Vizsga, Matematika I, 2013.jan.~~16~~ 23.   NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

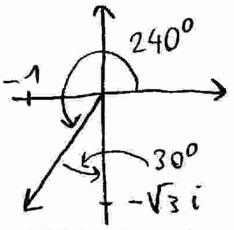
- Ird fel a $P(1,2,1)$ ponton atmeno, $\bar{n} = [2,3,4]$ normalvektoru sik egyenletet!

$$\bar{n} \cdot (\bar{r} - \bar{r}_0) = 0$$

$$2(x-1) + 3(y-2) + 4(z-1) = 0$$

$$2x + 3y + 4z - 12 = 0$$

- $z_1 = -1 - \sqrt{3}.i$ Mi z_1 trigonometrikus alakja?



$$z_1 = 2(\cos 240° + i \cdot \sin 240°)$$

$$\sqrt{\sqrt{3}^2 + 1^2}$$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \ln(2x+4)$

$$[\ln(2x+4)]' = \frac{1}{2x+4} \cdot 2$$

- $\bar{v}_1 = [2,1,3]$, $\bar{v}_2 = [-2,2,1]$. Mennyi $\bar{v}_1 \times \bar{v}_2$?

$$\bar{v}_1 \times \bar{v}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & 3 \\ -2 & 2 & 1 \end{vmatrix} = \bar{i} \underbrace{\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}}_{1\cdot 1 - 3\cdot 2} - \bar{j} \underbrace{\begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix}}_{2\cdot 1 - 3\cdot(-2)} + \bar{k} \underbrace{\begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix}}_{2\cdot 2 - 1\cdot(-2)} =$$

$$= -5\bar{i} - 8\bar{j} + 6\bar{k} = (-5, -8, 6)$$

- Szamold ki az alabbi fuggveny inverzet! $f(x) = \ln(x-3) + 1$ (Az ertelmezesi tartomanyokat nem kell megadni!)

$$y = \ln(x-3) + 1$$

$$y - 1 = \ln(x-3)$$

$$e^{y-1} = e^{\ln(x-3)} = x - 3$$

$$x = e^{y-1} + 3$$

$$f^{-1}(y) = e^{y-1} + 3$$

$$f^{-1}(x) = e^{x-1} + 3$$

2. (4+(1+2+3) pont)

$f(0) = 1$

- Legyen $f(x) = 1 - \sin(-2x)$! Szamold ki f negyedrendu Taylor-polinomjat az $x = 0$ pont korul!

$f'(x) = -\cos(-2x)\cdot(-2)$ $\qquad f'(0) = 2$

$f''(x) = \sin(-2x)\cdot(-2)^2$ $\qquad f''(0) = 0$

$f'''(x) = \cos(-2x)\cdot(-2)^3$ $\qquad f'''(0) = -8$

$f^{(4)}(x) = -\sin(-2x)\cdot(-2)^4$ $\qquad f^{(4)}(0) = 0$

$$f(x) \approx T_4(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 =$$

$$= 1 + 2x + 0x^3 - \frac{4}{3}x^3 + 0x^4$$

- Legyen $f(x) = 4x^2 - 2x^4$!

1. Keresd meg az f fuggveny gyokeit es hatarozd meg azok multiplicitasat!

$$4x^2 - 2x^4 = 2x^2(2 - x^2) = 2x^2(\sqrt{2} - x)(\sqrt{2} + x) = -2x^2(x - \sqrt{2})(x + \sqrt{2})$$

