1. Legyen $a = i + 2j - 3k,\ b = 2j - 3k,\ c = 2i + j + k.$ $(1 + 2 + 2 + 5 \cdot 1 \text{ pont})$

   (a) Mennyi ab ?
   (b) Mennyi $a \times b$?
   (c) Mennyi abc ?
   (d) Mennyi az a, b vektorok kozotti szog koszinusza?
   (e) Mennyi az a, b vektorok altal kifeszitett paraleogramma terulete?
   (f) Mennyi az a, b vektorok altal kifeszitett haromszog terulete?
   (g) Mennyi az a, b, c vektorok altal kifeszitett ferde teglatest (paralepipedon) terfogata?
   (h) Mennyi az a, b, c vektorok altal kifeszitett tetraeder terfogata?

2. (a) Keresd meg a kovetkezo egyenlet osszes megoldasat pivotalas segitsegevel!

$$\begin{aligned} 3x_2 + x_4 &= 10 \\ x_1 + x_2 + 2x_3 &= 11 \\ -x_1 + x_3 &= -2 \end{aligned}$$

   (b) Ellenorizd az eredmenyedet! $(7 + 3 \text{ pont})$

3. Adott harom pont: $A = (1, 0, 0)$, $B = (0, 0, 1)$, $C = (0, 1, 0)$. $(4 + 3 + 3 \text{ pont})$

   (a) Ird fel az oket tartalmazo siknak egy, a sikot meghatarozo egyenletet!
   (b) Mennyi a $D = (1, 0, 5)$ pont tavolsaga ettol a siktol?
   (c) Keresd meg az $E = (0, 1, 0)$ es a $F = (3, 4, 5)$ pontokon athalado egyenes es a sik metszespontjat!

4. Legyen $v = \sqrt{3}i - 1$, $w = 1 + i$. $(2 + 2 + 3 + 3 \text{ pont})$

   (a) Mennyi $vw + \bar{v}$?
   (b) Mennyi w/v algebrai alakja?
   (c) Mennyi v^5 ? (Add meg az eredmenyt algebrai alakban is!)
   (d) Mennyi $\sqrt[4]{w}$?

0.1 Megoldas

1. 13, $(0,3,2)$, 5, $\sqrt{\frac{13}{14}}$, $\sqrt{13}$, $\frac{\sqrt{13}}{2}$, 5, $\frac{5}{6}$

2.

$$\begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} & \begin{pmatrix} 0 & 3 & 0 & 1 & 10 \\ 1_p & 1 & 2 & 0 & 11 \\ -1 & 0 & 1 & 0 & -2 \end{pmatrix} \end{array}, \quad \begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} e_1 \\ a_1 \\ e_3 \end{array} & \begin{pmatrix} 0 & 3 & 0 & 1 & 10 \\ 1 & 1 & 2 & 0 & 11 \\ 0 & 1_p & 3 & 0 & 9 \end{pmatrix} \end{array},$$

$$\begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} e_1 \\ a_1 \\ a_2 \end{array} & \begin{pmatrix} 0 & 0 & -9 & 1_p & -17 \\ 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 3 & 0 & 9 \end{pmatrix} \end{array}, \quad \begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} a_4 \\ a_1 \\ a_2 \end{array} & \begin{pmatrix} 0 & 0 & -9 & 1 & -17 \\ 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 3 & 0 & 9 \end{pmatrix} \end{array}$$

vagy

$$\begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} & \begin{pmatrix} 0 & 3 & 0 & 1 & 10 \\ 1_p & 1 & 2 & 0 & 11 \\ -1 & 0 & 1 & 0 & -2 \end{pmatrix} \end{array}, \quad \begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} e_1 \\ a_1 \\ e_3 \end{array} & \begin{pmatrix} 0 & 3 & 0 & 1_p & 10 \\ 1 & 1 & 2 & 0 & 11 \\ 0 & 1 & 3 & 0 & 9 \end{pmatrix} \end{array},$$

$$\begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} a_4 \\ a_1 \\ e_3 \end{array} & \begin{pmatrix} 0 & 3 & 0 & 1 & 10 \\ 1 & 1 & 2 & 0 & 11 \\ 0 & 1_p & 3 & 0 & 9 \end{pmatrix} \end{array}, \quad \begin{array}{cc} & \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & b \end{array} \\ \begin{array}{c} a_4 \\ a_1 \\ a_2 \end{array} & \begin{pmatrix} 0 & 0 & -9 & 1 & -17 \\ 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 3 & 0 & 9 \end{pmatrix} \end{array}$$

Tehat a megoldas:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \\ 0 \\ -17 \end{pmatrix} - x_3 \begin{pmatrix} -1 \\ 3 \\ -1 \\ -9 \end{pmatrix}$$

Ellenorzes:

$$\begin{pmatrix} 0 & 3 & 0 & 1 \\ 1 & 1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 9 \\ 0 \\ -17 \end{pmatrix} = \begin{pmatrix} 10 \\ 11 \\ 9 \end{pmatrix}, \quad \begin{pmatrix} 0 & 3 & 0 & 1 \\ 1 & 1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ -1 \\ -9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

3. (a) $x + y + z = 1$
   (b) $\sqrt{3} \cdot (2 - 1/3) = 5/\sqrt{3}$
   (c) $(0, 1, 0)$

4. (a) $(-2 - \sqrt{3}) - i$
   (b) $\frac{-1+\sqrt{3}}{4} + \frac{-1-\sqrt{3}}{4} i$
   (c)

$$[2\left(\cos(2\pi/3) + i\sin(2\pi/3)\right)]^5 = 32\left(\cos(10\pi/3) + i\sin(10\pi/3)\right) =$$
$$32\left(\cos(4\pi/3) + i\sin(4\pi/3)\right) = 32(-1/2 - \sqrt{3}/2i) = -16 - 16\sqrt{3}i$$

   (d)

$$\sqrt[4]{\sqrt{2}}\left(\cos\left(\frac{\pi/4}{4} + \frac{2\pi \cdot k}{4}\right) + i\sin\left(\frac{\pi/4}{4} + \frac{2\pi \cdot k}{4}\right)\right) = \sqrt[8]{2}\left(\cos\left(\frac{\pi}{16} + \frac{2\pi \cdot k}{4}\right) + i\sin\left(\frac{\pi}{16} + \frac{2\pi \cdot k}{4}\right)\right),$$

   ahol $k = 0, 1, 2, 3$.