le x\le 0\\ 3-x, & 0\le x\le 3\end{cases}$$

$$\text{Felulet}= 2\pi\int_a^b f(x)\cdot\sqrt{1+[f'(x)]^2}\,dx = 2\pi\int_{-3}^{0}\left(3+\frac{2}{3}x\right)\underbrace{\sqrt{1+\left(\frac{2}{3}\right)^2}}_{\sqrt{13}/3}dx+2\pi\int_0^3(3-x)\sqrt{1+(-1)^2}\,dx$$

$$=2\pi\left[\frac{\left(3+\frac{2}{3}x\right)^2}{2\cdot\frac{2}{3}}\right]_{-3}^{0}\cdot\frac{\sqrt{13}}{3}+2\pi\left[\frac{(3-x)^2}{2\cdot(-1)}\right]_0^3\cdot\sqrt{2} =$$

$$=\frac{2\pi\sqrt{13}}{3}\cdot\left(\frac{3^2}{2\cdot\frac{2}{3}}-\frac{1^2}{2\cdot\frac{2}{3}}\right)+2\pi\sqrt{2}\left(\frac{0^2}{2\cdot(-1)}-\frac{3^2}{2\cdot(-1)}\right)$$

Legyen $f(x,y)=-x^2+y^2+x-y$. Hatarozd meg f kritikus pontjanak a helyet es a tipusat!
f parcialis derivaltjai:

$$f'_x=-2x+1 \qquad f''_{xx}=-2 \qquad f''_{yy}=2$$
$$f'_y=2y-1 \qquad f''_{xy}=f''_{yx}=0$$

A kritikus pont helye:

$$\left.\begin{array}{r}-2x+1=0\\ 2y-1=0\end{array}\right\}\longrightarrow \begin{array}{l}x=\frac{1}{2}\\ y=\frac{1}{2}\end{array} \qquad P_{krit}\left(\frac{1}{2},\frac{1}{2}\right)$$

A kritikus pont tipusanak a meghatarozasa:

$$H(f)=\begin{pmatrix}-2&0\\0&2\end{pmatrix},\quad \big(H(f)\big)(P_{krit})=\begin{pmatrix}-2&0\\0&2\end{pmatrix}$$

$\begin{pmatrix}-2&0\\0&2\end{pmatrix}$ sajátértékei: Mivel a mátrix diagonális, így $\lambda_1=-2$, $\lambda_2=2$.

Vagy $\begin{vmatrix}-2-\lambda & 0\\ 0 & 2-\lambda\end{vmatrix}=(-2-\lambda)(2-\lambda)-0\cdot 0=0 \longrightarrow \lambda_1=-2, \lambda_2=2.$

Mivel $\lambda_1=-2<0$, de $\lambda_2=2>0$, így a kritikus pont nyeregpont.

Pót ZH2 2013.05.14

~~Zh1~~, Matematika II, ~~2012.03.21~~. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1.

1. Ird fel a kovetkezo DE karakterisztikus egyenletet es keresd meg azok gyokeit!
   $y'' + 6y' - 9y = 0.$

   $$\lambda^2 + 6\lambda - 9 = 0$$

   $$\lambda_{1,2} = \frac{-6 \pm \sqrt{36+4\cdot 9}}{2} = -3 \pm 6\sqrt{2}$$

2. Oldd meg a kovetkezo DE-t! $y' = -x.$

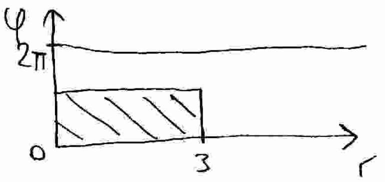
   $$y_{ált} = \int -x\,dx = -\frac{x^2}{2} + C$$

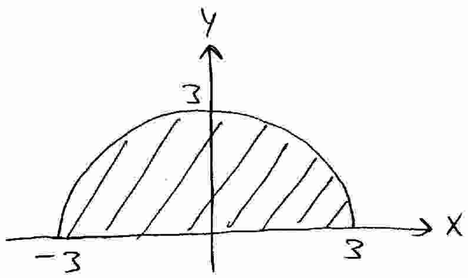
3. ~~Rajzold le a kovetkezo tartomanyt!~~ Mennyi $\int x^4y^6\,dx$ és $\int x^4y^6\,dy$?

   $$\int x^4y^6\,dx = \frac{x^5}{5}y^6 + C \qquad \int x^4y^6\,dy = x^4\frac{y^7}{7} + C$$

   Rajzold le D-t!

