Lin. Alg. Zh2, 2015 Megoldás

① A $A=\begin{bmatrix}0&2\\2&0\end{bmatrix}$

Sajátérték: $\begin{vmatrix}0-\lambda & 2\\ 2 & 0-\lambda\end{vmatrix}=0=\lambda^2-2^2 \longrightarrow \lambda_1=2,\ \lambda_2=-2$

Sajátvektor: $\lambda_1=2$

$\begin{bmatrix}0-2 & 2\\ 2 & 0-2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$-2x+2y=0$

$x=y$

altér: $\left\{\begin{bmatrix}x\\x\end{bmatrix}\right\}$ bázis: $\vec{v}_1=\begin{bmatrix}1\\1\end{bmatrix}$

$\lambda_2=-2$

$\begin{bmatrix}0-(-2) & 2\\ 2 & 0-(-2)\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$2x+2y=0$

$y=-x$

altér: $\left\{\begin{bmatrix}x\\-x\end{bmatrix}\right\}$ bázis: $\vec{v}_2=\begin{bmatrix}1\\-1\end{bmatrix}$

saját rendszer:

$\lambda_1=2,\ \vec{v}_1=\begin{bmatrix}1\\1\end{bmatrix},\quad \lambda_2=-2,\ \vec{v}_2=\begin{bmatrix}1\\-1\end{bmatrix}$

$A^{13}\begin{bmatrix}6\\8\end{bmatrix}=??$

$\begin{bmatrix}6\\8\end{bmatrix}=\alpha_1\vec{v}_1+\alpha_2\vec{v}_2=\alpha_1\begin{bmatrix}1\\1\end{bmatrix}+\alpha_2\begin{bmatrix}1\\-1\end{bmatrix}$

$\alpha_1+\alpha_2=6$

$\alpha_1-\alpha_2=8 \longrightarrow \alpha_1=7,\ \alpha_2=-1$

$A^{13}\begin{bmatrix}6\\8\end{bmatrix}=A^{13}\left[7\vec{v}_1-1\vec{v}_2\right]=7\cdot 2^{13}\cdot\vec{v}_1-1\cdot(-2)^{13}\vec{v}_2$

$=7\cdot 2^{13}\begin{bmatrix}1\\1\end{bmatrix}-1\cdot(-2)^{13}\begin{bmatrix}1\\-1\end{bmatrix}=2^{13}\begin{bmatrix}7-(-1)^{13}\\7+(-1)^{13}\end{bmatrix}=2^{13}\begin{bmatrix}8\\6\end{bmatrix}$

vagy: $A^2=\begin{bmatrix}0&2\\2&0\end{bmatrix}^2=4\cdot\begin{bmatrix}1&0\\0&1\end{bmatrix} \longrightarrow A^{13}=(A^2)^6\cdot A=4^6\cdot E\cdot A=4^6\cdot A$

$=\begin{bmatrix}0&2^{13}\\2^{13}&0\end{bmatrix}$

tehát $A^{13}\begin{bmatrix}6\\8\end{bmatrix}=2^{13}\begin{bmatrix}8\\6\end{bmatrix}$

B $A=\begin{bmatrix}1&2\\0&1\end{bmatrix}$

Saját érték: $\begin{vmatrix}1-\lambda & 2\\ 0 & 1-\lambda\end{vmatrix}=(1-\lambda)^2=0 \longrightarrow \lambda=1$

saját vektor: $\begin{bmatrix}1-1 & 2\\ 0 & 1-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0&2\\0&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$0x+2y=0 \longrightarrow y=0,\ x\in\mathbb{R}$

altér: $\left\{\begin{bmatrix}x\\0\end{bmatrix}\right\}$, bázis: $\vec{v}_1=\begin{bmatrix}1\\0\end{bmatrix}$

Saját rendszer: $\lambda_1=1,\ \vec{v}_1=\begin{bmatrix}1\\0\end{bmatrix}$

2A $A = \begin{bmatrix} 5 & 2 & 4 \\ 5 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ Mennyi A^{-1} ?

$$A^{-1} = \begin{bmatrix} -1 & 6 & -8 \\ 1 & -7 & 10 \\ 1 & -4 & 5 \end{bmatrix}$$

$$\begin{array}{c|ccc|ccc|}
 & a_1 & a_2 & a_3 & e_1 & e_2 & e_3 \\
\hline
e_1 & 5 & 2 & 4 & 1 & 0 & 0 \\
e_2 & 5 & 3 & 2 & 0 & 1 & 0 \\
e_3 & 3 & 2 & \boxed{1} & 0 & 0 & 1 \\
\hline
e_1 & -7 & -6 & 0 & 1 & 0 & -4 \\
e_2 & -1 & \boxed{-1} & 0 & 0 & 1 & -2 \\
a_3 & 3 & 2 & 1 & 0 & 0 & 1 \\
\hline
e_1 & \boxed{-1} & 0 & 0 & 1 & -6 & 8 \\
a_2 & 1 & 1 & 0 & 0 & -1 & 2 \\
a_3 & 1 & 0 & 1 & 0 & 2 & -3 \\
\hline
a_1 & 1 & 0 & 1 & -1 & 6 & -8 \\
a_2 & 0 & 1 & 0 & 1 & -7 & 10 \\
a_3 & 0 & 0 & 1 & 1 & -4 & 5 \\
\hline
\end{array} \leftarrow A^{-1}$$

