Econ. Anal. Test, Dec. 02 Solution

1A

$$\left(\sqrt[3]{(2x)^5} - \frac{x}{\sqrt[3]{5x}} + \ln(3(x-4))\right)' = \left((2x)^{5/3} - \frac{1}{\sqrt[3]{5}}\, x^{2/3} + \ln(3x-12)\right)' =$$

$$= \frac{5}{3}(2x)^{2/3}\cdot 2 - \frac{1}{\sqrt[3]{5}}\cdot\frac{2}{3}\, x^{-1/3} + \frac{1}{3x-12}\cdot 3$$

$$\left(\ln(x)\cdot e^{x^2-1}\right)' = \ln(x)'\cdot e^{x^2-1} + \ln(x)\cdot\left(e^{x^2-1}\right)' =$$

$$= \frac{1}{x}\cdot e^{x^2-1} + \ln(x)\cdot e^{x^2-1}\cdot \underbrace{2x}_{(x^2-1)'}$$

$$\left(\frac{\tan(2x)}{x+(3x)^2}\right)' = \frac{(\tan(2x))'\cdot(x+(3x)^2) - \tan(2x)\cdot(x+(3x)^2)'}{(x+(3x)^2)^2} =$$

$$= \frac{\frac{1}{\cos^2(2x)}\cdot(x+(3x)^2) - \tan(2x)\cdot(1+2\cdot(3x)\cdot 3)}{(x+(3x)^2)^2}$$

$$\left[\frac{y-x}{2y+3x}\right]'_y = \frac{(y-x)'_y\cdot(2y+3x) - (y-x)\cdot(2y+3x)'_y}{(2y+3x)^2} =$$

$$= \frac{1\cdot(2y+3x) - (y-x)\cdot 2}{(2y+3x)^2}$$

1B

$$\int_0^1 \underbrace{e^{3x}}_{f'}\cdot \underbrace{2x}_{g}\,dx = \left[\frac{e^{3x}}{3}\cdot 2x\right]_0^1 - \int_0^1 \frac{e^{3x}}{3}\cdot 2\,dx = \left[\frac{e^{3x}}{3}\cdot 2x\right]_0^1 - \left[\frac{e^{3x}}{3\cdot 3}\cdot 2\right]_0^1 =$$

$f' = e^{3x}$ $\quad g = 2x$

$f = \frac{e^{3x}}{3}$ $\quad g' = 2$

$$= \left(\frac{e^3}{3}\cdot 2\cdot 1 - 0\right) - \left[\frac{2e^3}{9} - \frac{2\cdot 1}{9}\right]$$

$$\int_1^2 e^{3x^2}\cdot 2x\,dx = \frac{1}{3}\int_1^2 e^{3x^2}\cdot \overbrace{(6x)}^{(3x^2)'}\,dx = \frac{1}{3}\cdot\left[e^{3x^2}\right]_1^2 =$$

$$= \frac{1}{3}\cdot\left(e^{12} - e^3\right)$$

2A

(1) $p(WWWBB) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5} \cdot \frac{2}{4}$ $\qquad \frac{5!}{3!(5-3)!}$

$p(\text{3 white and 2 black}) = p(WWWBB) \cdot \binom{5}{3} = p(WWWBB) \cdot \binom{5}{2}$

(2) $p(E) = p(\{1, 3, 5\}) = \frac{|\{1,3,5\}|}{|\{1,2,3,4,5,6\}|} = \frac{3}{6} = \frac{1}{2}$

$p(F) = p(\{1, 2, 3\}) = \frac{|\{1,2,3\}|}{|\{1,2,3,4,5,6\}|} = \frac{3}{6} = \frac{1}{2}$

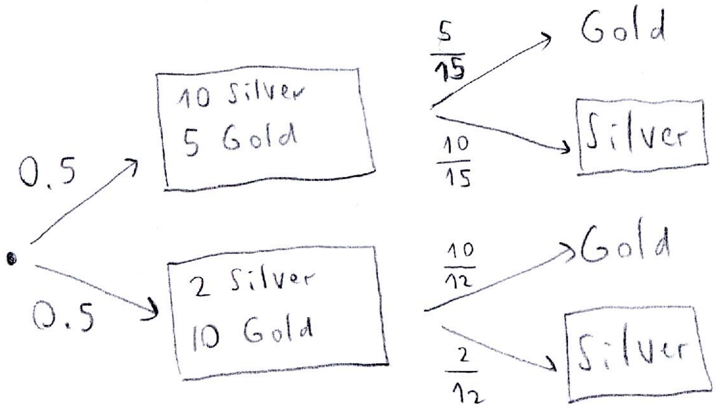
$p(E \cap F) = p(\{1, 3\}) = \frac{2}{6}$

