B. PotZh1, Matematika I, 2012.december. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5 × 2 pont.

- Ird fel az alabbi polinom gyokeit! $x^2 + 9$. Ird fel a polinom gyoktenyezos alakjat!

gyökök: $x^2 = -9 \rightarrow$

$\rightarrow x_1 = 3i, \; x_2 = -3i$

$$x^2+9 = (x-3i)(x-[-3i]) =$$
$$= (x-3i)(x+3i)$$

- Szamold ki: $\frac{4-3i}{3-2i}$, $= \frac{4-3i}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{12-6i^2-i}{3^2+2^2} = \frac{18}{13} - \frac{1}{13} i$

- $\bar{v}_1 = [2, 1, 3]$, $\bar{v}_2 = [-2, 1, 4]$. Mennyi $\bar{v}_1 \times \bar{v}_2$? $=$

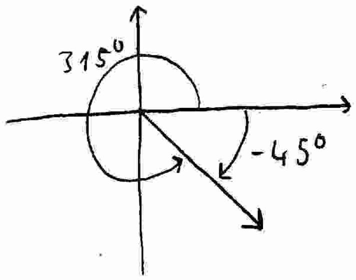
$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & 3 \\ -2 & 1 & 4 \end{vmatrix} = \bar{i} \underbrace{\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix}}_{1\cdot4-3\cdot1} - \bar{j} \underbrace{\begin{vmatrix} 2 & 3 \\ -2 & 4 \end{vmatrix}}_{2\cdot4-3\cdot(-2)} + k \underbrace{\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix}}_{2\cdot1-1\cdot(-2)} = \bar{i} - 14\bar{j} + 4\bar{k} =$$
$$= [1, -14, 4]$$

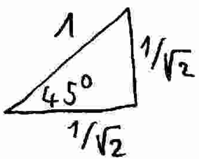
- Mennyi a kovetkezo maradekos osztas hanyadosa es maradeka?
  $(6x + 3) : (5x + 8) = 6/5$

$$\frac{-\left(6x + \frac{48}{5}\right)}{-\frac{33}{5}}$$

tehát $6x+3 = (5x+8)\cdot\frac{6}{5} - \frac{33}{5}$

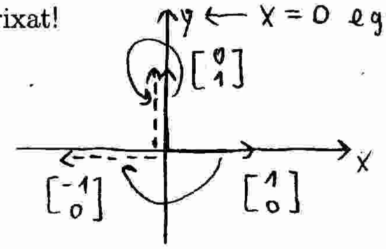
- Ird fel z algebrai alakjait, ahol
  $z = 4(\cos 315^\circ + i \sin 315^\circ)$ $= 4 \cdot \left(\frac{1}{\sqrt{2}} + i\left(-\frac{1}{\sqrt{2}}\right)\right) = \frac{4}{\sqrt{2}} - i\frac{4}{\sqrt{2}} = 2\sqrt{2} - 2\sqrt{2}\, i$





2. (3+2+5 pont)

- Ird fel az $x = 0$ egyenesre torteno meroleges tukrozes linearis transzformaciojat, illetve annak T matrixat!



$$\left.\begin{array}{l} T\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix} \\ T\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}\end{array}\right\} \to T = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$$

második oszlop

- Mennyi T^2 es T^{-1} ?

kétszeres tükrözés = identikus transzformáció, tehát $T^2 = E$

vagy $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$. $T^{-1} = T$, mivel $\left.\begin{array}{l} TT^{-1} = E \\ TT = E\end{array}\right\} T^{-1} = T$

- Keresd meg T sajatertekeit es sajatvektorait!

saját érték: $0 = \det(T - \lambda E) = \begin{vmatrix}-1-\lambda & 0\\ 0 & 1-\lambda\end{vmatrix} = (-1-\lambda)(1-\lambda) - 0\cdot 0 \to$

