B. Zh1, Matematika I, 2014.oktober 16. NEPTUN: Név:

1. Feladat

1. Oldd meg a Gauss-elimináció segítségével a következő egyenletrendszert! 5 pont

$$x + 2y - z = 6$$
$$-x + y + z = 3$$
$$2x - y + z = 0$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ -1 & 1 & 1 & 3 \\ 2 & -1 & 1 & 0 \end{array}\right| \xrightarrow[III \to III - 2 \cdot I]{II \to II + I} \left|\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 3 & 0 & 9 \\ 0 & -5 & 3 & -12 \end{array}\right| \xrightarrow{III \to III + \frac{5}{3} II} \left|\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 3 & 0 & 9 \\ 0 & 0 & 3 & 3 \end{array}\right|$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 3 & 0 & 9 \\ 0 & 0 & 3 & 3 \end{array}\right| \quad \begin{array}{l} 1x + 2 \cdot 3 - 1 \cdot 1 = 6 \longrightarrow x = 1 \\ \longrightarrow 3y + 0 \cdot 1 = 9 \longrightarrow y = 3 \\ \longrightarrow 3z = 3 \longrightarrow z = 1 \end{array}$$

$x = 1$ $y = 3$ $z = 1$

2. Oldd meg a következő egyenletrendszert! 5 pont

I. $ix + y = 1$

II. $2ix + 3iy = -3 + i$

2·I. $2ix + 2y = 2$

II.−2·I. $(-2+3i)y = -5+i$

$$y = \frac{-5+i}{-2+3i} = \frac{-5+i}{-2+3i} \cdot \frac{-2-3i}{-2-3i} =$$

$$= \frac{10-3i^2+13i}{2^2+3^2} = \frac{13+13i}{13} = 1+i$$

I. $\longrightarrow x = \frac{1-y}{i} = \frac{1-(1+i)}{i} = \frac{-i}{i} = -1$

$x = -1$ $y = 1+i$

3+3+4 pont

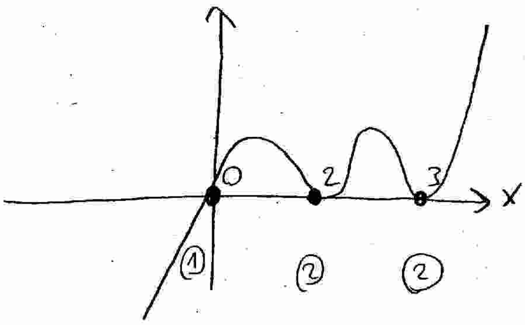
2. Feladat

1. Rajzold le a kovetkezo polinomot: $p(x) = (x-3)^2 x (2-x)^2$!

$p(x) = x^5 + \dots$

gyökök: 3 0 2

mult.: 2 1 2



2. Legyen $f(x) = 3x - 9$. Legyen $x_0 = 5$. Mennyi $f^6(x_0)$?

$f(x_f) = x_f$

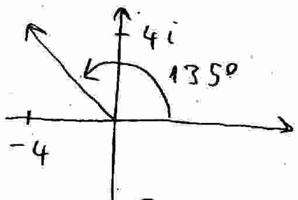
$3x_f - 9 = x_f$

$x_f = \frac{9}{2}$

$$f^6(5) = 3^6 \cdot \left(5 - \frac{9}{2}\right) + \frac{9}{2}$$

$$\left(= 729 \cdot \frac{1}{2} + \frac{9}{2} = 369\right)$$

3. Legyen $z = -4 + 4i$. Szamitsd ki $\sqrt[3]{z}$ trigonometrikus alakjat!



$$z = \underbrace{\sqrt{4^2 + 4^2}}_{\sqrt{2}\cdot 4} \left(\cos 135^\circ + i \cdot \sin 135^\circ\right)$$

$$\sqrt[3]{z} = \underbrace{\sqrt[3]{\sqrt{2}\cdot 4}}_{\sqrt[6]{2}\cdot\sqrt[3]{4}} \left(\cos\left[\frac{135^\circ}{3} + k\cdot 120^\circ\right] + i\cdot \sin\left[\frac{135^\circ}{3} + k \cdot 120^\circ\right]\right), \quad k = 0,1,2$$

$$w_1 = \sqrt[6]{2}\cdot\sqrt[3]{4}\left(\cos 45^\circ + i\cdot \sin 45^\circ\right)$$

$$w_2 = \sqrt[6]{2}\cdot\sqrt[3]{4}\left(\cos 165^\circ + i\cdot \sin 165^\circ\right)$$

$$w_3 = \sqrt[6]{2}\cdot\sqrt[3]{4}\left(\cos 285^\circ + i\cdot \sin 285^\circ\right)$$

3. Feladat

1. **Add meg a $P_1[0,5,0]$, $P_2[0,0,5]$ és a $P_3[5,0,0]$ pontokat tartalmazó sík egyenletet!** 5x2 pont

   a) Add meg a sik egy normalvektorat!

$$\bar{a} = P_2 - P_1 = (0,-5,5)$$
$$\bar{b} = P_3 - P_1 = (5,-5,0)$$
$$\bar{n} = \bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -5 & 5 \\ 5 & -5 & 0 \end{vmatrix} = \bar{i}\underbrace{\begin{vmatrix} -5 & 5 \\ -5 & 0 \end{vmatrix}}_{(-5)\cdot 0 - 5\cdot 5} - \bar{j}\begin{vmatrix} 0 & 5 \\ 5 & 0 \end{vmatrix} + \bar{k}\begin{vmatrix} 0 & -5 \\ 5 & -5 \end{vmatrix} =$$
$$= 25\bar{i} + 25\bar{j} + 25\bar{k} = (25, 25, 25)$$

   b) Mennyi a harom pont altal kifeszitett haromszog terulete?