4. $D = \{(r, \phi);\ 0 \le r \le 3,\ 0 \le \phi \le 2\pi/4\}$





5. Oldd meg a kovetkezo DE-t! $y' = -y.$

   $$y = C \cdot e^{-x}$$

2. Legyen $y'' + 6y' + 9y = 0$.

- Ird fel a DE karakterisztikus egyenletet es keresd meg a gyokeit!

$$\lambda^2 + 6\lambda + 9 = 0$$

$$\lambda_{1,2} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 9}}{2} = -3$$

- Ird fel a DE altalanos megoldasat!

$$y_{ált} = C_1 \cdot e^{-3x} + C_2 \cdot e^{-3x} \cdot x$$

- Ird fel a DE partikularis megoldasat, ha $y(0) = \overset{1}{\cancel{0}}$, $y'(0) = \overset{2}{\cancel{0}}$!

$$y'_{ált} = C_1 \cdot (-3) e^{-3x} + C_2 (-3) e^{-3x} \cdot x + C_2 e^{-3x}$$

$$y(0) = 0 \longrightarrow C_1 \cdot e^{-3 \cdot 0} + C_2 \cdot e^{-3 \cdot 0} \cdot 0 = C_1 = 1 \longrightarrow C_1 = 1$$

$$y'(0) = 0 \longrightarrow C_1 \cdot (-3) \cdot e^{-3 \cdot 0} + C_2 \cdot (-3) \cdot e^{-3 \cdot 0} \cdot 0 + C_2 \cdot e^{-3 \cdot 0} = 2$$

$$-3 + C_2 = 2 \longrightarrow C_2 = 5$$

$$y_{part} = 2 \cdot e^{-3x} + 5 \cdot e^{-3x} \cdot x$$

Ird fel f masodrendu kozelito Taylor-polinomjat az $(x, y) = (0, 0)$ pont korul, ha $f(x, y) = 2 + 3x + 4y$!

Mivel f egy elsőrendű polinom, így a közelítő másodrendű Taylor polinomja önmaga, vagyis $f(x,y) = T_2(x,y) = 2 + 3x + 4y$.

Vagy:

	Ezek értéke (0,0)-nál:
$f = 2 + 3x + 4y$	2
$f'_x = 3$	3
$f'_y = 4$	4
$f''_{xx} = 0$	0
$f''_{yy} = 0$	0
$f''_{xy} = 0$	0

$$f(x,y) \approx T_2(x,y) =$$

$$= 2 + (3\ \ 4)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x\ \ y)\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= 2 + 3x + 4y + \frac{1}{2}\left(0 \cdot x^2 + 2 \cdot 0 \cdot xy + 0 \cdot y^2\right)$$

3. Szamold ki a kovetkezo kettos integralt!

$$\int_{y=1}^{3}\int_{x=3}^{4} x - y^2 \; dxdy =$$

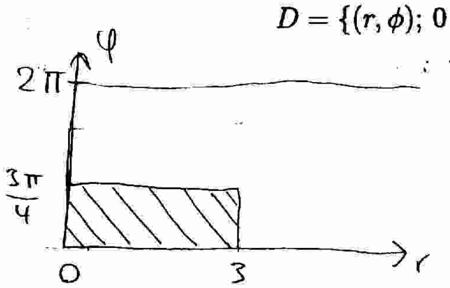
$$= \int_{y=1}^{3} \left[\frac{x^2}{2} - y^2 x \right]_{x=3}^{4} dy = \int_{y=1}^{3} \left(\frac{4^2}{2} - y^2 \cdot 4 \right) - \left(\frac{3^2}{2} - y^2 \cdot 3 \right) dy =$$

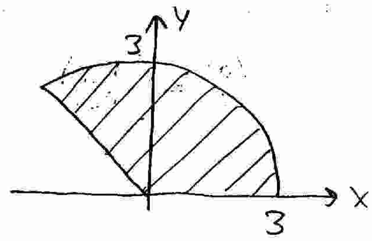
$$= \int_{y=1}^{3} \frac{5}{2} - y^2 dy = \left[\frac{5}{2} y - \frac{y^3}{3} \right]_{y=1}^{3} =$$

$$= \left(\frac{5}{2} \cdot 3 - \frac{3^3}{3} \right) - \left(\frac{5}{2} \cdot 1 - \frac{1^3}{3} \right) = -3\frac{2}{3}$$

Szamold ki a kovetkezo kettos integralokat es rajzold le a D integralasi tartomanyt!

