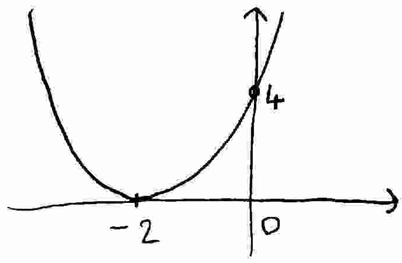
A. Zh2, Matematika I, 2012.december. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

- Rajzold le az alabbi fuggvenyt! $f(x) = (x+2)^2$



- Szamold ki az alabbi fuggveny inverzet! $f(x) = 2\ln(x+5)$

$y = 2\ln(x+5)$

$\frac{y}{2} = \ln(x+5)$

$e^{y/2} = e^{\ln(x+5)} = (x+5)$

$x = e^{y/2} - 5$

$f^{-1}(y) = e^{y/2} - 5$

$f^{-1}(x) = e^{x/2} - 5$

- Szamold ki a kovetkezo sorozat hatarerteket ahogy $n \to \infty$! $a_n = \frac{3n+1}{2n+4}$

$$\lim_{n\to\infty} \frac{3n+1}{2n+4} = \lim_{n\to\infty} \frac{3+1/n}{2+4/n} = \frac{3}{2}$$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \frac{1}{34x+1}$

$$\left[(34x+1)^{-1}\right]' = -1 \cdot (34x+1)^{-2} \cdot 34$$

- Ird fel az alabbi fuggveny elsorendu Taylor sorat (linearis kozeliteset) az $x = 0$ pont korul!
$f(x) = 1 + \sin(3x)$

$f(0) = 1 + \sin(3\cdot 0) = 1$

$f'(x) = \cos(3x)\cdot 3$

$f'(0) = \cos(3\cdot 0)\cdot 3 = 3$

$$f(x) \approx T_1(x) = 1 + 3x$$

2. (5 × 2 pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $\frac{2+x^3}{3+x^2}$

$$\left(\frac{2+x^3}{3+x^2}\right)' = \frac{(2+x^3)'\cdot(3+x^2)-(2+x^3)\cdot(3+x^2)'}{(3+x^2)^2} =$$

$$= \frac{3x^2\cdot(3+x^2)-(2+x^3)\cdot 2x}{(3+x^2)^2}$$

- $\left(\sqrt[3]{x^7}+\frac{1}{x^3}+\sin(3x)+\ln(2x)\right)' =$

$x^{7/3}$ $\quad x^{-3}$

$$= \frac{7}{3}x^{4/3}+(-3)x^{-4}+\cos(3x)\cdot 3+\frac{1}{2x}\cdot 2$$

- $\frac{\ln(4x)}{\frac{1}{x}+1}$

$$\left[\frac{\ln(4x)}{\frac{1}{x}+1}\right]' = \frac{[\ln(4x)]'\cdot\left(\frac{1}{x}+1\right)-\ln(4x)\left[\frac{1}{x}+1\right]'}{\left(\frac{1}{x}+1\right)^2} =$$

$$= \frac{\frac{1}{4x}\cdot 4\cdot\left(\frac{1}{x}+1\right)-\ln(4x)\cdot\left(-\frac{1}{x^2}\right)}{\left(\frac{1}{x}+1\right)^2}$$

- $\sin(\ln(2x))$

$$[\sin(\ln(2x))]' = \sin'(\ln(2x))\cdot(\ln(2x))' =$$

$$= \cos(\ln(2x))\cdot\frac{1}{2x}\cdot 2$$

- $\frac{\sqrt[8]{-x+2}}{ctg(2x)}$

$$\left[\frac{(-x+2)^{1/8}}{ctg(2x)}\right]' = \frac{\left[(-x+2)^{1/8}\right]'\cdot ctg(2x)-(-x+2)^{1/8}\cdot[ctg(2x)]'}{[ctg(2x)]^2} =$$

$$= \frac{\frac{1}{8}(-x+2)^{-7/8}\cdot(-1)\cdot ctg(2x)-(-x+2)^{1/8}\cdot\left[-\frac{1}{\sin^2(2x)}\cdot 2\right]}{[ctg(2x)]^2}$$

3. (5+5 pont)

$$f(0) = 2 + e^{0+1} = 2 + e$$

- Legyen $f(x) = 2 + e^{x+1}$! Szamold ki f harmadrendu Taylor-polinomjat az $x = 0$ pont korul!