gyök: $x_1 = 0$ $\qquad x_2 = \sqrt{2}$ $\qquad x_3 = -\sqrt{2}$

mult: 2 $\qquad$ 1 $\qquad$ 1

2. Keresd meg az f fuggveny szelsoertekeit es hatarozd meg azok tipusat!

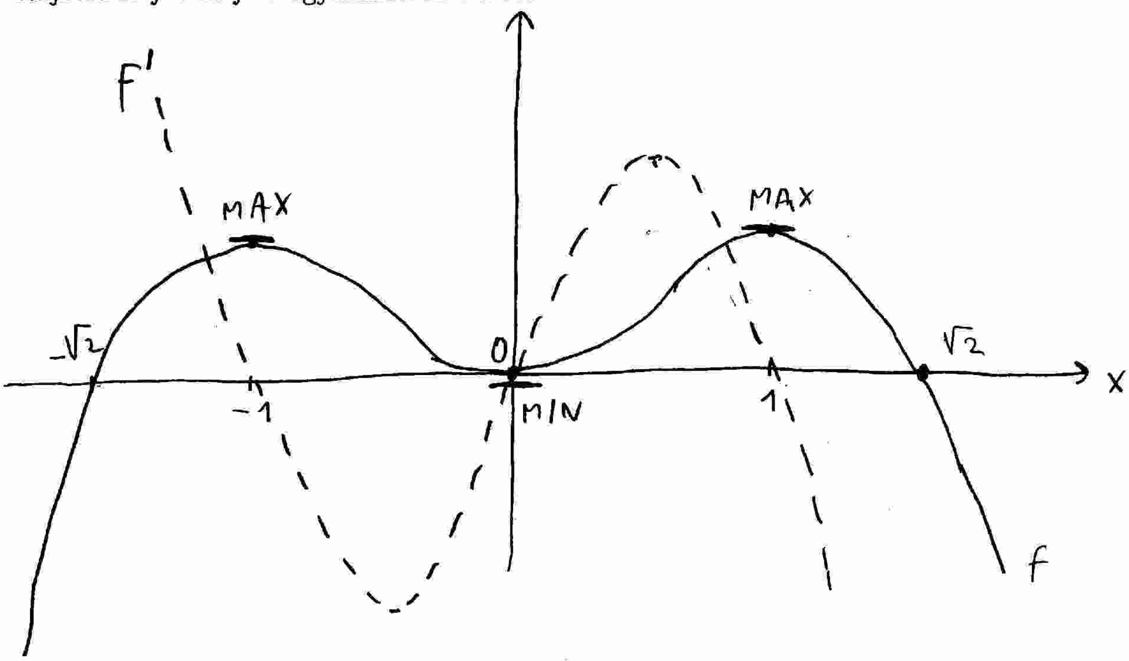
$$f'(x) = 8x - 8x^3 = 8x(1 - x^2) = 8x(1 - x)(1 + x) = 0$$

$x_1 = 0$ $\qquad x_2 = 1$ $\qquad x_3 = -1$

$f''(x) = 8 - 24x^2$

$f''(0) = 8 > 0$ MIN $\qquad f''(1) = -16 < 0$ MAX $\qquad f''(-1) = -16$ MAX

3. Rajzold le f-t es f'-t ugyanarra az abrara!



3. (1+2+3+2+2 pont)

- $$\phi\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4x+2y \\ 2y \end{bmatrix} = A\begin{bmatrix} x \\ y \end{bmatrix}.$$

  $0x+2y$

  Mennyi A?

$$A = \begin{bmatrix} 4 & 2 \\ 0 & 2 \end{bmatrix}$$

- Keresd meg A sajatertekeit!

$$0 = \det(A - \lambda E) = \begin{vmatrix} 4-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = (4-\lambda)(2-\lambda) - 2\cdot 0 =$$
$$= (4-\lambda)(2-\lambda) = 0$$

$\lambda_1 = 4$ $\qquad$ $\lambda_2 = 2$

- Keresd meg A sajatvektorait!

$$\begin{bmatrix} 4 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 4\cdot\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left.\begin{matrix} 4x+2y = 4x \\ 2y = 4y \end{matrix}\right\} y = 0$$

a sajátvektorok alakja: $\begin{bmatrix} x \\ 0 \end{bmatrix}$. egy ezek közül: $\bar{V}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 4 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left.\begin{matrix} 4x+2y = 2x \\ 2y = 2y \end{matrix}\right\} \rightarrow 2y = -2x \rightarrow y = -x$$

$\begin{bmatrix} x \\ -x \end{bmatrix}$, $\bar{V}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

- Ird fel $\bar{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ vektort a sajatvektorok linearis kombinaciojakent!