$\to \lambda_1 = 1, \lambda_2 = -1$

$\lambda_1 = 1$

$\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = 1\cdot\begin{bmatrix}x\\y\end{bmatrix}$

$\left.\begin{array}{l} -x = x \\ y = y\end{array}\right\} \to \begin{bmatrix}0\\y\end{bmatrix}$

$V_1 = \begin{bmatrix}0\\1\end{bmatrix}$

$\lambda_2 = -1$

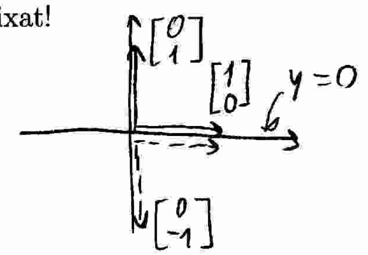
$\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = -1\cdot\begin{bmatrix}x\\y\end{bmatrix}$

$\left.\begin{array}{l} -x = -x \\ y = -y\end{array}\right\} \to \begin{bmatrix}x\\0\end{bmatrix}$

$V_2 = \begin{bmatrix}1\\0\end{bmatrix}$

Megjegyzés: mivel T diagonális, így a sajátértekek a diagonális elemek, a sajátvektorok pedig a standard bázist alkothatják.

- Ird fel az $y = 0$ egyenesre torteno meroleges tukrozes linearis transzformaciojat, illetve annak S matrixat!



$$\left.\begin{array}{l} S\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} \\ S\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix}\end{array}\right\} \to S = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

- Mennyi TS ?

$$TS = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix},$$

ez az origóra történő középpontos tükrözés mátrixa.

3. ((5+1)+4 pont)

- Oldd meg a Gauss-elimináció segítségével a következő egyenletrendszert! Ellenorizd az eredmenyt!

$$x - y + 2z = 2$$
$$x + y - z = 1$$
$$-2x - y + 2z = -1$$

$$\left|\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 1 & 1 & -1 & 1 \\ -2 & -1 & 2 & -1 \end{array}\right| \xrightarrow[\text{III.} \to \text{III.}+2\cdot\text{I.}]{\text{II.} \to \text{II.}-1\cdot\text{I.}} \left|\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 2 & -3 & -1 \\ 0 & -3 & 6 & 3 \end{array}\right| \xrightarrow{\text{III.} \to \text{III.}+\frac{3}{2}\text{II.}}$$

$$\left|\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 3/2 & 3/2 \end{array}\right| \begin{array}{l} \to 1x - 1\cdot 1 + 2\cdot 1 = 2 \longrightarrow x = 1 \\ \to 2y - 3\cdot 1 = -1 \longrightarrow y = 1 \\ \to \frac{3}{2}z = \frac{3}{2} \longrightarrow z = 1 \end{array}$$

Ell.: $1 - 1 + 2\cdot 1 = 2$
$1 + 1 - 1 = 1$
$-2\cdot 1 - 1 + 2\cdot 1 = -1$

Mennyi u es v, ha

$$\text{(I.)} \qquad u - iv = 2$$
$$\text{(II.)} \qquad (1+i)u - v = 1$$

$$U - iV = 2 \longrightarrow U = 2 + iV$$

$$\text{(II.)} \longrightarrow (1+i)(2+iV) - V = 1$$

$$(1\cdot i + i^2\cdot 1)V + 2 + 2i = 1$$

$$(-2+i)V = -1 - 2i$$

$$V = \frac{-1-2i}{-2+i}$$

$$V = \frac{-1-2i}{-2+i}\,\frac{-2-i}{-2-i} = \frac{\overbrace{2+2i^2}^{0}+5i}{2^2+1^2} = i$$

$$U = 2 + iV = 2 + i\cdot i = 2 - 1 = 1$$

4. (1+1+1+1+2+2+2 pont)
$\bar{v}_1 = [2, 2, 3]$, $\bar{v}_2 = [-3, 1, 0]$, $\bar{v}_3 = [3, 1, 1]$.