$f'(x) = e^{x+1}$ $\quad$ $f'(0) = e$

$f''(x) = e^{x+1}$ $\quad$ $f''(0) = e$

$f'''(x) = e^{x+1}$ $\quad$ $f'''(0) = e$

$$f(x) \approx T_3(x) = (2+e) + e \cdot x + \frac{e}{2!} x^2 + \frac{e}{3!} x^3 =$$

$$= 2 + e + ex + \frac{e}{2} x^2 + \frac{e}{6} x^3$$

- Keresd meg az $f(x) = 2x^2 - x^4$ fuggveny szelsoertekeit es hatarozd meg azok tipusat!

$$f'(x) = 4x - 4x^3 = 4x(1-x^2) = 4x(1-x)(1+x)$$

$$4x(1-x)(1+x) = 0 \longrightarrow$$

$\longrightarrow$ $x_1 = 0$ $\quad$ $x_2 = 1$ $\quad$ $x_3 = -1$

$$f''(x) = 4 - 12x^2$$

$f''(x_1) = f''(0) = 4 - 12 \cdot 0^2 = 4$ $\quad$ $f''(x_2) = f''(1) = 4 - 12 \cdot 1^2 = -8$ $\quad$ $f''(x_3) = f''(-1) = 4 - 12 \cdot (-1)^2 = -8$

$4 > 0$ $\quad$ $-8 < 0$ $\quad$ $-8 < 0$

MINIMUM $\quad$ MAXIMUM $\quad$ MAXIMUM

3. (5+5 pont) $f(0)=2+e^{0+1}=2+e$

- Legyen $f(x) = 2 + e^{x+1}$!. Szamold ki f harmadrendu Taylor-polinomjat az $x = 0$ pont korul!

$f'(x) = e^{x+1}$ $\qquad f'(0) = e$

$f''(x) = e^{x+1}$ $\qquad f''(0) = e$

$f'''(x) = e^{x+1}$ $\qquad f'''(0) = e$

$$f(x) \approx T_3(x) = (2+e) + e \cdot x + \frac{e}{2!}x^2 + \frac{e}{3!}x^3 =$$

$$= 2+e+ex+\frac{e}{2}x^2+\frac{e}{6}x^3$$

- Keresd meg az $f(x) = 2x^2 - x^4$ fuggveny szelsoertekeit es hatarozd meg azok tipusat!

$f'(x) = 4x - 4x^3 = 4x(1-x^2) = 4x(1-x)(1+x)$

$4x(1-x)(1+x) = 0 \longrightarrow$

$\longrightarrow x_1 = 0 \qquad x_2 = 1 \qquad x_3 = -1$

$f''(x) = 4 - 12x^2$

$f''(x_1) = f''(0) = 4 - 12 \cdot 0^2 = 4$

$4 > 0$

MINIMUM

$f''(x_2) = f''(1) = 4 - 12 \cdot 1^2 = -8$

$-8 < 0$

MAXIMUM

$f''(x_3) = f''(-1) = 4 - 12 \cdot (-1)^2 = -8$

$-8 < 0$

MAXIMUM

B. Zh2, Matematika I, 2012.december. NEPTUN: Gyak.Vez.:

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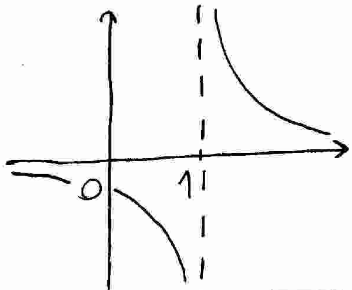
1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

- Rajzold le az alabbi fuggvenyt! $f(x) = \frac{1}{(x-1)}$



- Szamold ki az alabbi fuggveny inverzet! $f(x) = \sqrt{x-4}$

$y = \sqrt{x-4} \longrightarrow y \geq 0,\ R_f = [0, \infty) = D_{f^{-1}}$

$y^2 = x - 4 \qquad f^{-1}(y) = y^2 + 4$

$x = y^2 + 4 \qquad f^{-1}(x) = x^2 + 4, \quad x \geq 0$

- Szamold ki a kovetkezo sorozat hatarerteket ahogy $n \to \infty$! $a_n = (1 - \frac{2}{5n})^n$

$$\lim_{n\to\infty} \left(1 - \frac{2}{5n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{-2/5}{n}\right)^n = e^{-2/5}$$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \ln(-2x-9)$

$$\left[\ln(-2x-9)\right]' = \frac{1}{-2x-9} \cdot (-2)$$

- Legyen $f(x) = -x^2 + 6x$. Melyik x_{sz} pontban nulla f derivaltja? Mennyi $f''(x_{sz})$? Milyen szelsoerteke van f-nek az x_{sz} pontban?