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \alpha+\beta \\ -\beta \end{bmatrix} \longrightarrow \left.\begin{matrix} \alpha+\beta = 1 \\ -\beta = 1 \end{matrix}\right\} \rightarrow \begin{matrix} \beta = -1 \\ \alpha = 2 \end{matrix}$$

tehát $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2\cdot\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1)\cdot\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

- Mennyi $A^{33}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$? (A vegeredmenyt nem kell numerikusan kiszamitani!)

$$A^{33}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = A^{33}\left(2\cdot\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = 2\cdot A^{33}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1)A^{33}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= 2\cdot 4^{33}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-1)\cdot 2^{33}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

4. $(3 \times 2 + (1+3)$ pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- 1. $\left(\sqrt[4]{\operatorname{ctg}(-4x)}\right)' = \left(\left[ctg(-4x)\right]^{1/4}\right)' =$

$$= \frac{1}{4}\left[ctg(-4x)\right]^{-3/4} \cdot \left(ctg(-4x)\right)' =$$

$$= \frac{1}{4}\left[ctg(-4x)\right]^{-3/4} \cdot \left(-\frac{1}{\sin^2(-4x)} \cdot (-4)\right)$$

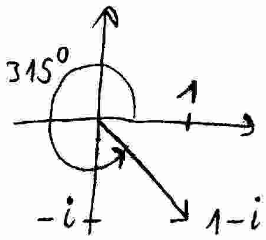
2. $\left(\ln(3x)\cos(2x-1)\right)' = \left[\ln(3x)\right]' \cdot \cos(2x-1) + \ln(3x) \cdot \left[\cos(2x-1)\right]'$

$$= \frac{1}{3x} \cdot 3 \cdot \cos(2x-1) + \ln(3x) \cdot \left(-\sin(2x-1) \cdot 2\right)$$

3. $\left(\ln\left(\frac{3-2x}{2+3x}\right)\right)' = \ln'\left(\frac{3-2x}{2+3x}\right) \cdot \left(\frac{3-2x}{2+3x}\right)' =$

$$= \frac{1}{\frac{3-2x}{2+3x}} \cdot \frac{(3-2x)' \cdot (2+3x) - (3-2x) \cdot (2+3x)'}{(2+3x)^2} =$$

$$= \frac{2+3x}{3-2x} \cdot \frac{-2 \cdot (2+3x) - (3-2x) \cdot 3}{(2+3x)^2}$$



- 1. $z_1 = 1 - i$. Mi z_1 trigonometrikus alakja?

$$z_1 = \underset{\uparrow\atop \sqrt{1^2+1^2}}{\sqrt{2}}\left(\cos 315^\circ + i \cdot \sin \underset{\uparrow\atop \text{vagy } -45^\circ}{315^\circ}\right)$$

2. Mennyi $\sqrt[2]{z_1}$?

$$\sqrt{\sqrt{2}\cos 315^\circ + i \cdot \sin 315^\circ} = \sqrt{\sqrt{2}}\left(\cos\frac{315^\circ + k \cdot 360^\circ}{2} + i \cdot \sin\frac{315^\circ + k \cdot 360^\circ}{2}\right)$$

ahol $k = 0, 1$

$$w_0 = \sqrt[4]{2}\left(\cos 157{,}5^\circ + i \cdot \sin 157{,}5^\circ\right)$$

$$w_1 = \sqrt[4]{2}\left(\cos 337{,}5^\circ + i \cdot \sin 337{,}5^\circ\right)$$