Vizsga, Matematika I, 2013.jan.16. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

- Ird fel a $P(1,5,3)$ ponton atmeno, $\bar{v} = [-2,4,1]$ iranyvektoru egyenes parametrikus egyenletet!

$$\bar{r}(t) = \bar{r}_0 + t \cdot \bar{v}$$

$$\bar{r}(t) = (1,5,3) + t \cdot (-2,4,1) = (1-2t, 5+4t, 3+t)$$

- $\bar{v}_1 = [2,1,0]$, $\bar{v}_2 = [-2,0,1]$, $\bar{v}_3 = [3,1,0]$. Mennyi $(\bar{v}_1\bar{v}_2\bar{v}_3)$?

$$(\bar{v}_1\bar{v}_2\bar{v}_3) = \begin{vmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & 0 \end{vmatrix} = 2 \cdot \underbrace{\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}}_{0\cdot 0 - 1\cdot 1} - 1 \cdot \underbrace{\begin{vmatrix} -2 & 1 \\ 3 & 0 \end{vmatrix}}_{(-2)\cdot 0 - 1\cdot 3} + 0 \cdot \underbrace{\begin{vmatrix} -2 & 0 \\ 3 & 1 \end{vmatrix}}_{(-2)\cdot 1 - 0\cdot 3} =$$

$$= 2 \cdot (-1) - 1 \cdot (-3) + 0 \cdot (-2) = 1$$

- Szamold ki: $\frac{4+3i}{3-2i} = \frac{4+3i}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{12+6i^2+17i}{3^2+2^2} = \frac{6}{13} + \frac{17}{13} i$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \ln(3x)$

$$[\ln(3x)]' = \ln'(3x) \cdot 3 = \frac{1}{3x} \cdot 3 = \frac{1}{x}$$

- Ird fel az alabbi fuggveny elsorendu Taylor polinomat (linearis kozeliteset) az $x = 0$ pont korul!
$f(x) = e^{2x+1}$

$$f(0) = e^{2\cdot 0+1} = e^1 = e$$

$$f'(x) = e^{2x+1} \cdot 2 \qquad f'(0) = e^{2\cdot 0+1} \cdot 2 = 2e$$

$$f(x) \approx T_1(x) = f(0) + f'(0)x = e + 2ex$$

2. (1+1+1+1+2+2+2 pont)
$\bar{v}_1 = [2, 1, 3]$, $\bar{v}_2 = [-3, 1, 1]$, $\bar{v}_3 = [2, 1, 0]$.

- Mennyi $3\bar{v}_1 + 2\bar{v}_2$?

$$3 \cdot [2,1,3] + 2 \cdot [-3,1,1] = [6,3,9] + [-6,2,2] =$$
$$= [0,5,11]$$

- Mennyi $\bar{v}_1\bar{v}_2$?

$$\bar{V}_1\bar{V}_2 = [2,1,3] \cdot [-3,1,1] = 2 \cdot (-3) + 1 \cdot 1 + 3 \cdot 1 = -2$$

- Mennyi $\bar{v}_1 \times \bar{v}_2$?

$$\bar{V}_1 \times \bar{V}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & 3 \\ -3 & 1 & 1 \end{vmatrix} = \bar{i} \overbrace{\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}}^{1 \cdot 1 - 3 \cdot 1} - \bar{j} \overbrace{\begin{vmatrix} 2 & 3 \\ -3 & 1 \end{vmatrix}}^{2 \cdot 1 - 3 \cdot (-3)} + \bar{k} \overbrace{\begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}}^{2 \cdot 1 - 1 \cdot (-3)}$$
$$= -2\bar{i} - \bar{j} \cdot 11 + \bar{k} \cdot 5 = (-2, -11, 5)$$

- Mennyi $(\bar{v}_1\bar{v}_2\bar{v}_3)$?

$$(\bar{V}_1\bar{V}_2\bar{V}_3) = \begin{vmatrix} 2 & 1 & 3 \\ -3 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 1 \cdot \begin{vmatrix} -3 & 1 \\ 2 & 0 \end{vmatrix} + 3 \begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} =$$
$$= 2 \cdot (-1) - 1 \cdot (-2) + 3 \cdot (-5) = -15$$

- Meroleges-e $\bar{v}_1$ es $\bar{v}_2$? Miert?

Nem, mert $\bar{V}_1 \cdot \bar{V}_2 = -2 \neq 0$

- Egy sikba esik-e $\bar{v}_1$, $\bar{v}_2$ es $\bar{v}_3$? Miert?