$f'(x) = -2x + 6 \longrightarrow -2x + 6 = 0 \longrightarrow x_{sz} = 3$

$f''(x) = -2 \qquad f''(x_{sz}) = f''(3) = -2 < 0$

$\downarrow$

MAXIMUM

2. (5 × 2 pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $tg(3x)\ln(-3x)$

$$[tg(3x)\cdot \ln(-3x)]' = [tg(3x)]'\cdot \ln(-3x) + tg(3x)\cdot[\ln(-3x)]' =$$
$$= \frac{1}{\cos^2(3x)}\cdot 3\cdot \ln(-3x) + tg(3x)\cdot\frac{1}{-3x}\cdot(-3)$$

- $\sin(\sqrt{x+1})$

$$[\sin(x+1)^{1/2}]' = \sin'((x+1)^{1/2})\cdot[(x+1)^{1/2}]' =$$
$$= \cos((x+1)^{1/2})\cdot\frac{1}{2}(x+1)^{-1/2}$$

- $\left(\sqrt[2]{8x} + \frac{1}{(3x)^4} + tg(x) + ctg(4x)\right)' = \frac{1}{2}(8x)^{-1/2}\cdot 8 + (-4)(3x)^{-5}\cdot 3 + \frac{1}{\cos^2 x} - \frac{1}{\sin^2(4x)}\cdot 4$

$\sqrt[2]{8x} \downarrow (8x)^{1/2}$

$\frac{1}{(3x)^4} \downarrow (3x)^{-4}$

- $\frac{\sqrt{4x-2}}{\sin(x-1)}$

$$\left[\frac{(4x-2)^{1/2}}{\sin(x-1)}\right]' = \frac{[(4x-2)^{1/2}]'\cdot\sin(x-1) - (4x-2)^{1/2}\cdot[\sin(x-1)]'}{(\sin(x-1))^2}$$
$$= \frac{\frac{1}{2}(4x-2)^{-1/2}\cdot\sin(x-1) - (4x-2)^{1/2}\cdot\cos(x-1)}{(\sin(x-1))^2}$$

- $2^{3x}e^{-x+2}$

$$[2^{3x}\cdot e^{-x+2}]' = [2^{3x}]'\cdot e^{-x+2} + 2^{3x}\cdot[e^{-x+2}]' =$$
$$= [\ln 2\cdot 2^{3x}\cdot 3]\cdot e^{-x+2} + 2^{3x}\cdot e^{-x+2}\cdot(-1)$$

vagy
$$= [e^{\ln 2\cdot(3x) + (-x+2)}]' = [e^{\ln 2\cdot(3x) - x+2}]' =$$
$$= e^{\ln 2\cdot(3x) - x+2}\cdot(\ln 2\cdot 3 - 1)$$

3. ((2+1+1+3)+(2+1) pont)

- Legyen $f(x) = \frac{(x-1)}{(2-x)x^2}$!
  - Hatarozd meg f szamlalojanak es nevezojenek gyokeit es azok multiplicitasat!