$\bar{a}$ és $\bar{b}$ által kifeszített paralelograma területe: $|\bar{n}| =$

$$= \sqrt{25^2+25^2+25^2} = \sqrt{3}\cdot 25$$

Δ területe ennek a fele: $\dfrac{\sqrt{3}\cdot 25}{2}$

   c) Ird fel a sik egyenletet!

$$\bar{n}\cdot(\bar{r}-\bar{r}_0) = 0 \qquad (\bar{r}_0 = P_1 = (0,5,0)$$
$$25(x-0) + 25\cdot(y-5) + 25\cdot(z-0) = 0$$
$$25x + 25y + 25z - 125 = 0$$

2. **Adott két pont: $Q_1[2,2,0]$ és $Q_2[4,4,0]$. Add meg a rajtuk keresztülmenő egyenes paraméteres egyenletet!**

$$\bar{v} = Q_2 - Q_1 = (2,2,0), \qquad \bar{r}_0 = (2,2,0) = Q_1$$
$$\bar{r}(t) = \bar{r}_0 + t\cdot\bar{v} = (2,2,0) + t\cdot(2,2,0) = (2+2t,\ 2+2t,\ 0)$$

3. **Keresd meg a sik es az egyenes metszespontjat!**

$$(x,y,z) = (2+2t, 2+2t, 0)$$
$$25(2+2t) + 25(2+2t) + 25\cdot 0 - 125 = 0$$
$$100t - 25 = 0$$
$$t = \frac{1}{4}$$
$$\bar{r}\left(\frac{1}{4}\right) = \left(2+2\cdot\frac{1}{4},\ 2+2\cdot\frac{1}{4},\ 0\right) = (2.5,\ 2.5,\ 0)$$

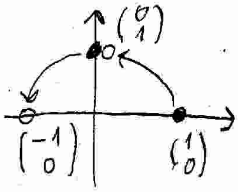
nyomdahiba. Helyes megoldásnak számít, ha valaki rámutatott a feladat értelmetlenségére

4. Feladat

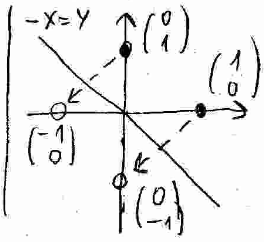
1. Legyen $\phi : \begin{pmatrix} x \\ y \end{pmatrix} \to A \begin{pmatrix} x \\ y \end{pmatrix}$ az $\cancel{x = y}$ 90°-os elforgatas transzformacioja, illetve legyen 3 + 2 pont

$\psi : \begin{pmatrix} x \\ y \end{pmatrix} \to B \begin{pmatrix} x \\ y \end{pmatrix}$ az $-x = y$ egyenesre torteno meroleges tukrozes transzformacioja.

Mennyi A es B ?



$$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



$$B\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad B\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

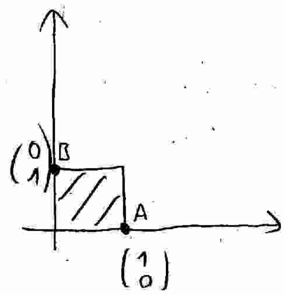
Ha $\underset{\uparrow B}{\psi}(\underset{\uparrow A}{\phi}(\bar{v})) = C\bar{v}$ es $\phi(\psi(\bar{v})) = D\bar{v}$ akkor mennyi C es D ?

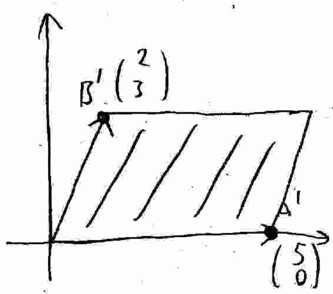
$$C = BA = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2. Legyen $\phi : \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 5 & 2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$. 2 + 1 + 2 pont

Rajzold le, hogy hova kepezi le ϕ az egysegnegyzetet!

$$\varphi\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \qquad \varphi\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$





Ird fel az $\begin{pmatrix} 5 & 2 \\ 0 & 3 \end{pmatrix}^{-1}$ inverz matrixot definialo egyenletet!

$$A A^{-1} = E$$

$$\begin{pmatrix} 5 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x & u \\ y & v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 5x+2y & 5u+2v \\ 0x+3y & 0u+3v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

I. $5x+2y=1$ $5u+2v=0$

II. $0x+3y=0$ $0u+3v=1$

II. $\to y=0$ II. $\to v=\frac{1}{3}$

I. $\to x=\frac{1}{5}$ I. $\to u=-\frac{2}{15}$

Keresd meg az inverz matrixot!

$$\begin{pmatrix} 5 & 2 \\ 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1/5 & -2/15 \\ 0 & 1/3 \end{pmatrix}$$

1: 3 + 2 pont

2: 2 + (3) pont