1. Legyen $a = i - j + 3k,\ b = 2j - k,\ c = i + 3j + 2k.$ (4 × 1 + 2 × 3 pont)

(a) Mennyi ab ?

$$(1,-1,3)(0,2,-1) = 1\cdot 0 + (-1)\cdot 2 + 3\cdot(-1) = -5$$

(b) Mennyi $a \times b$?

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 3 \\ 0 & 2 & -1 \end{vmatrix} = \bar{i}\cdot\begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix} - \bar{j}\cdot\begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} + \bar{k}\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix}$$

$$= \underbrace{-5}_{(-1)\cdot(-1)-3\cdot 2}\bar{i} - \underbrace{(-1)}_{1\cdot(-1)-3\cdot 0}\bar{j} + 2\bar{k} = -5\bar{i} + \bar{j} + 2\bar{k}$$

$$= (-5,1,2)$$

(c) Mennyi abc ?

$$\bar{a}\bar{b}\bar{c} = \begin{vmatrix} 1 & -1 & 3 \\ 0 & 2 & -1 \\ 1 & 3 & 2 \end{vmatrix} = 1\cdot\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} - (-1)\cdot\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} + 3\cdot\begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= 1\cdot 7 - (-1)\cdot 1 + 3\cdot(-2) = 2$$

vagy

$$= (\bar{a}\times b)\bar{c} = (-5,1,2)(1,3,2) = -5\cdot 1 + 1\cdot 3 + 2\cdot 2 = 2$$

(d) Mennyi $(a \times b)c$?

$$(a\times b)c = \bar{a}\bar{b}\bar{c} = 2$$

(e) Ird fel a $A = a$, $B = b$ pontokon athalado egyenes egy parameteres egyenletet!

$$\bar{v} = \overrightarrow{AB} = B - A = (0,2,-1) - (1,-1,3) = (-1,3,-4)$$

$$\bar{r}(t) = \underbrace{P}_{P=A} + t\cdot\bar{v} = (1,-1,3) + t\cdot(-1,3,-4) = (1-t,\ -1+3t,\ 3-4t)$$

(f) Ird fel a $P = a$ pontot tartalmazo, $n = c$ normalvektoru sik egy egyenletet!

$\bar{n}(\bar{r} - P) = 0$ vagy $\bar{n}\cdot\bar{r} = \bar{n}\cdot P$

$P = (1,-1,3)$, $n = (1,3,2)$

$$1\cdot(x-1) + 3(y-(-1)) + 2\cdot(z-3) = 0 \iff x + 3y + 2z - 4 = 0$$

vagy $1x + 3y + 2z = (1,3,2)\cdot(1,-1,3) = 4$

Megoldas

2.

$$\left(\begin{array}{ccccccc} & a_1 & a_2 & a_3 & e_1 & e_2 & e_3 \\ e_1 & 4 & 2 & 16 & 1 & 0 & 0 \\ e_2 & 1_p & 2 & 3 & 0 & 1 & 0 \\ e_3 & 2 & 5 & 4 & 0 & 0 & 1 \end{array}\right), \left(\begin{array}{ccccccc} & a_1 & a_2 & a_3 & e_1 & e_2 & e_3 \\ e_1 & 0 & -6 & 4 & 1 & -4 & 0 \\ a_1 & 1 & 2 & 3 & 0 & 1 & 0 \\ e_3 & 0 & 1_p & -2 & 0 & -2 & 1 \end{array}\right),$$

$$\left(\begin{array}{ccccccc} & a_1 & a_2 & a_3 & e_1 & e_2 & e_3 \\ e_1 & 0 & 0 & -8_p & 1 & -16 & 6 \\ a_1 & 1 & 0 & 7 & 0 & 5 & -2 \\ a_2 & 0 & 1 & -2 & 0 & -2 & 1 \end{array}\right), \left(\begin{array}{ccccccc} & a_1 & a_2 & a_3 & e_1 & e_2 & e_3 \\ a_3 & 0 & 0 & 1 & -\frac{1}{8} & 2 & -\frac{3}{4} \\ a_1 & 1 & 0 & 0 & \frac{7}{8} & -9 & \frac{13}{4} \\ a_2 & 0 & 1 & 0 & -\frac{1}{4} & 2 & -\frac{1}{2} \end{array}\right)$$

$$\text{tehat } A^{-1} = \begin{pmatrix} \frac{7}{8} & -9 & \frac{13}{4} \\ -\frac{1}{4} & 2 & -\frac{1}{2} \\ -\frac{1}{8} & 2 & -\frac{3}{4} \end{pmatrix}$$

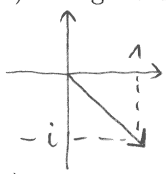
$det(A) = 1_p \cdot 1_p \cdot (-8)_p$, mivel a $(1,2,3) \to (2,3,1)$ permutacio paros.