gyök:	$x_1 = 1$	$x_1 = 0$	$x_2 = 2$
	számláló	nevező	
mult:	1	2	1

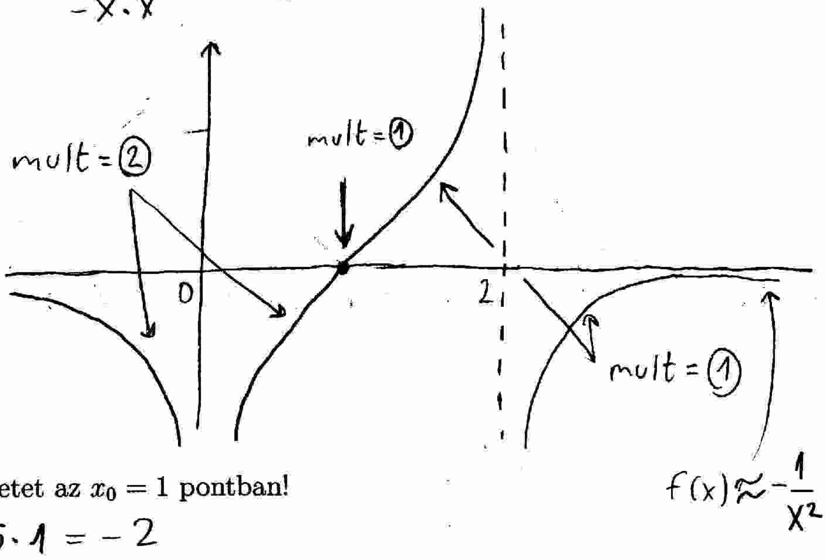
  - Mennyi $\lim_{x\to\infty} f(x)$?

$$\lim_{x\to\infty} \frac{(x-1)}{(2-x)x^2} = \lim_{x\to\infty} \frac{x}{-x\cdot x^2} = \lim_{x\to\infty} -\frac{1}{x^2} = 0$$

  - Milyen f elojele ha x nagyon nagy?

ha $|x|$ nagy, akkor $f(x) \approx \frac{x}{-x\cdot x^2} = -\frac{1}{x^2} < 0$, ~~pozitiv~~ negativ

  - Rajzold le f-t!



- Legyen $f(x) = 3x^2 - 5x$!
  - Ird fel f erintojenek az $y(x)$ egyenletet az $x_0 = 1$ pontban!

$$f(x_0) = f(1) = 3\cdot 1^2 - 5\cdot 1 = -2$$

$$f'(x) = 6x - 5, \quad f'(x_0) = f'(1) = 6\cdot 1 - 5 = 1$$

$$y(x) = f(x_0) + f'(x_0)\cdot(x - x_0) = -2 + 1\cdot(x-1) = x - 3$$

  - Mennyi $f(1+\Delta x) - y(1+\Delta x)$?

$$\left[3\cdot(1+\Delta x)^2 - 5\cdot(1+\Delta x)\right] - \left[(1+\Delta x) - 3\right] =$$

$$= \left[3\cdot(1 + 2\Delta x + \Delta x^2) - 5 - 5\Delta x\right] - \left[-2 + \Delta x\right] = 3\Delta x^2$$

4.((3+2)+3+2 pont)

- – Vizsgald meg az $a_n = \frac{2^{2n+1}3^{n-1}}{7^{2n-5}}$ sorozat monotonitasat!

$$a_n = \left(\frac{2^2 \cdot 3}{7^2}\right)^n \cdot \underbrace{\frac{2 \cdot 3^{-1}}{7^{-5}}}_{\frac{2 \cdot 3^{-1}}{7^{-5}} > 0} . \qquad \frac{2^2 \cdot 3}{7^2} = \frac{12}{49} \qquad 0 < \frac{12}{49} < 1 ,$$

$\Rightarrow$ a_n szig. mon. csökkenő

  – Mennyi $\lim_{n\to\infty} a_n$?

$$\lim_{n\to\infty} \left(\frac{12}{49}\right)^n \cdot \frac{2 \cdot 7^5}{3} = 0$$

- Mennyi $\lim_{n\to\infty} \left(\frac{3n-5}{3n}\right)^{2n-1}$?

$$\lim_{n\to\infty} \left(1 + \frac{-5/3}{n}\right)^{2n-1} = \lim_{n\to\infty} \left[\left(1 + \frac{-5/3}{n}\right)^n\right]^2 \cdot \left(1 + \frac{-5/3}{n}\right)^{-1} =$$

$$= \left[e^{-5/3}\right]^2 \cdot 1 = e^{-10/3}$$

- Mennyi $\lim_{x\to\infty} \frac{e^{2x}}{x}$?

$$\lim_{x\to\infty} \frac{e^{2x}}{x} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{e^{2x} \cdot 2}{1} = \infty$$

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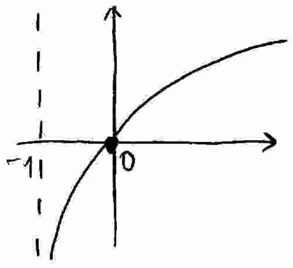


