Matematika 1. Pót Zh 1. Javítókulcs 14. dec. 11.

I

①
$$\begin{aligned} x-y+z&=0\\ x+y+z&=4\\ 2x+y-z&=12\end{aligned}$$
$\boxed{1}$
$$\left|\begin{array}{ccc|c} 1 & -1 & 1 & 0\\ 1 & 1 & 1 & 4\\ 2 & 1 & -1 & 12\end{array}\right| \xrightarrow[III\to III-2I]{II\to II-I} \left|\begin{array}{ccc|c} 1 & -1 & 1 & 0\\ 0 & 2 & 0 & 4\\ 0 & 3 & -3 & 12\end{array}\right| \boxed{1}$$

$$\xrightarrow{III\to III-\frac{3}{2}II} \left|\begin{array}{ccc|c} 1 & -1 & 1 & 0\\ 0 & 2 & 0 & 4\\ 0 & 0 & -3 & 6\end{array}\right| \boxed{1}$$

$1x-1\cdot 2+1\cdot(-2)=0 \to \underline{x=4}$

$2y+0\cdot(-2)=4 \longrightarrow \underline{y=2}$

$-3z=6 \to \underline{z=-2}$ $\boxed{2}$

②
$$\left.\begin{aligned} ix+y&=1\\ -(1+i)x-iy&=0\end{aligned}\right\} \longrightarrow \left.\begin{aligned} -x+iy&=i\\ -(1+i)x-iy&=0\end{aligned}\right\} \longrightarrow \begin{aligned} &(-2-i)x=i\\ &x=\frac{i}{-2-i}\cdot\frac{-2+i}{-2+i}=\frac{-1-2i}{2^2+1^2}=-\frac{1}{5}-\frac{2}{5}i \quad \boxed{2}\end{aligned}$$

$$y=1-ix=1-i\left(-\frac{1}{5}-\frac{2}{5}i\right)=\frac{3}{5}+\frac{1}{5}i \quad \boxed{3}$$

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① $P_3-P_1=(0,2,0)-(4,0,0)=(-4,2,0)$

$P_2-P_1=(0,0,2)-(4,0,0)=(-4,0,2)$

$$\bar n=(P_2-P_1)\overset{\boxed{1}}{\times}(P_3-P_1)=\begin{vmatrix}\bar i & \bar j & \bar k\\ -4 & 0 & 2\\ -4 & 2 & 0\end{vmatrix}= \boxed{1}$$

$$=\bar i\begin{vmatrix}0 & 2\\ 2 & 0\end{vmatrix}-\bar j\begin{vmatrix}-4 & 2\\ -4 & 0\end{vmatrix}+\bar k\begin{vmatrix}-4 & 0\\ -4 & 2\end{vmatrix}=$$

$$=-4\,\bar i-\bar j\cdot 8+\bar k(-8)=(-4,-8,-8) \quad \boxed{1}$$

$$T_\Delta=\frac{|\bar n|}{2}=\frac{1}{2}\sqrt{4^2+8^2+8^2}=6 \quad \boxed{1}$$

sík egyenlete:

$-4(x-4)-8(y-0)-8(z-0)=0$

$-4x-8y-8z+16=0$ $\boxed{2}$

② $\bar r_0=Q_1=(2,0,0)$

$\bar V=Q_2-Q_1=(1,1,0)-(2,0,0)=(-1,1,0)$

$\bar r(t)=\bar r_0+\bar v t=$ $\boxed{1}$

$=(2,0,0)+t\cdot(-1,1,0)=$

$=(2-t,\ t,\ 0)$ $\boxed{1}$

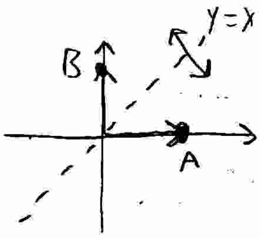
II

③ Metszéspont: $\boxed{1}$

$$-4(2-t)-8\cdot t-8\cdot 0+16=0 \longrightarrow -12t+8=0$$

$$t=\frac{2}{3},\quad r\left(\frac{2}{3}\right)=\left(2-\frac{2}{3},\ \frac{2}{3},\ 0\right)$$

