Vizsga, Matematika I, 2012.dec.21. NEPTUN: Gyak.Vez.:

Név: Aláírás:

1. Beugro feladatok (otbol legalabb harom helyes megoldas szukseges) 5×2 pont.

- Szamold ki: $\frac{4+3i}{3-2i}$, $= \frac{4+3i}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{12+6i^2+17i}{3^2+2^2} = \frac{6}{13} + \frac{17i}{13}$

$\uparrow 6i^2 = -6$

- $\bar{v}_1 = [2,2,3]$, $\bar{v}_2 = [-2,1,4]$. Mennyi $\bar{v}_1 \times \bar{v}_2$? $=$

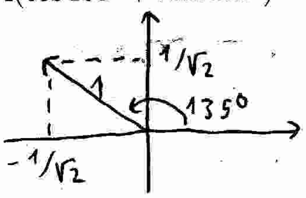
$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 2 & 3 \\ -2 & 1 & 4 \end{vmatrix} = \bar{i} \underbrace{\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}}_{2\cdot 4-3\cdot 1} - \bar{j} \underbrace{\begin{vmatrix} 2 & 3 \\ -2 & 4 \end{vmatrix}}_{2\cdot 4-3\cdot(-2)} + \bar{k} \underbrace{\begin{vmatrix} 2 & 2 \\ -2 & 1 \end{vmatrix}}_{2\cdot 1-2\cdot(-2)} = 5\bar{i} - 14\bar{j} + 6\bar{k}$$

$$= [5, -14, 6]$$

- Ird fel z algebrai alakjait, ahol $z = 4(\cos 135° + i \sin 135°)$ $= 4\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right) = -2\sqrt{2} + 2\sqrt{2} i$

$\cos 135° = -\frac{1}{\sqrt{2}}$

$\sin 135° = \frac{1}{\sqrt{2}}$



- Szamold ki az alabbi fuggveny inverzet! $f(x) = 2e^{x-3} + 1$

$y = 2e^{x-3} + 1$

$\frac{y-1}{2} = e^{x-3}$

$\ln \frac{y-1}{2} = \ln e^{x-3} = x - 3$

$x = \ln \frac{y-1}{2} + 3$

$f^{-1}(y) = \ln \frac{y-1}{2} + 3$

$f^{-1}(x) = \ln \frac{x-1}{2} + 3$

- Szamold ki a kovetkezo fuggveny x szerinti derivaltjat! $f(x) = \mathrm{tg}(-2x-9)$

$$\left[\mathrm{tg}(-2x-9)\right]' = \frac{1}{\cos^2(-2x-9)} \cdot (-2)$$

2. $(3 \times 2 + (1+3)$ pont) Szamold ki a kovetkezo fuggvenyek derivaltjait!

- 1. $\left(\sqrt[3]{\operatorname{tg}(-4x)}\right)' = \left(\left[tg(-4x)\right]^{1/3}\right)' = \frac{1}{3}\cdot\left[tg(-4x)\right]^{-2/3}\cdot\left[tg(-4x)\right]' =$

$$= \frac{1}{3}\left[tg(-4x)\right]^{-2/3}\cdot\frac{1}{\cos^2(-4x)}\cdot(-4)$$

2. $\left(x^3\cos(2x-1)\right)' = (x^3)'\cdot\cos(2x-1) + x^3\left(\cos(2x-1)\right)' =$

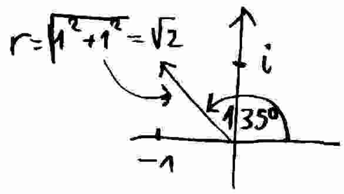
$$= 3x^2\cdot\cos(2x-1) + x^3\cdot(-\sin(2x-1))\cdot 2$$

3. $\left(\ln\left(\frac{3-2x}{2+3x}\right)\right)' = \ln'\left(\frac{3-2x}{2+3x}\right)\cdot\left(\frac{3-2x}{2+3x}\right)' =$

$$= \frac{1}{\frac{3-2x}{2+3x}}\cdot\frac{(3-2x)'\cdot(2+3x)-(3-2x)\cdot(2+3x)'}{(2+3x)^2} =$$

$$= \frac{1}{\frac{3-2x}{2+3x}}\cdot\frac{-2\cdot(2+3x)-(3-2x)\cdot 3}{(2+3x)^2}$$

- 1. $z_1 = -1 + i$. Mi z_1 trigonometrikus alakja?



$$z_1 = \sqrt{2}(\cos 135^\circ + i\cdot\sin 135^\circ)$$

2. Mennyi $\sqrt[2]{z_1}$?

$$\sqrt{\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)} = \sqrt{\sqrt{2}}\cdot\left(\cos\left[\frac{135^\circ}{2}+k\cdot 180^\circ\right] + i\cdot\sin\left[\frac{135^\circ}{2}+k\cdot 180^\circ\right]\right),$$

$k=0,1.$

$$W_0 = \sqrt[4]{2}(\cos 67{,}5^\circ + i\cdot\sin 67{,}5^\circ)$$

$$W_1 = \sqrt[4]{2}(\cos 247{,}5^\circ + i\cdot\sin 247{,}5^\circ)$$

3. (1+2+3+2+2 pont)

- $$\phi\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4x - 2y \\ 2y \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}.$$

  Mennyi A?

$$A = \begin{bmatrix} 4 & -2 \\ 0 & 2 \end{bmatrix}$$

- Keresd meg A sajatertekeit!

$$0 = \det(A - \lambda E) = \begin{vmatrix} 4-\lambda & -2 \\ 0 & 2-\lambda \end{vmatrix} = (4-\lambda)(2-\lambda) - (-2)\cdot 0 \longrightarrow$$

$$\lambda_1 = 4 \quad , \quad \lambda_2 = 2$$

- Keresd meg A sajatvektorait!