- Mennyi $3\bar{v}_1 - 2\bar{v}_2$?

$$3[2,2,3]-2[-3,1,0]=[6,6,9]-[-6,2,0]=[12,4,9]$$

- Mennyi $\bar{v}_1\bar{v}_2$?

$$[2,2,3][-3,1,0]=2\cdot(-3)+2\cdot 1+3\cdot 0=-4$$

- Mennyi $\bar{v}_1 \times \bar{v}_2$? =

$$=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 2 & 3 \\ -3 & 1 & 0 \end{vmatrix} = \bar{i}\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} - \bar{j}\begin{vmatrix} 2 & 3 \\ -3 & 0 \end{vmatrix} + \bar{k}\begin{vmatrix} 2 & 2 \\ -3 & 1 \end{vmatrix} = -3\bar{i}-9\bar{j}+8\bar{k}$$

- Mennyi $(\bar{v}_1\bar{v}_2\bar{v}_3)$? =

$$\begin{vmatrix} 2 & 2 & 3 \\ -3 & 1 & 0 \\ 3 & 1 & 1 \end{vmatrix} = 2\cdot\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 2\cdot\begin{vmatrix} -3 & 0 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} -3 & 1 \\ 3 & 1 \end{vmatrix} = 2\cdot 1-2\cdot(-3)+3\cdot(-6) = = -10$$

vagy $=(\bar{V}_1 \times \bar{V}_2)\cdot V_3 = [-3,-9,8]\cdot[3,1,1] = -3\cdot 3-9\cdot 1+8\cdot 1$

- Meroleges-e $\bar{v}_1$ es $\bar{v}_2$? Miert?

Nem, mert $\bar{V}_1\cdot\bar{V}_2=-4 \neq 0$

- Egy sikba esik-e $\bar{v}_1$, $\bar{v}_2$ es $\bar{v}_3$? Miert?

Nem, mert $(\bar{V}_1\bar{V}_2\bar{V}_3)=-10 \neq 0$

- Mekkora a $\bar{v}_1$ es $\bar{v}_2$ vektorok altal bezart szog koszinusza??

$$\bar{V}_1\cdot\bar{V}_2 = |\bar{V}_1|\cdot|\bar{V}_2|\cdot\cos\alpha \longrightarrow \cos\alpha = \frac{\bar{V}_1\cdot\bar{V}_2}{|\bar{V}_1|\cdot|V_2|}$$

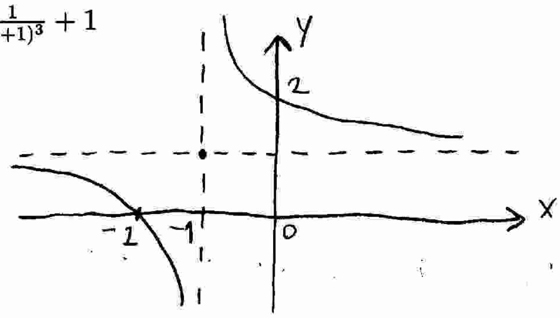
$$\cos\alpha = \frac{-4}{\sqrt{2^2+2^2+3^2}\,\sqrt{(-3)^2+1^2+0^2}} = \frac{-4}{\sqrt{17}\sqrt{10}}$$

B. PotZh2, Matematika I, 2012.december. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

- Rajzold le az alabbi fuggvenyt! $f(x) = \frac{1}{(x+1)^3} + 1$



- Szamold ki az alabbi fuggveny inverzet! $f(x) = \sqrt{5x+6}$

$y = \sqrt{5x+6}$ $\quad f^{-1}(y) = \frac{y^2-6}{5}$

$y^2 = 5x+6$ $\quad f^{-1}(x) = \frac{x^2-6}{5}$

$x = \frac{y^2-6}{5}$

$\left[R_f = [0, \infty)\text{, tehát } D_{f^{-1}} = R_f = [0, \infty)\text{, vagyis } f^{-1}(x) = \frac{x^2-6}{5}\text{, ahol } x \geq 0 \right]$

- Szamold ki a kovetkezo sorozat hatarerteket ahogy $n \to \infty$! $a_n = \left(1 - \frac{3}{4n}\right)^n$

$$\lim_{n\to\infty} \left(1 + \frac{-3/4}{n}\right)^n = e^{-3/4}$$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = ctg(-3x+1)$

$$\left[ctg(-3x+1)\right]' = \frac{1}{\sin^2(-3x+1)} \cdot (-3)$$

- Ird fel az alabbi fuggveny elsorendu Taylor polinomat (linearis kozeliteset) az $x = 0$ pont korul!

$f(x) = \frac{1}{3x+2} = (3x+2)^{-1}$ $\quad f(0) = 1/2$

$f'(x) = -1 \cdot (3x+2)^{-2} \cdot 3$ $\quad f'(0) = -3/4$

$$f(x) \approx T_1(x) = \frac{1}{2} - \frac{3}{4}x$$

2. (5 × 2 pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $\left(\sqrt[4]{tg(-4x+4)}\right)' = \left[\left(tg(-4x+4)\right)^{1/4}\right]' = \frac{1}{4}\cdot\left(tg(-4x+4)\right)^{-3/4}\cdot\left[tg(-4x+4)\right]'$

$$= \frac{1}{4}\left(tg(-4x+4)\right)^{-3/4}\cdot\frac{1}{\cos^2(-4x+4)}\cdot 4$$

- $\left(\sqrt[3]{5x}\cdot ctg(2x-1)\right)' = \left[(5x)^{1/3}\right]'\cdot ctg(2x-1) + \sqrt[3]{5x}\cdot\left(ctg(2x-1)\right)' =$

$$= \frac{1}{3}(5x)^{-2/3}\cdot 5\cdot ctg(2x-1) + \sqrt[3]{5x}\cdot\left(-\frac{1}{\sin^2(2x-1)}\cdot 2\right)$$

- $\left(\ln\left(\frac{3x+2x^2}{\sin(x)-5x}\right)\right)' = \ln'\left(\frac{3x+2x^2}{\sin x-5x}\right)\cdot\left(\frac{3x+2x^2}{\sin x-5x}\right)' =$

$$= \frac{1}{\frac{3x+2x^2}{\sin x-5x}}\cdot\frac{\underbrace{(3+4x)}_{(3x+2x^2)'}(\sin x-5x)-(3x+2x^2)\underbrace{(\cos x-5)}_{(\sin x-5x)'}}{(\sin x-5x)^2}$$

- $\left[\underbrace{\frac{3}{\sqrt{3x-4}}}_{3\cdot(3x-4)^{-1/2}} + \underbrace{\sqrt{4x^7}}_{\sqrt{4}\cdot x^{7/4}} + arctg(-2x) + arcsin(3x)\right]' =$

$$= 3\cdot\left(-\frac{1}{2}\right)\cdot(3x-4)^{-3/2}\cdot 3 + \sqrt{4}\cdot\frac{7}{4}\cdot x^{3/4} +$$

$$+ \frac{1}{1+(-2x)^2}\cdot 2 + \frac{1}{\sqrt{1-(3x)^2}}\cdot 3$$

- $\left[\ln(\ln(5x+1))\right]' = \ln'(\ln(5x+1))\cdot\left[\ln(5x+1)\right]' =$

$$= \frac{1}{\ln(5x+1)}\cdot \ln'(5x+1)\cdot(5x+1)' =$$

$$= \frac{1}{\ln(5x+1)}\cdot\frac{1}{5x+1}\cdot 5$$

3. (2+3+2+3 pont) Legyen $f(x) = x^2 - x^4$.

