① $\lg 100 = 2 \iff 10^2 = 100$, $\quad \lg 0.00001 = -5 \iff 10^{-5} = \frac{1}{10^5} = 0.00001$

$\lg \sqrt[2]{1000} = \frac{1}{2} \lg 1000 = \frac{3}{2}$, $\quad \log_2(2\cdot\sqrt{2}) = \log_2 2 + \log_2 \sqrt{2} = 1 + \frac{1}{2} = \frac{3}{2}$

$\log_{\sqrt{2}} \frac{1}{2} = -2 \iff \sqrt{2}^{\,-2} = \frac{1}{\sqrt{2}^{\,2}} = \frac{1}{2}$, $\quad 1000^{2/3} = \left(\sqrt[3]{1000}\right)^2 = 10^2 = 100$

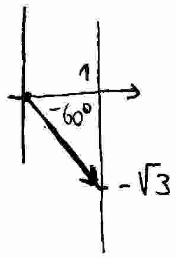
$\arcsin \frac{1}{2} = 60^\circ = \frac{\pi}{3}$, $\quad \arccos -\frac{\sqrt{3}}{2} = 150^\circ = \frac{5}{6}\pi$

$\arcsin -\frac{\sqrt{3}}{2} = 120^\circ = \frac{2\pi}{3}$, $\quad \arccos -\frac{1}{\sqrt{2}} = 135^\circ = \frac{3\pi}{4}$

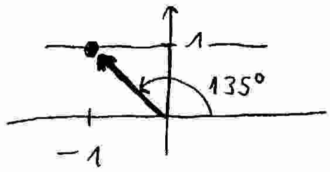
$\mathrm{arctg}\, 1 = 45^\circ = \frac{\pi}{4}$, $\quad \mathrm{arctg}(-\sqrt{3}) = -60^\circ = -\frac{\pi}{3}$

$\mathrm{arcctg}\, 1 = 45^\circ = \frac{\pi}{4}$, $\quad \mathrm{arctg}\,(-1) = 135^\circ = \frac{3\pi}{4}$

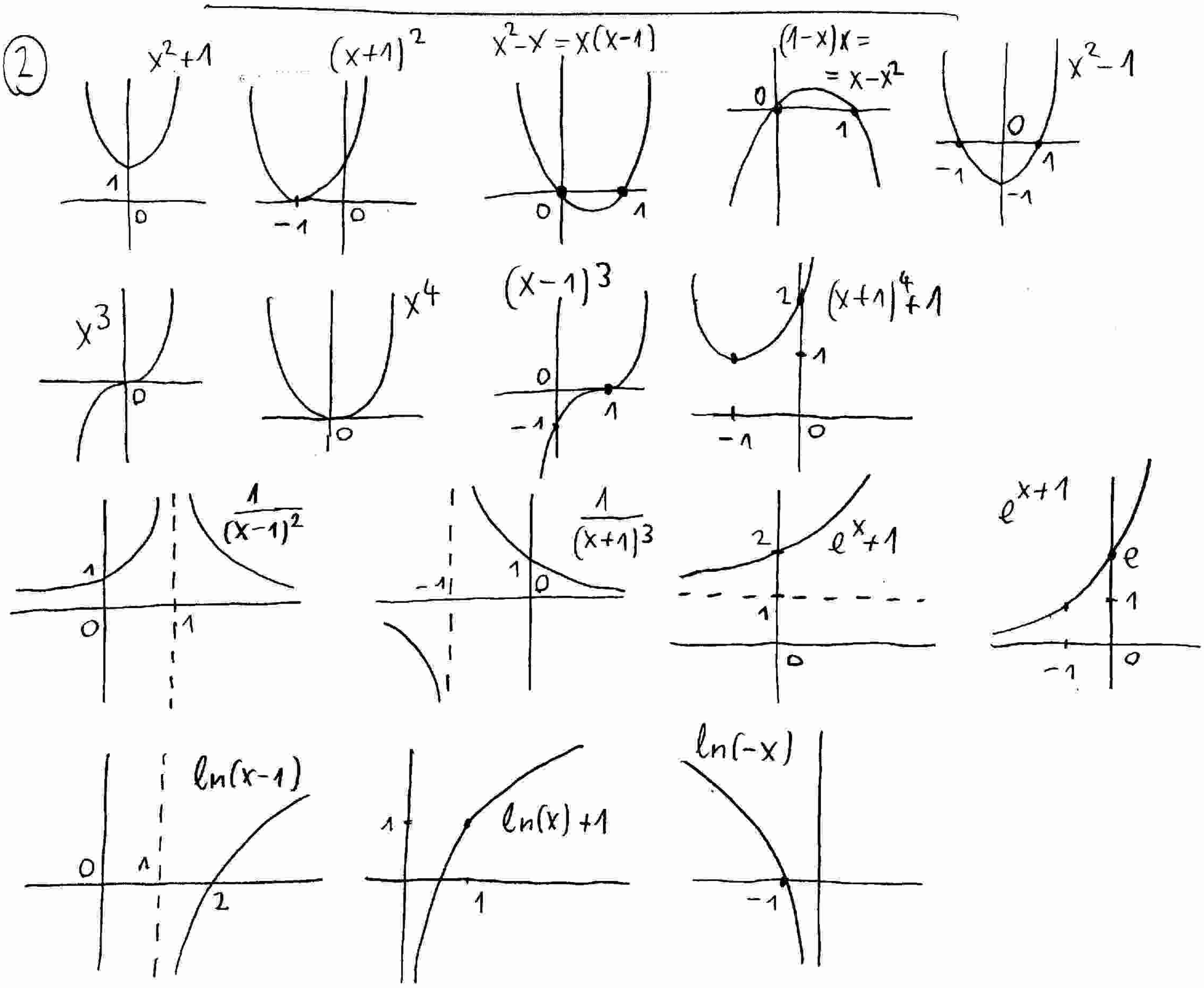
$\mathrm{arctg}(-\sqrt{3})$:



$\mathrm{arcctg}(-1)$:



②



③ $\frac{1}{(x-1)^2}$: $D_f: x \neq 1$, $R_f: (0,\infty)$ $\qquad$ $\frac{1}{(x+1)^3}$: $D_f: x \neq 1$, $R_f: (-\infty,0) \cup (0,\infty)$

e^x+1: $D_f: \mathbb{R}$, $R_f: (1,\infty)$ $\qquad$ e^{x+1}: $D_f: \mathbb{R}$, $R_f: (0,\infty)$ $\qquad$ $\ln(x-1)$: $D_f: (1,\infty)$, $R_f: \mathbb{R}$

$(\ln x)+1$: $D_f: (0,\infty)$, $R_f: \mathbb{R}$ $\qquad$ $\ln(-x)$: $D_f: (-\infty,0)$, $R_f: \mathbb{R}$

④

$$f(x) = 3x-1$$
$$y = 3x-1$$
$$x = \frac{y+1}{3}$$
$$f^{-1}(y) = \frac{y+1}{3}$$
$$f^{-1}(x) = \frac{x}{3}+\frac{1}{3}$$

$$f(x) = \ln(x-1)$$
$$y = \ln(x-1)$$
$$e^y = e^{\ln(x-1)}$$
$$e^y = x-1$$
$$x = e^y+1$$
$$f^{-1}(y) = e^y+1$$
$$f^{-1}(x) = e^x+1$$

$$f(x) = e^{x+1}$$
$$y = e^x+1$$
$$y-1 = e^x$$
$$\ln(y-1) = \ln(e^x) = x$$
$$f^{-1}(y) = \ln(y-1)$$
$$f^{-1}(x) = \ln(x-1)$$

⑤

$$\lim_{n\to\infty} \frac{2n+1}{3n+4} = \lim_{n\to\infty} \frac{2+1/n}{3+4/n} = \frac{2+0}{3+0} = \frac{2}{3}$$

$$\lim_{n\to\infty} \frac{2}{3n+4} = \lim_{n\to\infty} \frac{2/n}{3+4/n} = \frac{0}{3} = 0$$

$$\lim_{n\to\infty} \frac{2n^2}{3n+4} = \lim_{n\to\infty} \frac{2n}{3+4/n} = \frac{\infty}{3} = \infty$$

$$\lim_{n\to\infty} \left(\frac{n-3}{n}\right)^n = \lim_{n\to\infty} \left(1+\frac{-3}{n}\right)^n = e^{-3}$$

$$\lim_{n\to\infty} \left(\frac{n-3}{2n}\right)^n = \lim_{n\to\infty} \left(\frac{1}{2}-\frac{3}{2n}\right)^n = \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$$

$$\lim_{n\to\infty} n^3-n = \lim_{n\to\infty} n^3 = \infty$$

$$\lim_{n\to\infty} n^3-n^4 = \lim_{n\to\infty} -n^4 = -\infty$$

⑤ folyt.

$$\lim_{n\to\infty}\left(1-\frac{4}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-4}{n}\right)^n=e^{-4}$$

$$\lim_{n\to\infty}\left(2-\frac{4}{n}\right)^n=\lim_{n\to\infty}2^n=\infty$$

$$\lim_{n\to\infty}\frac{2^{2n+1}}{5^n}=\lim_{n\to\infty}\left(\frac{2^2}{5}\right)^n\cdot 2^1=0 \quad, \text{mivel } \left|\frac{2^2}{5}\right|<1$$

$$\lim_{n\to\infty}\frac{6^n}{3^{2n-5}}=\lim_{n\to\infty}\left(\frac{6}{3^2}\right)^n\cdot\frac{1}{3^{-5}}=0 \,, \text{mivel } \left|\frac{6}{3^2}\right|<1$$