Nem, mert $(\bar{V}_1\bar{V}_2\bar{V}_3) = -15 \neq 0$

- Mekkora a $\bar{v}_1$ es $\bar{v}_2$ vektorok altal bezart szog koszinusza?

$$\bar{V}_1\bar{V}_2 = |\bar{V}_1| \cdot |\bar{V}_2| \cdot \cos\alpha$$
$$-2 = \sqrt{2^2+1^2+3^2} \cdot \sqrt{(-3)^2+1^2+1^2} \cdot \cos\alpha$$
$$\cos\alpha = \frac{-2}{\sqrt{14} \cdot \sqrt{11}} = -\frac{\sqrt{2}}{\sqrt{7} \cdot \sqrt{11}}$$

3.(5+(3+1+1) pont)

- Oldd meg a Gauss-elimináció segítségével a következő egyenletrendszert! Ellenorizd az eredmenyt!

$$x + y + z = 6$$
$$2x + y - z = 4$$
$$-x - y + 2z = 0$$

$$\left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & -1 & 4 \\ -1 & -1 & 2 & 0 \end{array}\right| \xrightarrow[III. \to III. + I.]{II. \to II. - 2 \cdot I.} \left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & -8 \\ 0 & 0 & 3 & 6 \end{array}\right|$$

$III. \Rightarrow 3z = 6 \qquad z = 2$

$II. \Rightarrow -1y - 3 \cdot 2 = -8 \qquad y = 2$

$I. \Rightarrow 1x + 1 \cdot 2 + 1 \cdot 2 = 6 \qquad x = 2$

Ell. $2 + 2 + 2 = 6$

$2 \cdot 2 + 2 - 2 = 4$

$-2 - 2 + 2 \cdot 2 = 0$

- Legyen $f(x) = e^{3x} + 2$.
  - Mennyi $f^{-1}(x)$?

$y = e^{3x} + 2$

$y - 2 = e^{3x}$

$\ln(y-2) = \ln(e^{3x}) = 3x$

$x = \dfrac{\ln(y-2)}{3}$

$f^{-1}(y) = \dfrac{\ln(y-2)}{3}$

$f^{-1}(x) = \dfrac{\ln(x-2)}{3}$

  - Mennyi D_f es R_f?

$D_f = \mathbb{R} \quad , \quad R_f = (2, \infty)$

  - Mennyi $D_{f^{-1}}$ es $R_{f^{-1}}$?

$D_{f^{-1}} = (2, \infty) \qquad R_{f^{-1}} = \mathbb{R}$

4.(3 × 2 +(2+2) pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- 1. $\left(\sqrt[3]{\sin(3x)}\right)' = \left[\left(\sin(3x)\right)^{1/3}\right]' = \frac{1}{3}\left(\sin(3x)\right)^{-2/3} \cdot \left[\sin(3x)\right]' =$

$$= \frac{1}{3}\left(\sin(3x)\right)^{-2/3} \cdot \cos(3x) \cdot 3$$

2. $\left(\sqrt[3]{x}\, tg(2x-1)\right)' = \left(x^{1/3}\right)' \cdot tg(2x-1) + \sqrt[3]{x} \cdot \left[tg(2x-1)\right]' =$

$$= \frac{1}{3} x^{-2/3} \cdot tg(2x-1) + \sqrt[3]{x}\, \frac{1}{\cos^2(2x-1)} \cdot 2$$

3. $\left(\frac{x^7}{sin(3x)}\right)' = \frac{(x^7)' \cdot \sin(3x) - x^7(\sin(3x))'}{\left[\sin(3x)\right]^2} =$

$$= \frac{7x^6 \cdot \sin(3x) - x^7 \cdot \cos(3x) \cdot 3}{\left[\sin(3x)\right]^2}$$

- 1. Mennyi $\lim_{n\to\infty}\left(\frac{n+5}{n}\right)^{2n-1}$? $= \lim_{n\to\infty}\left(1+\frac{5}{n}\right)^{2n-1} =$

$$= \lim_{n\to\infty}\left[\left(1+\frac{5}{n}\right)^n\right]^2 \cdot \left(1+\frac{5}{n}\right)^{-1} = \left[e^5\right]^2 \cdot 1^{-1} = e^{10}$$

2. Mennyi $\lim_{x\to\infty}\frac{e^{2x}}{x}$? $= \frac{\infty}{\infty} = \lim_{x\to\infty}\frac{e^{2x}\cdot 2}{1} = \frac{\infty}{1} = \infty$