$$D = \{(r,\phi);\ 0 \le r \le 3,\ 0 \le \phi \le 3\pi/4\}, \qquad \iint_D x - y\, dA$$





$$\iint_D x - y \, dA = \int_{r=0}^{3} \int_{\varphi=0}^{3\pi/4} (r\cos\varphi - r\sin\varphi)\, \underbrace{r\, d\varphi\, dr}_{dA} =$$

$$= \int_{r=0}^{3} r^2 \left[\sin\varphi - (-\cos\varphi) \right]_{\varphi=0}^{3\pi/4} dr = \int_{r=0}^{3} r^2 \left(\overbrace{(\sin 135^\circ + \cos 135^\circ)}^{0} - \underbrace{(\sin 0^\circ + \cos 0^\circ)}_{1} \right) dr$$

$$= \int_{r=0}^{3} r^2 \cdot (-1)\, dr = \left[-\frac{r^3}{3} \right]_{r=0}^{3} = \left(-\frac{3^3}{3} \right) - \left(\frac{0^3}{3} \right) = -9$$

4. Legyen

$$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

- Keresd meg A sajatertekeit!

$$0 = \begin{vmatrix} 2-\lambda & 0 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 0\cdot 1 = (2-\lambda)(3-\lambda)$$

$$\lambda_1 = 2 \qquad \lambda_2 = 3$$

- Keresd meg A sajatvektorait!

① $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 2\cdot\begin{pmatrix} x \\ y \end{pmatrix}$

$$\left.\begin{array}{l} 2x = 2x \\ 1x + 3y = 2y \end{array}\right\} \rightarrow y = -x$$

$$\overline{V}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

② $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 3\begin{pmatrix} x \\ y \end{pmatrix}$

$$\left.\begin{array}{l} 2x = 3x \\ 1x + 3y = 3y \end{array}\right\} \rightarrow x = 0$$

$$\overline{V}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Ird fel a DE altalanos megoldasat!

$$\overline{Y}_{ált} = C_1 \cdot e^{2x}\cdot\begin{pmatrix} 1 \\ -1 \end{pmatrix} + C_2 e^{3x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Ird fel a DE partikularis megoldasat, ha

$$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$C_1 e^{2\cdot 0}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + C_2\cdot e^{3\cdot 0}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$C_1\begin{pmatrix} 1 \\ -1 \end{pmatrix} + C_2\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix} \longrightarrow \begin{pmatrix} C_1 \\ -C_1 + C_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$C_1 = 6 \qquad C_2 = 13$$

$$\overline{Y}_{part} = 6\cdot e^{2x}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + 13 e^{3x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

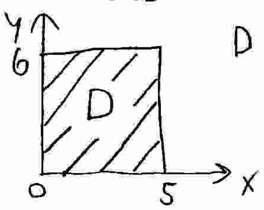
PótZh2 2013.05.14

~~Zh1,~~ Matematika II, ~~2012.03.21.~~ NEPTUN: Gyak.Vez.:

Név: Aláírás:

1.

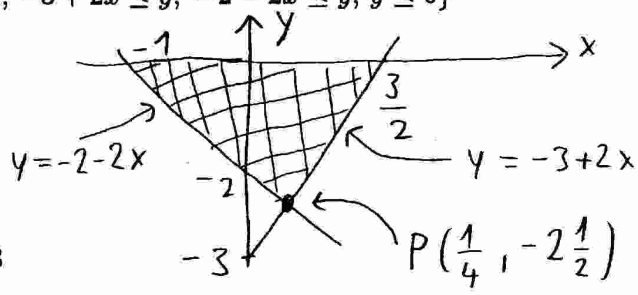
1. Mennyi $\int\int_D -3\,dA$, ha $D = \{(x,y);\ 0 \le x \le 5,\ 0 \le y \le 6\}$?