$\lambda_1 = 4 \quad \begin{bmatrix} 4 & -2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 4\begin{bmatrix} x \\ y \end{bmatrix}$

$4x - 2y = 4x \to x \in \mathbb{R}$

$2y = 4y \to y = 0$

sajátaltér:

$\left\{\begin{bmatrix} x \\ 0 \end{bmatrix}; x \in \mathbb{R}\right\} \qquad \bar{V}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda_2 = 2 \quad \begin{bmatrix} 4 & -2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$

$4x - 2y = 2x \longrightarrow x = y$

$2y = 2y$

sajátaltér:

$\left\{\begin{bmatrix} x \\ x \end{bmatrix}; x \in \mathbb{R}\right\} \qquad \bar{V}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

- Ird fel $\bar{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ vektort a sajatvektorok linearis kombinaciojakent!

$$\bar{V} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \end{bmatrix} \longrightarrow \alpha = 0 \, , \, \beta = 1$$

$$\bar{V} = 1 \cdot \bar{V}_2$$

- Mennyi $A^{33}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$? (A vegeredmenyt nem kell numerikusan kiszamitani!)

$$A^{33}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = A^{33} \cdot V_2 = 2^{33} V_2 = 2^{33} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

4. (4+(1+2+3) pont)

$f(0) = 2 + \sin(3\cdot 0) = 2$

- Legyen $f(x) = 2 + \sin 3x$! Szamold ki f harmadrendu Taylor-polinomjat az $x = 0$ pont korul!

$f'(x) = \cos(3x)\cdot 3 \qquad f'(0) = \cos(3\cdot 0)\cdot 3 = 3$

$f''(x) = -\sin(3x)\cdot 3^2 \qquad f''(0) = -\sin(3\cdot 0)\cdot 3^2 = 0$

$f'''(x) = -\cos(3x)\cdot 3^3 \qquad f'''(0) = -\cos(3\cdot 0)\cdot 3^3 = -27$

$$f(x) \approx T_3(x) = 2 + 3\cdot x + \frac{0}{2!}x^2 + \frac{-27}{3!}x^3 = 2 + 3x - \frac{9}{2}x^3$$

- Legyen $f(x) = x^2 - x^3$!

1. Keresd meg az f fuggveny gyokeit es hatarozd meg azok multiplicitasat!

$x^2 - x^3 = x^2(1-x) = -x^2(x-1)$ gyökök: $x_1 = 0 \quad x_2 = 1$

mult: ② ①

2. Keresd meg az f fuggveny szelsoertekeit es hatarozd meg azok tipusat!

$f'(x) = 2x - 3x^2 \longrightarrow 2x - 3x^2 = 0 \longrightarrow x(2-3x) = 0$

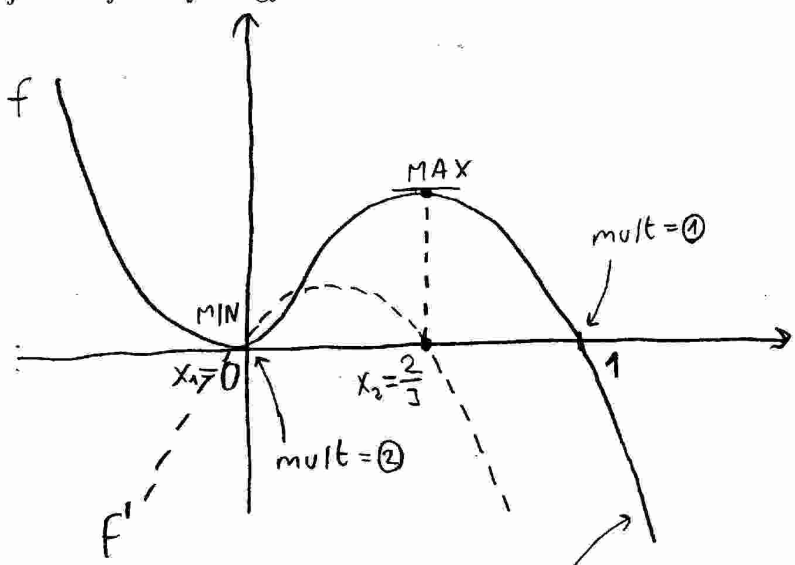
$f''(x) = 2 - 6x \qquad x_1 = 0 \qquad x_2 = 2/3$

$f''(x_1) = 2 - 6\cdot 0 = \qquad f''(x_2) = 2 - 6\cdot\frac{2}{3} = -2$

$= 2 > 0 \qquad -2 < 0$

MIN MAX

3. Rajzold le f-t es f'-t ugyanarra az abrara!



$\lim_{x\to\infty} x^2 - x^3 = \lim_{x\to\infty} -x^3 = -\infty$

jegy:	
0–15	1
16–21	2
22–27	3
28–33	4
34–40	5

Alpót:	
0–14	NEM
15–40	IGEN

Alpót + jegy:	
19–24	2
25–30	3
31–36	4
37–40	5