- Mennyi f, f' f''?

$$f'(x) = 2x - 4x^3$$

$$f''(x) = 2 - 12x^2$$

- Hol vannak f szelsoertekei es milyen tipusuak?

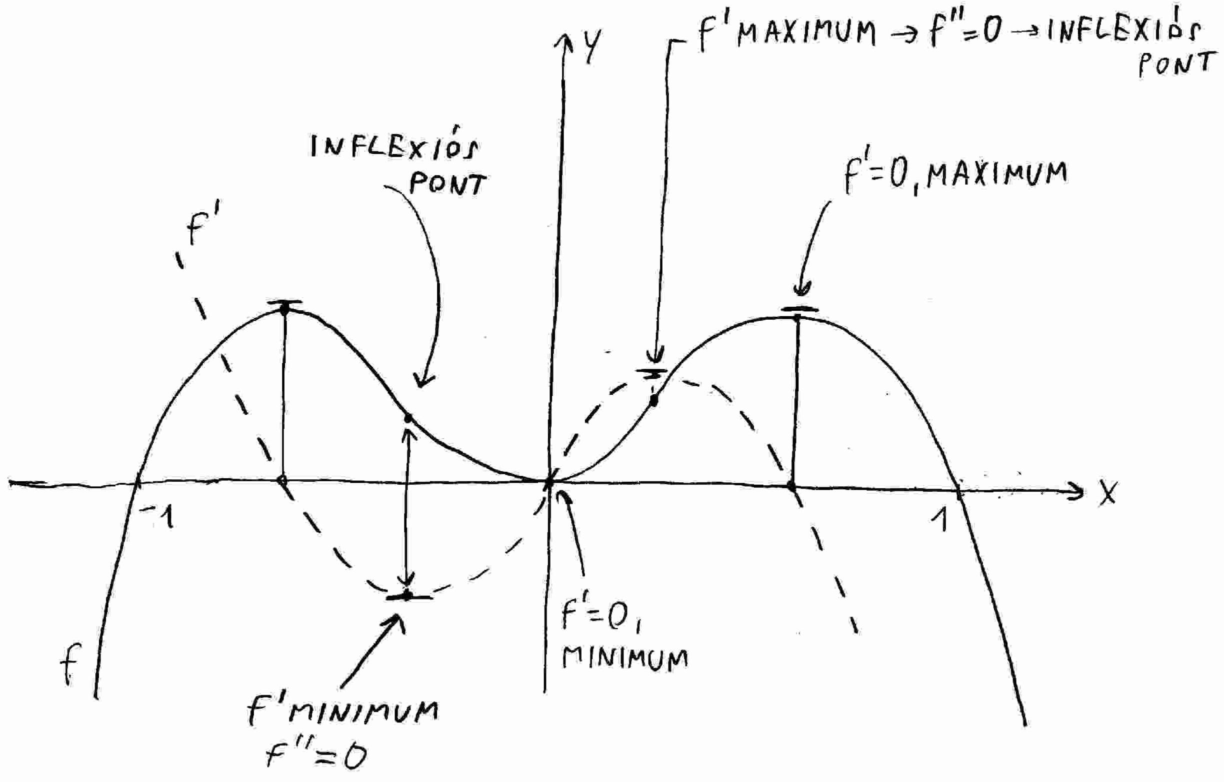
$$f' = 0 = 2x - 4x^3 = 2x(1 - 2x^2) \longrightarrow$$

$\longrightarrow x_1 = 0$	$x_2 = \frac{1}{\sqrt{2}}$	$x_3 = -\frac{1}{\sqrt{2}}$
$f''(x_1) = 2 - 12 \cdot 0^2 = 2$	$f''(\frac{1}{\sqrt{2}}) = 2 - 12 \cdot (\frac{1}{\sqrt{2}})^2 = -4$	$f''(-\frac{1}{\sqrt{2}}) = 2 - 12 \cdot (-\frac{1}{\sqrt{2}})^2 = -4$
$2 > 0$	$-4 < 0$	$-4 < 0$
MIN	MAX	MAX

- Hol van f inflexios pontja?

$$f''(x_{inf}) = 0 = 2 - 12x_{inf}^2 \longrightarrow x_{inf} = \pm \frac{1}{\sqrt{6}}$$

- Rajzold le f-t es f'-t (utobbit szaggatott vonallal) ugyanarra az abrara!