D területe $= 5 \cdot 6 = 30$

$$\iint_D -3\,dA = -3 \cdot 30 = -90$$

2. Rajzold le a kovetkezo tartomanyt! $D = \{(x,y);\ -3 + 2x \le y,\ -2 - 2x \le y,\ y \le 0\}$



3. Oldd meg a kovetkezo DE-t! $y' = 5y$, $y(0) = 13$

$$y_{ált} = C \cdot e^{5x} \qquad C \cdot e^{5 \cdot 0} = 13 \longrightarrow C = 13$$

$$y_{part} = 13 e^{5x}$$

4. $y'(x) = e^{-x}$, $y(0) = 3$. Mennyi $y(3)$?

$$y = \int e^{-x} dx = \frac{e^{-x}}{-1} + C = -e^{-x} + C$$

$$y(0) = 3 \longrightarrow -e^{-0} + C = 3 \longrightarrow C = 4 \qquad y(3) = -e^{-3} + 4$$

5. Szamitsd ki a kovetkezo integralokat!
$\int x/(y-1)\,dx$, $\int x/(y-1)\,dy$.

$$\int \frac{x}{y-1} dx = \frac{x^2}{2} \cdot \frac{1}{y-1} + C$$

$$\int \frac{x}{y-1} dy = x \cdot \ln|y-1| + C$$

2. Legyen

$$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

- Keresd meg A sajaterteket!

$$0 = \begin{vmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 2\cdot 0 \longrightarrow \lambda_1 = 1, \quad \lambda_2 = 3$$

- Keresd meg A sajatvektorait!

$\lambda_1 = 1$

$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1\cdot\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left.\begin{aligned} x+2y &= x \\ 3y &= y \end{aligned}\right\} \rightarrow y = 0, \ x \text{ tetszőleges}$$

$$\overline{V}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\lambda_2 = 3$

$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 3\cdot\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left.\begin{aligned} x+2y &= 3x \\ 3y &= 3y \end{aligned}\right\} x = y$$

$$\overline{V}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- Ird fel a DE altalanos megoldasat!

$$\overline{y}_{ált}(x) = C_1\cdot e^{1\cdot x}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2\cdot e^{3x}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- Ird fel a DE partikularis megoldasat, ha

$$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$\overline{y}(0) = C_1\cdot e^{1\cdot 0}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2\, e^{3\cdot 0}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} C_1 + C_2 \\ C_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

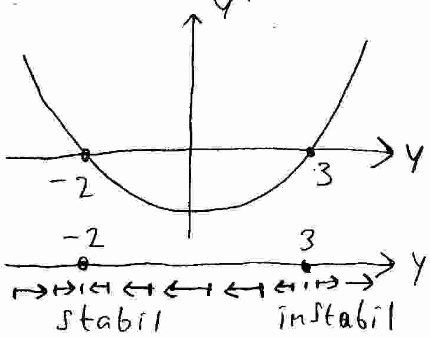
$$\left.\begin{aligned} C_1 + C_2 &= 6 \\ C_2 &= 7 \end{aligned}\right\} \rightarrow C_1 = -1$$

$$\overline{y}_{part}(x) = -1\cdot e^{1\cdot x}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 7\cdot e^{3x}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

3. Legyen $y' = f(y) = (y+2)(y-3)$.

- Keresd meg a DE fixpontjait!

$$(y+2)(y-3)=0 \longrightarrow y_1=-2, \quad y_2=3$$

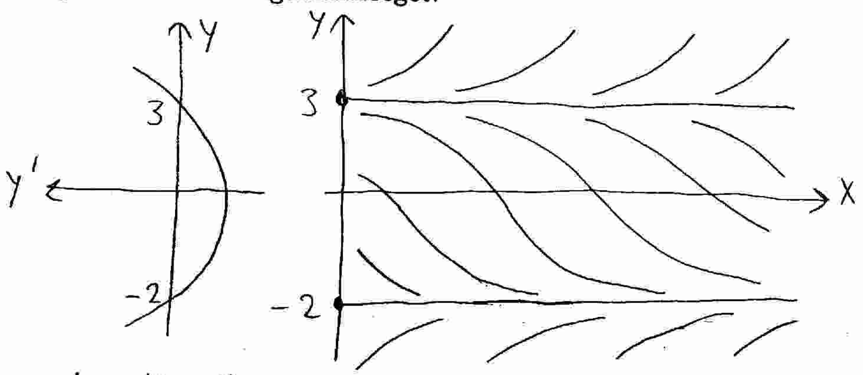


- Vizsgald meg azok stabilitasat! (indokold valaszodat!)

-2: stabil

3: instabil

- Rajzold le a DE megoldassereget!



Legyen $y' = -(3y+1)$.

- Keresd meg a DE altalanos megoldasat!

$$\frac{dy}{dx} = -(3y+1)$$

$$-\frac{dy}{3y+1} = dx$$

$$-\int \frac{dy}{3y+1} = \int dx$$

$$-\frac{\ln|3y+1|}{3} = x + C$$

$$\ln|3y+1| = -3(x+C)$$

$$|3y+1| = e^{-3(x+C)}$$

$$3y+1 = \pm e^{-3(x+C)}$$

$$y = \pm\frac{1}{3}e^{-3(x+C)} - \frac{1}{3}$$

$$= \pm\frac{1}{3}e^{-3C}\cdot e^{-3x} - \frac{1}{3}$$

$$= K\cdot e^{-3x} - \frac{1}{3}$$

- Keresd meg a DE partikularis megoldasat, ha $y(1) = 3$!

$$y(1)=3 \longrightarrow +\frac{1}{3}e^{-3(1+C)} - \frac{1}{3} = 3 \longrightarrow e^{-3(1+C)} = 10 \longrightarrow$$

$$-3(1+C) = \ln(10) \longrightarrow C = \frac{\ln(10)}{-3} - 1 \longrightarrow y_{part} = \frac{1}{3}e^{-3\left(x+\left[\frac{\ln 10}{-3}-1\right]\right)} - \frac{1}{3}$$

Vagy: $K\cdot e^{-3\cdot 1} - \frac{1}{3} = 3 \longrightarrow K = \frac{3\frac{1}{3}}{e^{-3\cdot 1}} = \frac{10}{3}\cdot e^{3}$

$$\longrightarrow y_{part} = \frac{10}{3}\cdot e^{3}\cdot e^{-3x} - \frac{1}{3}$$

4. Legyen $y'' + 9y = 0$.

- Ird fel a DE karaktèrisztikus egyenletet es keresd meg a gyokeit!

$$\lambda^2 + 9 = 0 \qquad \lambda_1 = 0 + 3i$$
$$\lambda_2 = 0 - 3i$$

- Ird fel a DE altalanos megoldasat!

$$y_{ált} = e^{0 \cdot x}\left(C_1 \cdot \cos(3x) + C_2 \cdot \sin(3x)\right) =$$
$$= C_1 \cos(3x) + C_2 \sin(3x)$$

- Ird fel a DE partikularis megoldasat, ha $y(0) = 3$, $y'(0) = 4$!

$$y'_{ált} = -C_1 \cdot 3\sin(3x) + C_2 \cdot 3 \cdot \cos(3x)$$
$$y(0) = 3 \longrightarrow C_1 \cdot \cos(3 \cdot 0) + C_2 \sin(3 \cdot 0) = C_1 \cdot 1 + C_2 \cdot 0 = 3 \longrightarrow C_1 = 3$$
$$y'(0) = 4 \longrightarrow -C_1 \cdot 3\sin(3 \cdot 0) + C_2 \cdot 3\cos(3 \cdot 0) = 3C_2 = 4 \longrightarrow C_2 = \frac{4}{3}$$
$$y_{part} = 3 \cdot \cos(3x) + \frac{4}{3}\sin(3x)$$

Ird fel f masodrendu kozelito Taylor-polinomjat az $(x, y) = (0, 0)$ pont korul, ha $f(x, y) = \cos(-3x - 7y^2 + 2xy)$!

$$f'_x = -\sin(-3x - 7y^2 + 2xy) \cdot (-3 + 2y)$$
$$f'_y = -\sin(-3x - 7y^2 + 2xy) \cdot (-14y + 2x)$$
$$f''_{xx} = -\cos(-3x - 7y^2 + 2xy) \cdot (-3 + 2y)^2$$
$$f''_{xy} = -\cos(-3x - 7y^2 + 2xy)(-14y + 2x)(-3 + 2y) - \sin(-3x - 7y^2 + 2xy) \cdot 2$$
$$f''_{yy} = -\cos(-3x - 7y^2 + 2xy)(-14y + 2x)^2 - \sin(-3x - 7y^2 + 2xy) \cdot (-14)$$

Ezek értéke, ha $x = y = 0$:

$f \to 1$

$f'_x \to 0$

$f'_y \to 0$

$f''_{xx} \to -9$

$f''_{xy} \to 0$

$f''_{yy} \to 0$

$$f(x,y) \approx T_2(x,y) = 1 + (0\ 0)\begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2}(x\ y)\begin{pmatrix} -9 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= 1 + 0x + 0y + \frac{1}{2}(-9x^2 + 2 \cdot 0 \cdot xy + 0y^2) = 1 - \frac{9}{2}x^2$$

VAGY: Mivel $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$, így

$$f = 1 - \frac{1}{2!}(-3x - 7y^2 + 2xy)^2 + \ldots \approx 1 - \frac{1}{2!}(-3x)^2$$