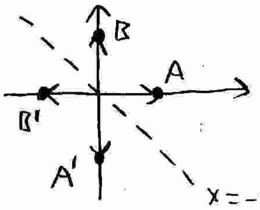
$\boxed{1}$

① 

$A \to A' = B$
$B \to B' = A$
$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}0\\1\end{pmatrix}$
$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix}$

$\bar{\bar{A}} = \left(\boxed{\begin{matrix}0\\1\end{matrix}}\boxed{\begin{matrix}1\\0\end{matrix}}\right)$ $\boxed{1}$

III



$A \to A'$
$B \to B'$
$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix}$
$x=-y$ $\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}-1\\0\end{pmatrix}$

$\bar{\bar{B}} = \begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$ $\boxed{1}$

$\psi(\varphi(v)) = B(Av) = (BA)v = Cv$ $\qquad C = \begin{pmatrix}0 & -1\\ \boxed{-1\ \ 0}\end{pmatrix}\left(\boxed{\begin{matrix}0\\1\end{matrix}}\begin{matrix}1\\0\end{matrix}\right) = \begin{pmatrix}-1 & 0\\ \boxed{0} & -1\end{pmatrix}$ $\boxed{1}$

$-1\cdot 0 + 0\cdot 1 = 0$

$\varphi(\psi(v)) = A(Bv) = (AB)v = Dv$ $\boxed{1}$ $\qquad D = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix} = \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$ $\boxed{1}$

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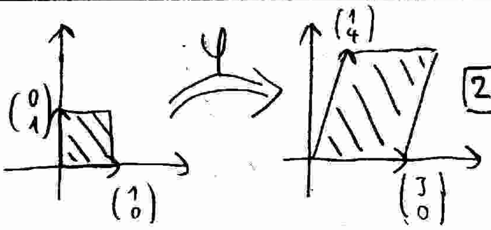
② $\varphi: \begin{pmatrix}x\\y\end{pmatrix} \to \begin{pmatrix}3 & 1\\0 & 4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$

$\varphi\left(\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}3\\0\end{pmatrix}, \quad \varphi\left(\begin{pmatrix}0\\1\end{pmatrix}\right) = \begin{pmatrix}1\\4\end{pmatrix}$



$\boxed{2}$

$AA^{-1} = E$ $\quad \begin{pmatrix}2 & 0\\-3 & 4\end{pmatrix}\begin{pmatrix}x & u\\y & v\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ $\boxed{1}$

$2x + 0y = 1$
$-3x + 4y = 0$ $\boxed{1}$
$x = \frac{1}{2} \quad y = \frac{3}{8}$

$2u + 0v = 0$
$-3u + 4v = 1$
$u = 0 \quad v = \frac{1}{4}$

$A^{-1} = \begin{pmatrix}\frac{1}{2} & 0\\ 3/8 & 1/4\end{pmatrix}$ $\boxed{1}$

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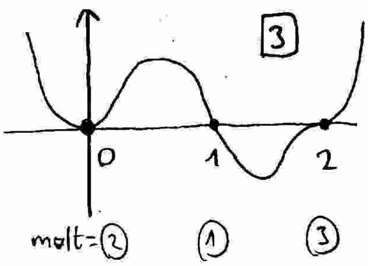
① $(1-x)x^2(2-x)^3 = x^6 + \ldots$



$\boxed{3}$

IV

② $f(x_f) = x_f$
$9 - 2x_f = x_f \to x_f = 3$ $\boxed{1}$

$f^{10}(8) = (-2)^{10}(8-3) + 3 = 1024 \cdot 5 + 3 = 5123$ $\boxed{2}$

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③ $z = 5 + 5i = (\sqrt{2}\cdot 5)\cdot(\cos 45^\circ + i\sin 45^\circ)$ $\boxed{1}$

$\sqrt[3]{5+5i} = \sqrt[3]{(\sqrt{2}\cdot 5)\cdot(\cos 45^\circ + i\cdot\sin 45^\circ)} =$

$= \sqrt[3]{\sqrt{2}\cdot 5}\cdot\left(\cos\left[\frac{45^\circ}{3} + k\cdot\frac{360^\circ}{3}\right] + i\cdot\sin\left[\frac{45^\circ}{3} + k\cdot\frac{360^\circ}{3}\right]\right), \quad k = 0, 1, 2$ $\boxed{1}$