3. Legyen $u = 1 - i$, $v = 2\left(\cos\left(\frac{2\pi}{4}\right) + i\sin\left(\frac{2\pi}{4}\right)\right)$. Mennyi (1 + 1 + 3 + 1 + 1 + 3 pont)

(a) v algebrai alakja,

$$v = 2\left(\cos\frac{\pi}{2} + i\cdot\sin\frac{\pi}{2}\right) = 2\left(\cos 90^\circ + i\cdot\sin 90^\circ\right) = 2\cdot(0 + i\cdot 1) = 2i$$

(b) u trigonometrikus alakja,



$$u = \underbrace{\sqrt{2}}_{\sqrt{1^2+(-1)^2}}\left(\cos\left[-\frac{\pi}{4}\right] + i\cdot\sin\left[-\frac{\pi}{4}\right]\right)$$

(itt $-\frac{\pi}{4} = -45^\circ$ ekvivalens $\frac{7\pi}{4} = 315^\circ$-kal)

(c) $\bar{u}$,

$$\bar{u} = \overline{1-i} = 1+i$$

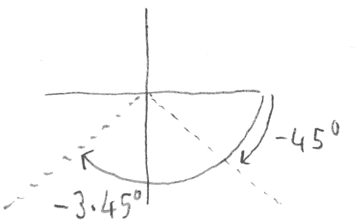
(d) $u\bar{u}$,

$$(1-i)(1+i) = 1^2 + (-1)^2 = 2$$

(e) u^3 trigonometrikus es algebrai alakja,

$$u^3 = \left[\sqrt{2}\left(\cos\left[-\frac{\pi}{4}\right] + i\cdot\sin\left[-\frac{\pi}{4}\right]\right)\right]^3 = (\sqrt{2})^3\cdot\left(\cos\left[-\frac{3\pi}{4}\right] + i\cdot\sin\left[-\frac{3\pi}{4}\right]\right)$$

$$= 2\sqrt{2}\cdot\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = -2-2i$$



$\left(-\frac{3\pi}{4} = -135^\circ \text{ ekvivalens } \frac{5\pi}{4} = 225^\circ\text{-kal}\right)$

(f) $\sqrt[3]{u}$ trigonometrikus alakja?

$$\sqrt[3]{u} = \sqrt[3]{\sqrt{2}\left(\cos\left[-\frac{\pi}{4}\right] + i\cdot\sin\left[-\frac{\pi}{4}\right]\right)} =$$

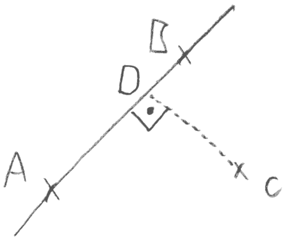
$3\cdot 2 \longrightarrow 6$

$$= \sqrt[6]{2}\left(\cos\left[\frac{-\pi/4}{3} + k\cdot\frac{2\pi}{3}\right] + i\cdot\sin\left[\frac{-\pi/4}{3} + k\cdot\frac{2\pi}{3}\right]\right), \text{ ahol } k = 0,1,2$$

$$= \sqrt[6]{2}\left(\cos\left[-\frac{\pi}{12} + \frac{2k\pi}{3}\right] + i\cdot\sin\left[-\frac{\pi}{12} + \frac{2k\pi}{3}\right]\right)$$

4. Adott harom pont: $A = (1, 0, 0)$, $B = (0, 0, 1)$, $C = (0, 1, 0)$. (7 + 3 pont)

(a) Mennyi a C pont tavolsaga az A es B pontokon athalado egyenestol?



egyenes: $\overline{v} = \overrightarrow{AB} = B - A = (0,0,1) - (1,0,0) = (-1,0,1)$

$\overline{r}(t) = A + t \cdot \overline{v} = (1,0,0) + t \cdot (-1,0,1) = (1-t, 0, t)$

$\overrightarrow{CD} \perp \overline{v}$, tehát

$\overrightarrow{CD} = D - C = (1-t, 0, t) - (0,1,0) = (1-t, -1, t)$

$0 = \overrightarrow{CD} \cdot \overline{v} = (1-t, -1, t) \cdot (-1, 0, 1) = (1-t)\cdot(-1) + (-1)\cdot 0 + t \cdot 1$

$= 2t - 1 \implies t = \frac{1}{2}$

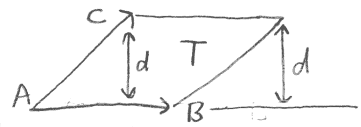
$D = \overline{r}\left(\frac{1}{2}\right) = \left(1 - \frac{1}{2}, 0, \frac{1}{2}\right) = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$

$\overrightarrow{CD} = D - C = \left(\frac{1}{2}, -1, \frac{1}{2}\right)$

$|\overrightarrow{CD}| = \sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}}$

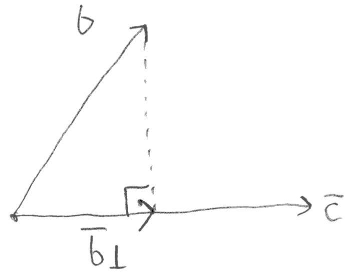
a keresett távolság

II. megoldás:
paraleogramma T területe: $|\overrightarrow{AB} \times \overrightarrow{AC}|$
$T = |\overrightarrow{AB}| \cdot d$, tehát
$d = \dfrac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{|\overrightarrow{AB}|}$



(b) Mennyi az $b = B - A$ vektor meroleges vetulete a $c = C - A$ vektorra?

$\overline{b}_\perp = \dfrac{(\overline{b}\,\overline{c})}{|\overline{c}|^2} \cdot \overline{c}$



$b = (0,0,1) - (1,0,0) = (-1,0,1)$
$c = (0,1,0) - (1,0,0) = (-1,1,0)$

$\overline{b}_\perp = \dfrac{(-1,0,1)\cdot(-1,1,0)}{(-1)^2 + 1^2 + 0^2} \cdot (-1,1,0) = \dfrac{1}{2} \cdot (-1,1,0) = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$

Megjegyzés: Ezeket az eredményeket számolás nélkül is ki lehet találni: