Vizsga, Matematika I, 2013.jan.09. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

- Ird fel a $P(1,0,1)$ ponton atmeno, $\bar{n} = [-2,3,1]$ normalvektoru sik egyenletet!

$$-2(x-1)+3(y-0)+1\cdot(z-1)=0$$

- Mennyi $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$?

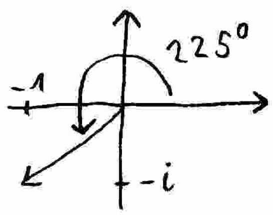
$2\cdot 2+3\cdot 1+4\cdot 1$

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 17 \\ 0 & 0 \\ 8 & 11 \end{bmatrix}$$

$0\cdot 3+0\cdot 1+0\cdot 2$

$2\cdot 3+3\cdot 1+1\cdot 2$

- $z_1 = -1 - i$. Mi z_1 trigonometrikus alakja?



$$z_1 = \sqrt{2}\,(\cos 225^\circ + i\cdot\sin 225^\circ)$$

$\sqrt{(-1)^2+(-1)^2}$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \frac{1}{x^2+1}$

$$\left[(x^2+1)^{-1}\right]' = -1\cdot(x^2+1)^{-2}\cdot(x^2+1)' = -1\cdot(x^2+1)^{-2}\cdot 2x$$

- $\bar{v}_1 = [2,1,3]$, $\bar{v}_2 = [-2,2,1]$. Mennyi $\bar{v}_1\bar{v}_2$?

$$\bar{v}_1\cdot\bar{v}_2 = 2\cdot(-2)+1\cdot 2+3\cdot 1 = 1$$

2. $(3 \times 2 + (3+1)$ pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- $\left(\sqrt[3]{(2x)^7} + \frac{1}{2x^3} + ctg(3x) + \ln(2x)\right)' = \frac{7}{3}(2x)^{4/3} \cdot 2 + \frac{1}{2}(-3)x^{-4} - \frac{1}{\sin^2(3x)} \cdot 3 + \frac{1}{2x} \cdot 2$

$\sqrt[3]{(2x)^7} = (2x)^{7/3}$, $\frac{1}{2x^3} = \frac{1}{2}x^{-3}$

- $\left(\ln(\sin(-2x))\right)' = \ln'(\sin(-2x)) \cdot (\sin(-2x))' =$

$$= \frac{1}{\sin(-2x)} \cdot \cos(-2x) \cdot (-2)$$

- $\left(\frac{\sqrt[3]{-2x+2}}{tg(2x)}\right)' = \frac{\left[(-2x+2)^{1/3}\right]' \cdot tg(2x) - \sqrt[3]{-2x+2} \cdot [tg(2x)]'}{[tg(2x)]^2} =$

$$= \frac{\frac{1}{3}(-2x+2)^{-2/3} \cdot tg(2x) - \sqrt[3]{-2x+2} \cdot \frac{1}{\cos^2(2x)} \cdot 2}{[tg(2x)]^2}$$

Mennyi $\lim_{n\to\infty} \left(\frac{3n-2}{3n}\right)^{5n-6}$? $= \lim_{n\to\infty} \left[1 + \frac{-2/3}{n}\right]^{5n-6} =$

$$= \lim_{n\to\infty} \Big[\underbrace{\left(1 + \frac{-2/3}{n}\right)^n}_{e^{-2/3}}\Big]^5 \cdot \left(1 + \frac{-2/3}{n}\right)^{-6} = \left[e^{-2/3}\right]^5 \cdot 1^{-6} = e^{-10/3}$$

Mennyi $\lim_{n\to\infty} \frac{3n^2-2n+1}{4n^2-5n+13}$? $=$

$$= \lim_{n\to\infty} \frac{3 - 2/n + 1/n^2}{4 - 5/n + 13/n^2} = \frac{3-0+0}{4-0+0} = \frac{3}{4}$$

3. (6+4 pont)

- Oldd meg a Gauss-elimináció segítségével a következő egyenletrendszert! Ellenorizd az eredmenyt!