$w_0 = \sqrt[6]{2}\cdot\sqrt[3]{5}\cdot(\cos 15^\circ + i\cdot\sin 15^\circ)$
$w_1 = \quad$ —‖— 135° —‖—
$w_2 = \quad$ —‖— 255° —‖— $\boxed{2}$

① $f(x)=\sin(3x)$ $\boxed{2}$ $f(0)=\sin(3\cdot 0)=0$ $\boxed{2}$ I

$f'(x)=\cos(3x)\cdot 3$ $f'(0)=1\cdot 3=3$

$f''(x)=-\sin(3x)\cdot 3^2$ $f''(0)=-0\cdot 3^2=0$

$f'''(x)=-\cos(3x)\cdot 3^3$ $f'''(0)=-1\cdot 3^3=-27$

$$\sin(3x)\approx 0+3x+\frac{0}{2!}x^2+\frac{-27}{3!}x^3$$

$$=3x-\frac{9}{2}x^3 \quad \boxed{1}$$

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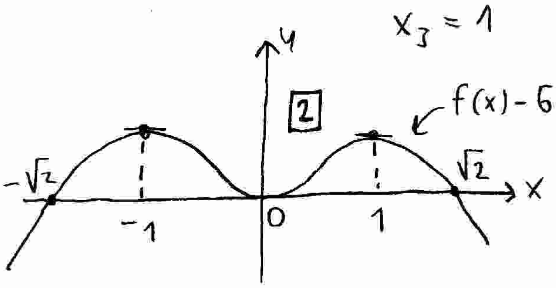
② $f(x)=2x^2-x^4+6$ MIN/MAX: $f'=0=4x-4x^3=4x(1-x)(1+x)$

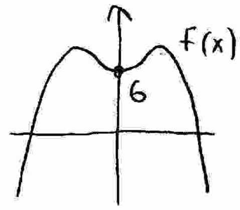
$\boxed{1}$ $f'=4x-4x^3$ $x_1=-1$ $f''(-1)=4-12\cdot(-1)^2=-8<0$ MAX

$f''=4-12x^2$ $x_2=0$ $\boxed{1}$ $f''(0)=4-12\cdot 0^2=4>0$ MIN $\boxed{1}$

$x_3=1$ $f''(1)=4-12\cdot 1^2=-8<0$ MAX

$\boxed{2}$





II

① $\left[\ln(3x)\ln(-3x)\right]'=\left(\frac{1}{3x}\cdot 3\right)\cdot\ln(-3x)+\ln(3x)\cdot\left(\frac{1}{-3x}\right)\cdot(-3)$

vagy: Értelmezési tartomány $=\emptyset=\{x;3x>0\}\cap\{x;-3x>0\}$,

tehát nincs ilyen függvény

③ $\left[\sqrt[2]{(8x)^3}+\frac{1}{3x^4}+\mathrm{tg}\,x+\mathrm{ctg}(1-4x)\right]'=\left[(8x)^{2/3}+\frac{1}{3}x^{-4}+\mathrm{tg}\,x+\mathrm{ctg}(1-4x)\right]'$

$$=\frac{2}{3}(8x)^{-1/3}\cdot 8+\frac{1}{3}(-4)x^{-5}+\frac{1}{\cos^2 x}-\frac{1}{\sin^2(1-4x)}\cdot(-4)$$

④ $\left[\frac{\ln(4x-2)}{\mathrm{tg}(x-1)}\right]'=\frac{[\ln(4x-2)]'\cdot\mathrm{tg}(x-1)-\ln(4x-2)\cdot[\mathrm{tg}(x-1)]'}{[\mathrm{tg}(x-1)]^2}$

$$=\frac{\frac{1}{4x-2}\cdot 4\cdot\mathrm{tg}(x-1)-\ln(4x-2)\cdot\frac{1}{\cos^2(x-1)}}{[\mathrm{tg}(x-1)]^2}$$

$\boxed{5\times 2}$

② $\left[\sin(x-1)^5\right]'=\cos(x-1)^5\cdot\left[(x-1)^5\right]'=\cos(x-1)^5\cdot 5\cdot(x-1)^4$