4. ((2+2+3+1)+2 pont)

- Legyen $f(x) = (1-x)^2 x^2 (6-x)^3$!
  - Hatarozd meg f gyokeit es azok multiplicitasat!

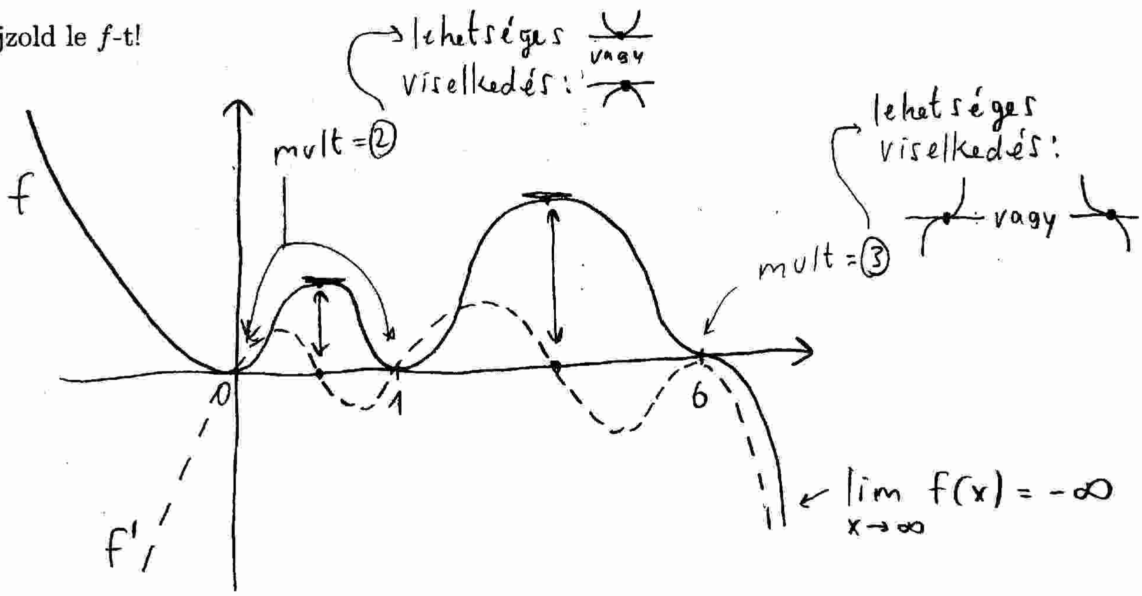
gyök: $x_1 = 1$ $x_2 = 0$ $x_3 = 6$

mult: 2 2 3

  - Mennyi $\lim_{x\to\infty} f(x)$?

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (-x)^2 x^2 (-x)^3 = \lim_{x\to\infty} -x^7 = -\infty$$

  - Rajzold le f-t!



  - Rajzold le ugyanarra az abrara szaggatott vonallal f'-t is!
- Mennyi $\lim_{x\to\infty} \frac{3x^2+2}{4x^3+5}$? $= \lim_{x\to\infty} \frac{3/x + 2/x^3}{4 + 5/x^3} = \frac{0+0}{4+0} = 0$

vagy

$$\lim_{x\to\infty} \frac{3x^2+2}{4x^3+5} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{6x}{12x^2} = \lim_{x\to\infty} \frac{1}{2x} = 0$$

L'Hospital szabály