⑥

$$\lim_{x\to\infty}\left(\frac{1}{2}\right)^x=0\,,\quad \lim_{x\to\infty}2^x=\infty\,,\quad \lim_{x\to-\infty}2^x=0\,,$$

$$\lim_{x\to\infty}\frac{x}{x^2+1}=\lim_{x\to\infty}\frac{1/x}{1+1/x^2}=\frac{0}{1}=0$$

$$\lim_{x\to\infty}\frac{3x^2}{x^2+1}=\lim_{x\to\infty}\frac{3}{1+1/x^2}=3$$

$$\lim_{x\to\infty}\frac{x^3}{x^2+1}=\lim_{x\to\infty}\frac{x}{1+1/x^2}=\infty$$

⑦

$$(fg)'=f'g+fg' \qquad \left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}$$

$$(f(g(x))'=f'(g(x))\cdot g'(x) \qquad \left(\frac{1}{f}\right)'=\frac{1'\cdot f-1\cdot f'}{f^2}=-\frac{f'}{f^2}$$

⑧

$$\left(\sqrt[3]{x}\right)'=\left(x^{1/3}\right)'=\frac{1}{3}x^{-2/3} \qquad \left(\frac{1}{x^6}\right)'=\left(x^{-6}\right)'=-6\cdot x^{-7}$$

$$\left(\sqrt[4]{x^3}\right)'=\left(x^{4/3}\right)'=\frac{4}{3}x^{1/3} \qquad \left(e^{3x+2}\right)'=e^{3x+2}\cdot 3$$

$$\left(\sin(3x-1)\right)'=\cos(3x-1)\cdot 3 \qquad \left(\frac{1}{3x-1}\right)'=\left((3x-1)^{-1}\right)'=-1\cdot(3x-1)^{-2}\cdot 3$$

$$\left(\ln(4x-9)\right)'=\frac{1}{4x-9}\cdot 4 \qquad \left(e^{-x}\right)'=e^{-x}\cdot(-1)=-e^{-x}$$

8 folyt.

$$(\cos(3x))' = -\sin(3x)\cdot 3 \qquad (\sin(-3x))' = \cos(-3x)\cdot(-3)$$

$$(\mathrm{tg}\,(3x))' = \frac{1}{\cos^2(3x)}\cdot 3 \qquad (\mathrm{ctg}(7x))' = -\frac{1}{\sin^2(7x)}\cdot 7$$

9 Elsőrendű Taylor-sor (lineáris közelítés) $x=0$ körül.

a) $f(x)=1+x$ Mivel $f(x)$ elsőfokú polinom, $T_1(x)=f(x)=1+x$

Vagy: $f(x)=1+x$ $f(0)=1$ $T_1=f(0)+f'(0)x=1+1\cdot x$

$f'(x)=1$ $f'(0)=1$

b) $f(x)=1+x+x^2$. Mivel a sorfejtést $x=0$ körül végezzük, így $T_1(x)$ az $f(x)$ polinom elsőfokú része: $T_1(x)=1+x$

Vagy: $f(x)=1+x+x^2$ $f(0)=1$ $T_1=1+x$

$f'(x)=1+2x$ $f'(0)=1$

c) $f(x)=e^{2x}$ $f(0)=e^{2\cdot 0}=1$ $T_1=1+2x$

$f'(x)=2e^{2x}$ $f'(0)=2\cdot e^{2\cdot 0}=2$

d) $f(x)=\sin(3x)$ $f(0)=\sin(3\cdot 0)=0$ $T_1=0+3x=3x$

$f'(x)=3\cdot\cos(3x)$ $f'(0)=3\cdot\cos(3\cdot 0)=3$

e) $f(x)=\cos(4x)$ $f(0)=1$ $T_1=1+0x=1$

$f'(x)=-4\cdot\sin(4x)$ $f'(0)=0$

f) $f(x)=\frac{1}{x-1}=(x-1)^{-1}$ $f(0)=1$ $T_1=1+(-1)\cdot x=1-x$

$f'(x)=-1\cdot(x-1)^{-2}=-\frac{1}{(x-1)^2}$ $f'(0)=-1$

g) $f(x)=\mathrm{tg}\,x$ $f(0)=0$ $T_1=0+1\cdot x=x$

$f'(x)=\frac{1}{\cos^2 x}$ $f'(0)=1$

h) $f(x)=\ln(x+1)$ $f(0)=\ln 1=0$ $T_1=0+1\cdot x=x$

$f'(x)=\frac{1}{x+1}$ $f'(0)=1$

⑩ Szélsőérték

a) $f(x) = x - x^2$

$f'(x) = 1 - 2x \longrightarrow 1 - 2x = 0 \longrightarrow x_1 = \frac{1}{2}$ lehetséges helye a szélsőértéknek

$f''(x) = -2$ $\qquad f''\left(\frac{1}{2}\right) = -2 < 0$ MAXIMUM

b) $f(x) = x^2 - 1$

$f'(x) = 2x \longrightarrow 2x = 0 \longrightarrow x_1 = 0$

$f''(x) = 2$ $\qquad f''(0) = 2 > 0$ MINIMUM

c) $f(x) = x^3 + 1$

$f'(x) = 3x^2 \longrightarrow 3x^2 = 0 \longrightarrow x_1 = 0$

$f''(x) = 6x$ $\qquad f''(0) = 6 \cdot 0 = 0$ ebből nem lehet megállapitaní a tipust.