⑤ $\left[2^{3x}\ln(-x+2)\right]'=\left[2^{3x}\right]'\cdot\ln(-x+2)+2^{3x}\cdot\left[\ln(-x+2)\right]'$

$$=2^{3x}\cdot\ln 2\cdot 3\cdot\ln(-x+2)+2^{3x}\cdot\frac{1}{-x+2}\cdot(-1)$$

III

① $f(x)=\sqrt{x+2}+1$ $\qquad f^{-1}(x)=(x-1)^2-2$ $\boxed{2}$

$y=\sqrt{x+2}+1$

$(y-1)^2=x+2$

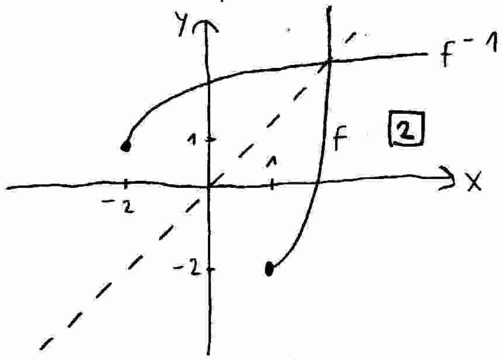
$x=(y-1)^2-2$

$f^{-1}(y)=(y-1)^2-2$

$D_f=[-2,\infty)$ $\quad D_{f^{-1}}=R_f=[1,\infty)$

$\boxed{1}$

$R_f=[1,\infty)$ $\quad R_{f^{-1}}=[-2,\infty)$



$\boxed{2}$

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② $$\lim_{n\to\infty}\left(1-\frac{3n}{4}\right)^{2n-33}=1^{\infty}=\lim_{n\to\infty}\left[\left(1+\frac{-3/4}{n}\right)^n\right]^2\cdot\left(1-\frac{3n}{4}\right)^{22}$$ $\boxed{3}$

$$=\left[e^{-3/4}\right]^2\cdot 1^{22}=e^{-3/2}$$

③ $$\lim_{n\to\infty}\left(\frac{1}{2}-\frac{3n}{4}\right)^{2n-33}=$$

$$=\lim_{n\to\infty}\left(\frac{1}{2}-0\right)^{2n-33}=\lim_{n\to\infty}\left[\left(\tfrac{1}{2}\right)^2\right]^n\cdot\left(\tfrac{1}{2}\right)^{33}=0\cdot\left(\tfrac{1}{2}\right)^{33}=0$$

$\boxed{2}$

IV

① $$\frac{3^{2+\Delta x}-3^2}{\Delta x}=3^2\cdot\frac{3^{\Delta x}-1}{\Delta x}$$ $\boxed{1}$

② $$\lim_{\Delta x\to 0}3^2\cdot\frac{3^{\Delta x}-1}{\Delta x}=3^2\cdot\frac{0}{0}=3^2\cdot\lim_{\Delta x\to 0}\frac{3^0\cdot\ln(3)}{1}=3^2\cdot\ln 3$$ $\boxed{2}$

③ $$\lim_{x\to 0}\frac{3^x-1}{x}=\frac{0}{0}=\lim_{x\to 0}\frac{3^x\ln 3}{1}=\frac{3^0\cdot\ln 3}{1}=\ln 3$$ $\boxed{2}$

④ $$\lim_{x\to\infty}\frac{\ln x}{e^x}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{1/x}{e^x}=\frac{0}{\infty}=0$$ $\boxed{1}$

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⑤ $$a_{n+1}-a_n=\frac{3-5(n+1)}{2(n+1)+1}-\frac{3-5n}{2n+1}=\frac{-2-5n}{2n+3}-\frac{3-5n}{2n+1}=\frac{-11}{(2n+3)(2n+1)}<$$

<0, szig. mon. csökk $\boxed{2}$ $\qquad$ ha $n=1,2,3\ldots$

⑥ $$\lim_{n\to\infty}\frac{3-5n}{2n+1}=\lim\frac{3/n-5}{2+1/n}=\frac{-5}{2}=-\frac{5}{2}$$

$\boxed{1}$