2B $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ {}^{-}0 & {}^{+}1 & {}^{-}0 & {}^{+}0 \\ 5 & 0 & 1 & 0 \\ 6 & 0 & 0 & 7 \end{bmatrix}$ Mennyi $(A^{-1})_{32}$?

$$\det A = -0 \cdot \begin{vmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 7 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 3 & 0 \\ 5 & 1 & 0 \\ 6 & 0 & 7 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 5 & 0 & 0 \\ 6 & 0 & 7 \end{vmatrix} + 0 \cdot \begin{vmatrix} 1 & 2 & 3 \\ 5 & 0 & 1 \\ 6 & 0 & 0 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 3 & {}^{+}0 \\ 5 & 1 & {}^{-}0 \\ 6 & 0 & {}^{+}7 \end{vmatrix} = 7 \cdot \begin{vmatrix} 1 & 3 \\ 5 & 1 \end{vmatrix} = 7 \cdot (-14) = -98$$

$$(A^{-1})_{32} = \frac{1}{\det A} \cdot (-1)^{3+2} \cdot \begin{vmatrix} 1 & 2 & {}^{+}0 \\ 5 & 0 & {}^{-}0 \\ 6 & 0 & {}^{+}7 \end{vmatrix} = \frac{1}{-98} \cdot (-1) \cdot \underbrace{(-70)}_{7 \cdot \begin{vmatrix} 1 & 2 \\ 5 & 0 \end{vmatrix}} = -\frac{5}{7}$$

$$\begin{vmatrix} 1 & 2 & \boxed{3} & 0 \\ \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} \\ 5 & 0 & \boxed{1} & 0 \\ 6 & 0 & \boxed{0} & 7 \end{vmatrix}$$

A_{23}

Töröld ki!

3A $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 2 & 5 \\ 1 & 2 & 3 \end{bmatrix}$. Számold ki pivotálással det(A)-t!

	a_1	a_2	a_3
e_1	2	3	5
e_2	4	2	5
e_3	$\boxed{1}$	2	3
e_1	0	$\boxed{-1}$	-1
e_2	0	-6	-7
a_1	1	2	3
a_2	0	1	1
e_2	0	0	$\boxed{-1}$
a_1	1	0	1
a_2	0	1	0
a_3	0	0	1
a_1	1	0	0

$$\det A = \boxed{1} \cdot \boxed{-1} \cdot \boxed{-1} \cdot (-1)^2 = 1$$

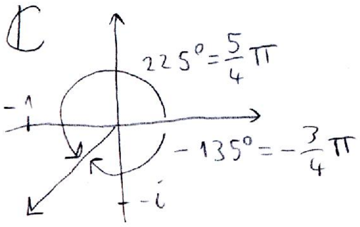
$$\det = \operatorname{sgn}\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = \text{"} \begin{matrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{matrix} \text{"} = (-1)^{\overset{2 \text{ db } \times}{2}} = 1$$

3B $x_0 = 5555$, $x_{n+1} = 24 - 3x_n$. Mennyi x_n?

$$x_{fix} = 24 - 3x_{fix} \longrightarrow x_{fix} = 6$$

$$x_n = (-3)^n \cdot (5555 - 6) + 6$$

3C Mennyi $(-1-i)^4$?



$$(-1-i) = \underbrace{\sqrt{(-1)^2 + (-1)^2}}_{\sqrt{2}} \cdot \left(\cos\left[-\tfrac{3}{4}\pi\right] + i \cdot \sin\left[-\tfrac{3}{4}\pi\right] \right)$$

$$(-1-i)^4 = (\sqrt{2})^4 \cdot \left(\cos\left[4 \cdot \left(-\tfrac{3}{4}\pi\right)\right] + i \cdot \sin\left[4 \cdot \left(-\tfrac{3}{4}\pi\right)\right] \right) =$$

$$= 4 \cdot \left(\cos[-3\pi] + i \cdot \sin[-3\pi] \right) =$$

$$= 4 \cdot \left(\cos[\pi] + i \cdot \sin[\pi] \right) =$$

$$= 4 \cdot (-1 + i \cdot 0) = -4$$

4A

$$2x_1+4x_2-6x_3=8$$
$$1x_1+2x_2-3x_3=4$$
$$3x_1+6x_2-9x_3=12$$

	a_1	a_2	a_3	b
e_1	2	4	-6	8
e_2	$\boxed{1}$	2	-3	4
e_3	3	6	-9	12
e_1	0	0	0	0
a_1	1	2	-3	4
e_3	0	0	0	0

partikuláris megoldás: "b" oszlop:

$$\vec{b}=0\vec{e}_1+4\vec{a}_1+0\vec{e}_3=4\vec{a}_1+0\vec{a}_2+0\vec{a}_3$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\0\\0\end{bmatrix}=\vec{X}_{part}$$

homogén egyenlet megoldásainak egy bázisa:

"a_2" oszlop:

$$\vec{a}_2=0\vec{e}_1+2\vec{a}_1+0\vec{e}_3=2\vec{a}_1$$
$$2\vec{a}_1-1\vec{a}_2+0\vec{a}_3=\vec{0}$$
$$\vec{X}^{(2)}_{hom}=\begin{bmatrix}2\\-1\\0\end{bmatrix}$$

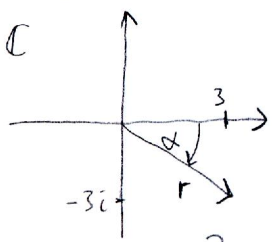
"a_3" oszlop:

$$\vec{a}_3=0\vec{e}_1-3\vec{a}_1+0\vec{e}_3=-3\vec{a}_1$$
$$-3\vec{a}_1+0\vec{a}_2-1\vec{a}_3=\vec{0}$$
$$\vec{X}^{(3)}_{hom}=\begin{bmatrix}-3\\0\\-1\end{bmatrix}$$

$$\vec{X}_{általános}=\vec{X}_{part}+t^{(2)}\vec{X}^{(2)}_{hom}+t^{(3)}\vec{X}^{(3)}_{hom}=\begin{bmatrix}4\\0\\0\end{bmatrix}+t^{(2)}\begin{bmatrix}2\\-1\\0\end{bmatrix}+t^{(3)}\begin{bmatrix}-3\\0\\-1\end{bmatrix}$$

4B

$z=3-3i$. Mennyi $\sqrt[3]{z}$?



$$\alpha=-45^\circ=-\frac{\pi}{4}$$
$$r=\sqrt{3^2+(-3)^2}=3\sqrt{2}$$

$$z=3\sqrt{2}\left(\cos[-\pi/4]+i\cdot\sin[-\pi/4]\right)$$

vagy $\frac{7}{4}\pi=315^\circ$

$$\sqrt[3]{z}=\sqrt[3]{(3\sqrt{2})}\cdot\left(\cos\left[\frac{-\pi/4+2k\pi}{3}\right]+i\cdot\sin\left[\frac{-\pi/4+2k\pi}{3}\right]\right),\quad k=0,1,2$$

$$W_1=\sqrt[3]{3}\cdot\sqrt[6]{2}\cdot\left(\cos[-\pi/12]+i\cdot\sin[-\pi/12]\right)\qquad -\frac{\pi}{12}=-15^\circ\sim 345^\circ$$

$$W_2=\sqrt[3]{3}\cdot\sqrt[6]{2}\cdot\left(\cos\left[\frac{-\pi}{12}+\frac{2\pi}{3}\right]+i\cdot\sin\left[-\frac{\pi}{12}+\frac{2\pi}{3}\right]\right)\qquad -\frac{\pi}{12}+\frac{2\pi}{3}=105^\circ$$

$$W_3=\sqrt[3]{3}\cdot\sqrt[6]{2}\cdot\left(\cos\left[\frac{-\pi}{12}+2\cdot\frac{2\pi}{3}\right]+i\cdot\sin\left[-\frac{\pi}{12}+2\cdot\frac{2\pi}{3}\right]\right)\qquad -\frac{\pi}{12}+2\cdot\frac{2\pi}{3}=225^\circ$$

4C

$$z_1+iz_2=3 \xrightarrow{I.\to i\cdot I.} iz_1-z_2=3i$$
$$iz_1+2z_2=i \xrightarrow{II.\to II.} iz_1+2z_2=i \xrightarrow{II.\to II.-I.} 3z_2=-2i\to z_2=-\frac{2}{3}i\to$$
$$\to z_1+i\left(-\frac{2}{3}i\right)=3\longrightarrow z_1=3-i\left(-\frac{2}{3}i\right)=\frac{7}{3}$$

Tehát $\begin{bmatrix}z_1\\z_2\end{bmatrix}=\begin{bmatrix}7/3\\-\frac{2}{3}i\end{bmatrix}$