- Szamold ki az alabbi fuggveny inverzet! $f(x) = e^{x-3}$

$y = e^{x-3}$

$\ln y = \ln(e^{x-3}) = x-3$

$x = \ln y + 3$

$f^{-1}(y) = \ln y + 3$

$f^{-1}(x) = \ln x + 3$

- Szamold ki a kovetkezo sorozat hatarerteket ahogy $n \to \infty$! $a_n = \left(2 - \frac{2}{5n}\right)^n$

$\lim\limits_{n\to\infty} \left(2 - \frac{2}{5n}\right)^n = \lim\limits_{n\to\infty} 2^n = \infty$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \cos(-2x-9)$

$\left[\cos(-2x+9)\right]' = -\sin(-2x+9)\cdot(-2)$

- Legyen $f(x) = 4x + x^2$. Melyik x_{sz} pontban nulla f derivaltja? Mennyi $f''(x_{sz})$? Milyen szelsoerteke van f-nek az x_{sz} pontban?

$f'(x) = 4 + 2x \longrightarrow 4 + 2x_{sz} = 0 \longrightarrow x_{sz} = -2$

$f''(x) = 2$

$f''(x_{sz}) = f''(-2) = 2 > 0$

$\downarrow$

MINIMUM

2. (5 × 2 pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $\sqrt[4]{\cos(-4x)}$

$$\left(\left[\cos(-4x)\right]^{1/4}\right)' = \frac{1}{4}\left[\cos(-4x)\right]^{-3/4} \cdot \left[\cos(-4x)\right]' =$$
$$= \frac{1}{4}\left[\cos(-4x)\right]^{-3/4} \cdot \left[-\sin(-4x)\right] \cdot (-4)$$

- $x^3 tg(2x-1)$

$$\left[x^3 tg(2x-1)\right]' = (x^3)' \cdot tg(2x-1) + x^3 \cdot \left[tg(2x-1)\right]' =$$
$$= 3x^2 tg(2x-1) + x^3 \frac{1}{\cos^2(2x-1)} \cdot 2$$

- $\ln\left(\frac{3+2x}{2-5x}\right)$

$$\left[\ln\left(\frac{3+2x}{2-5x}\right)\right]' = \ln'\left(\frac{3+2x}{2-5x}\right) \cdot \left(\frac{3+2x}{2-5x}\right)' = \frac{1}{\frac{3+2x}{2-5x}} \cdot \left(\frac{3\cdot(2-5x)-(3+2x)\cdot(-5)}{(2-5x)^2}\right)$$

$(3+2x)'$ $(2-5x)'$

- $\left(\frac{1}{\sqrt{3x}} + \sqrt{4x} + arctg(-2x) + arcsin(3x)\right)' = -\frac{1}{2}(3x)^{-3/2}\cdot 3 + \frac{1}{2}(4x)^{-1/2}\cdot 4 +$

$$+ \frac{1}{1+(-2x)^2}\cdot(-2) + \frac{1}{\sqrt{1-(3x)^2}}\cdot 3$$

$(3x)^{-1/2}$

$(4x)^{1/2}$

- $ctg(\ln(-x+1))$

$$\left[ctg(\ln(-x+1))\right]' = ctg'(\ln(-x+1)) \cdot \left[\ln(-x+1)\right]' =$$
$$= -\frac{1}{\sin^2(\ln(-x+1))} \cdot \frac{1}{-x+1} \cdot (-1)$$

3. (2+3+2+3 pont) Legyen $f(x) = x^2 - x^3$. $= x^2(1-x) = -1 \cdot x^2 \cdot (x-1)$

- Mennyi f, f' f''?

$$f'(x) = 2x - 3x^2$$
$$f''(x) = 2 - 6x$$

- Hol vannak f szelsoertekei es milyen tipusuak?

$$f'(x) = 2x - 3x^2 = x(2-3x) = 0 \longrightarrow x_1 = 0 \qquad x_2 = \frac{2}{3}$$

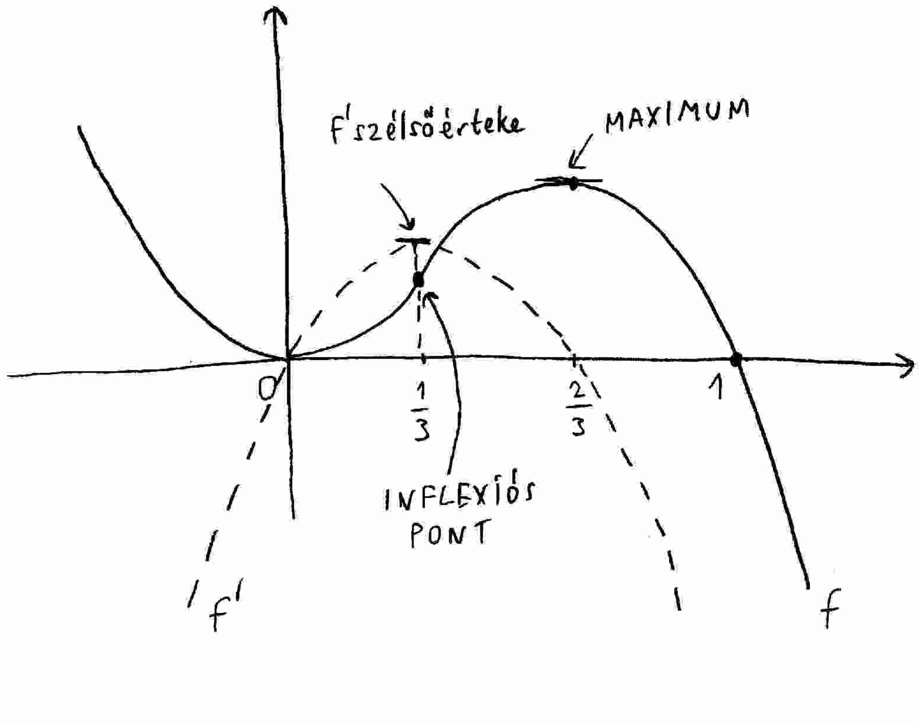
$$f''(x_1) = f''(0) = 2 - 6 \cdot 0 = 2 > 0 \qquad f''\left(\frac{2}{3}\right) = 2 - 6 \cdot \frac{2}{3} = -2 < 0$$

(lokális) MINIMUM MAXIMUM

- Hol van f inflexios pontja?

$$f''(x_{inf}) = 2 - 6x = 0 \longrightarrow x_{infl} = \frac{1}{3}$$

- Rajzold le f-t es f'-t (utobbit szaggatott vonallal) ugyanarra az abrara!



4.((2+2+1+3)+2 pont)

- Legyen $f(x) = \ln(x-1) + 2$.
  - Mennyi $f^{-1}(x)$?

$$y = \ln(x-1) + 2$$
$$y - 2 = \ln(x-1)$$
$$e^{y-2} = e^{\ln(x-1)} = x - 1$$
$$x = e^{y-2} + 1$$

$$f^{-1}(y) = e^{y-2} + 1$$
$$f^{-1}(x) = e^{x-2} + 1$$

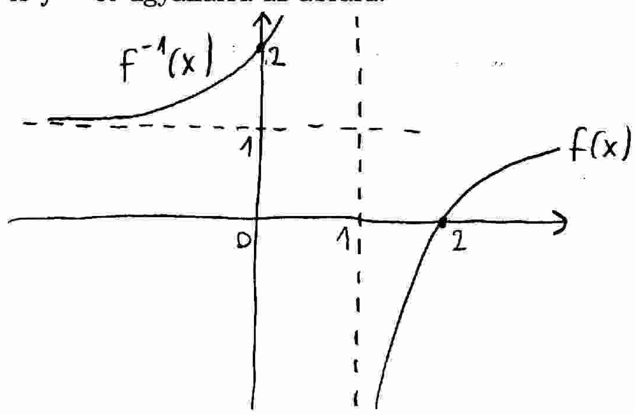
  - Mennyi D_f es R_f?

$$D_f = (1, \infty), \quad R_f = (-\infty, \infty) = \mathbb{R}$$

  - Mennyi $D_{f^{-1}}$ es $R_{f^{-1}}$?

$$D_{f^{-1}} = \mathbb{R} \qquad R_{f^{-1}} = (1, \infty)$$

  - Rajzold le f-t es f^{-1}-et ugyanarra az abrara!



Mennyi $\lim_{n\to\infty} \frac{(-1)^n}{2n+3}$?

$$\lim_{n\to\infty} (-1)^n \cdot \frac{1}{2n+3} = \lim_{n\to\infty} (-1)^n \cdot 0 = \lim_{n\to\infty} 0 = 0$$

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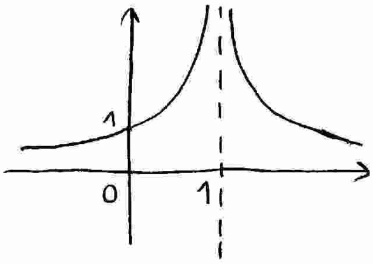
- Rajzold le az alabbi fuggvenyt! $f(x) = \frac{1}{(x-1)^2}$



- Szamold ki az alabbi fuggveny inverzet! $f(x) = 5x + 6$

$y = 5x + 6 \qquad f^{-1}(y) = \frac{y-6}{5}$

$y - 6 = 5x$

$x = \frac{y-6}{5} \qquad f^{-1}(x) = \frac{x-6}{5}$

- Szamold ki a kovetkezo sorozat hatarerteket ahogy $n \to \infty$! $a_n = \left(0.5 - \frac{2}{5n}\right)^n$

$$\lim_{n \to \infty} \left(0.5 - \frac{2}{5n}\right)^n = \lim_{n \to \infty} 0.5^n = 0$$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = tg(3x)$

$$\left[tg(3x)\right]' = \frac{1}{\cos^2(3x)} \cdot 3$$

- Ird fel az alabbi fuggveny elsorendu Taylor polinomat (linearis kozeliteset) az $x = 0$ pont korul! $f(x) = \frac{1}{x+2}$

$f(0) = \frac{1}{0+2} = 1/2$

$f'(x) = \left[(x+2)^{-1}\right]' = -1 \cdot (x+2)^{-2} \qquad f'(0) = -1 \cdot \frac{1}{(0+2)^2} = -\frac{1}{4}$

$$f(x) \approx T_1(x) = \frac{1}{2} - \frac{1}{4} x$$

2. (5 × 2 pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $\left(3x^5 + (2x)^4 + \frac{1}{x^{1/3}} + e^{-2x}\right)' = 3 \cdot 5 \cdot x^4 + 4 \cdot (2x)^3 \cdot 2 + (-\frac{1}{3})x^{-4/3} + (-2)e^{-2x}$

$\downarrow$

$x^{-1/3}$

- $\frac{\cos(2x)}{x^2+1}$

$$\left[\frac{\cos(2x)}{x^2+1}\right]' = \frac{[\cos(2x)]' \cdot (x^2+1) - \cos(2x) \cdot [x^2+1]'}{(x^2+1)^2} =$$

$$= \frac{-\sin(2x) \cdot 2 \cdot (x^2+1) - \cos(2x) \cdot 2x}{(x^2+1)^2}$$

- $x \cdot arctg(x)$

$$[x \cdot arctg\, x]' = x' \cdot arctg\, x + x \cdot (arctg\, x)' =$$

$$= 1 \cdot arctg\, x + x \frac{1}{1+x^2}$$

- $tg(\ln(-2x))$

$$[tg(\ln(-2x))]' = tg'(\ln(-2x)) \cdot (\ln(-2x))' =$$

$$= \frac{1}{\cos^2(\ln(-2x))} \cdot \frac{1}{-2x} \cdot (-2)$$

- $\left(\frac{3}{\sqrt{2x}} + \sqrt{2x-1} + arctg(-x) + arcsin(2x)\right)' = 3 \cdot (-\frac{1}{2})(2x)^{-3/2} \cdot 2 + \frac{1}{2}(2x-1)^{-1/2} \cdot 2$

$$+ \frac{1}{1+(-x)^2} \cdot (-1) + \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

$\downarrow$

$3 \cdot (2x)^{-1/2}$

$\downarrow$

$(2x-1)^{1/2}$

3. (((2+1+2)+(2+3) pont)

- Legyen $f(x) = x^3 - 5x$.
  - Mennyi $\frac{f(1+\Delta x)-f(1)}{\Delta x}$? (Segitseg: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$)

$$= \frac{[(1+\Delta x)^3 - 5(1+\Delta x)] - [1^3 - 5\cdot 1]}{\Delta x} =$$

$$= \frac{[(1+3\Delta x+3\Delta x^2+\Delta x^3) - (5+5\Delta x)] - [1^3-5]}{\Delta x} = \frac{-2\Delta x + 3\Delta x^2 + \Delta x^3}{\Delta x} =$$

$$= -2 + 3\Delta x + \Delta x^2$$

  - Mennyi az elozo kifejezes limesze ahogy $\Delta x \to 0$?

$$\lim_{\Delta x \to 0} -2 + 3\Delta x + \Delta x^2 = -2$$

  - Ird fel f erintojenek az $y(x)$ egyenletet az $x_0 = 1$ pontban!

$$f'(1) = -2, \quad f(1) = 1^3 - 5\cdot 1 = -4$$

$$y(x) = f(1) + f'(1)\cdot(x-1) = -4 - 2(x-1) =$$

$$= -2x - 2$$

- Mennyi $\lim_{x\to 0} \frac{e^{-5x}-1}{tg(7x)}$?

$$\lim_{x\to 0} \frac{e^{-5x}-1}{tg(7x)} = \frac{0}{0} = \lim_{x\to 0} \frac{e^{-5x}\cdot(-5)}{\frac{1}{\cos^2(7x)}\cdot 7} = \frac{e^{-5\cdot 0}\cdot(-5)}{\frac{1}{\cos^2(7\cdot 0)}\cdot 7} = -\frac{5}{7}$$

- Mennyi $\lim_{x\to\infty} \frac{x}{\ln(x)}$?

$$\lim_{x\to\infty} \frac{x}{\ln x} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{1}{1/x} = \lim_{x\to\infty} x = \infty$$

4. ((2+2+3+1)+2 pont)

- Legyen $f(x) = (x + 2)x^2(6 - x)^3$!
  - Hatarozd meg f gyokeit es azok multiplicitasat!

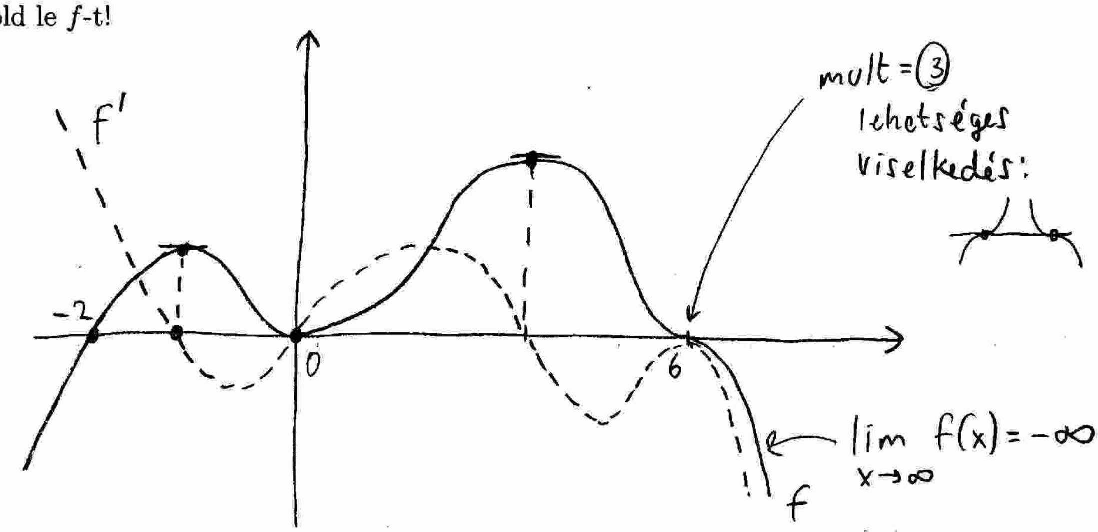
gyök: $x_1 = -2$ $x_2 = 0$ $x_3 = 6$

mult: 1 2 3

  - Mennyi $\lim_{x\to\infty} f(x)$?

$$\lim_{x\to\infty} (x+2)x^2(6-x)^3 = \lim_{x\to\infty} x \cdot x^2 \cdot (-x)^3 = \lim_{x\to\infty} -x^6 = -\infty$$

  - Rajzold le f-t!



  - Rajzold le ugyanarra az abrara szaggatott vonallal f'-t is!

- Mennyi $\lim_{x\to\infty} \frac{3x^2+2}{4x+5}$?

$$\lim_{x\to\infty} \frac{3x^2+2}{4x+5} = \lim_{x\to\infty} \frac{3x + 2/x}{4 + 5/x} = \frac{\infty}{4} = \infty$$