$$x - 2y + z = 0$$
$$2x + 2y - z = 9$$
$$-x - y + 2z = -3$$

$$\left|\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & 2 & -1 & 9 \\ -1 & -1 & 2 & -3 \end{array}\right| \xrightarrow[\text{III.} \to \text{III.} + \text{I.}]{\text{II.} \to \text{II.} - 2\cdot \text{I.}} \left|\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 6 & -3 & 9 \\ 0 & -3 & 3 & -3 \end{array}\right|$$

$$\xrightarrow{\text{III.} \to \text{III.} + \frac{1}{2}\text{II.}} \left|\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 6 & -3 & 9 \\ 0 & 0 & 3/2 & 3/2 \end{array}\right|$$

$$\frac{3}{2}z = \frac{3}{2} \to z = 1$$

$$6y - 3\cdot 1 = 9 \to y = 2$$

$$1x - 2\cdot 2 + 1\cdot 1 = 0 \to x = 3$$

Ell.:

$$3 - 2\cdot 2 + 1 = 0$$
$$2\cdot 3 + 2\cdot 2 - 1 = 9$$
$$-3 - 2 + 2\cdot 1 = -3$$

- Oldd meg a következő kétismeretlenes egyenletrendszert! Ellenorizd az eredményt!

$$(1) \quad z_1 + iz_2 = 1 + i$$
$$(2) \quad (1+i)z_1 + z_2 = i$$

$$(1) \longrightarrow z_1 = 1 + i - iz_2$$

$$(2) \longrightarrow (1+i)(1+i-iz_2) + z_2 = i$$

$$2i + (2-i)z_2 = i$$

$$z_2 = \frac{-i}{2-i} = \frac{-i}{2-i}\cdot\frac{2+i}{2+i} = \frac{1-2i}{2^2+1^2} = \boxed{\frac{1}{5} - \frac{2}{5}i}$$

$$z_1 = 1 + i - i\left(\frac{1}{5} - \frac{2}{5}i\right) = \boxed{\frac{3}{5} + \frac{4}{5}i}$$

Ell.: $\left(\frac{3}{5} + \frac{4}{5}i\right) + i\left(\frac{1}{5} - \frac{2}{5}i\right) = 1 + i$

$$(1+i)\left(\frac{3}{5} + \frac{4}{5}i\right) + \left(\frac{1}{5} - \frac{2}{5}i\right) = i$$

4.(4+(1+2+3) pont)

- Legyen $f(x) = 1 - e^{-2x}$! Szamold ki f negyedrendu Taylor-polinomjat az $x = 0$ pont korul!

$$f(x) = 1 - e^{-2x}$$
$$f'(x) = -e^{-2x}\cdot(-2) = e^{-2x}\cdot 2$$
$$f''(x) = e^{-2x}\cdot(-2)\cdot 2 = -4e^{-2x}$$
$$f'''(x) = e^{-2x}\cdot(-2)\cdot(-2)\cdot 2 = 8e^{-2x}$$
$$f^{(4)}(x) = 8e^{-2x}\cdot(-2) = -16e^{-2x}$$

$$f(0) = 1 - e^{-2\cdot 0} = 1 - 1 = 0$$
$$f'(0) = e^{-2\cdot 0}\cdot 2 = 2$$
$$f''(0) = -4$$
$$f'''(0) = 8$$
$$f^{(4)}(0) = -16$$

$$f(x) \approx T_4(x) = 0 + 2\cdot x + \frac{-4}{2!}x^2 + \frac{8}{3!}x^3 + \frac{-16}{4!}x^4$$
$$= 2x - 2x^2 + \frac{4}{3}x^3 - \frac{2}{3}x^4$$

- Legyen $f(x) = 2x^2 - x^4$!

1. Keresd meg az f fuggveny gyokeit es hatarozd meg azok multiplicitasat!

$$2x^2 - x^4 = -x^2(x^2-2) = -x^2(x-\sqrt{2})(x+\sqrt{2})$$

gyökök: $x_1 = 0$ $\quad x_2 = \sqrt{2}$ $\quad x_3 = -\sqrt{2}$

mult.: 2 $\quad$ 1 $\quad$ 1

2. Keresd meg az f fuggveny szelsoertekeit es hatarozd meg azok tipusat!

$$f'(x) = 4x - 4x^3 \qquad 0 = 4x - 4x^3 = 4x(1-x^2) = 4x(1-x)(1+x)$$
$$f''(x) = 4 - 12x^2$$

$x_1 = 0$	$x_2 = 1$	$x_3 = -1$
$f''(x_1) = f''(0) =$	$f''(1) =$	$f''(-1) =$
$= 4 - 12\cdot 0^2 = 4 > 0$	$4 - 12\cdot 1^2 = -8 < 0$	$4 - 12\cdot(-1)^2 = -8 < 0$
MIN	MAX	MAX

3. Rajzold le f-t es f'-t ugyanarra az abrara!