$\frac{2}{6} = p(E \cap F) \neq p(E) \cdot p(F) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ $\longrightarrow$ E and F are not independent

$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{2/6}{1/2} = \frac{2}{3}$

$p(F|E) = \frac{p(E \cap F)}{p(F)} = \frac{2/6}{1/2} = \frac{2}{3}$

(3)



$\text{chance} = 0.5 \cdot \frac{10}{15}$

$\text{chance} = 0.5 \cdot \frac{2}{12}$

$p(\text{my choosem box is "more silver"}) = \frac{0.5 \cdot \frac{10}{15}}{0.5 \cdot \frac{10}{15} + 0.5 \cdot \frac{2}{12}}$

$p(\text{other box is "more gold"}) = p(\text{my choosen box is "more silver"}) = \frac{10/15}{10/15 + 2/12}$

$\frac{2/3}{2/3 + 1/6} = \frac{4/6}{5/6} =$

$\boxed{3A}$ $f(x) = 5x - 2x^2$

$$\frac{f(3+\Delta x_n) - f(3)}{\Delta x_n} = \frac{[5\cdot(3+\Delta x_n) - 2\cdot(3+\Delta x_n)^2] - [5\cdot 3 - 2\cdot 3^2]}{\Delta x_n} =$$

$$= \frac{5\Delta x_n - 2\cdot 2\cdot 3\cdot \Delta x_n - 2\Delta x_n^2}{\Delta x_n} = -7 - 2\Delta x_n$$

$$\lim_{n\to\infty}\left(-7 + 2\cdot\left(\frac{1}{n^2}\right)\right) = -7$$

$$f'(3) = -7$$

$\boxed{3B}$ $\boxed{1}$ $a_n = \dfrac{3+4n}{2n+1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\frac{3}{n}+4}{2+1/n} = \frac{4}{2} = 2 \longrightarrow a_n \text{ is bounded}$$

Monotonicity: $a_{n+1} - a_n = \dfrac{3+4(n+1)}{2(n+1)+1} - \dfrac{3+4n}{2n+1} =$

upper and lower bounds:

$\boxed{3B}$ $\boxed{2}$ $\lim_{n\to\infty} \dfrac{(-1)^n}{2n^3+1} = 0$, alternating sequence no $\nearrow$ no $\searrow$

limit exists $\to$ bounded

$\boxed{3C}$ $f(x,y) = 5 - y^2 + 3x - x^3$

$f'_x = 3 - 3x^2$, $f'_y = -2y$

$f''_{xx} = -6x$, $f''_{yy} = -2$, $f''_{xy} = f''_{yx} = 0$

Critical point: $f'_y = f'_x = 0 \longrightarrow 3 - 3x^2 = 0$ $-2y = 0$

$x_1 = 1,\ x_2 = -1$ $y = 0$

$P_A(-1,0)$, $P_B(1,0)$

$$\text{Hessian}_f(P_A) = \begin{bmatrix} -6\cdot(-1) & 0 \\ 0 & -2 \end{bmatrix} \qquad \text{Hessian}(P_B) = \begin{bmatrix} -6\cdot 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$[-6\cdot[-1]]\cdot(-2) - 0^2 < 0$

Saddle point

$[-6\cdot 1]\cdot[-2] - 0^2 > 0$

MIN or MAX

$[-6\cdot 1] < 0 \longrightarrow$ MAX

$\boxed{4}$ $f(x) = x^2 - x^3 = x^2(1-x)$

domain: $D_f = \mathbb{R} = (-\infty, \infty)$

roots: $x^2(1-x) = 0 \longrightarrow x_1 = 0, \quad x_2 = 1$

$f(x) = x^2 - x^3$

$f'(x) = 2x - 3x^2 = x(2-3x)$

$f''(x) = 2 - 6x$

ⓐ $\nearrow \searrow$ MIN, MAX

$f'(x_{crit}) = 0 = x(2-3x) \longrightarrow x_1 = 0, \ x_2 = \frac{2}{3},$

type: $f''(x_{crit}) \begin{matrix} < 0 \ \text{MAX} \\ > 0 \ \text{MIN} \end{matrix}$ $\qquad f''(0) = 2 > 0$ MIN $\qquad f''\left(\frac{2}{3}\right) = 2 - 6 \cdot \frac{2}{3} < 0$ MAX

ⓑ $\cap \cup$ convex/concave

$f''(x_{infl}) = 0$

$2 - 6x = 0 \longrightarrow x = \frac{1}{3}$

$x < \frac{1}{3} \longrightarrow 2 - 6x > 0 \longrightarrow$ convex

$x > \frac{1}{3} \longrightarrow 2 - 6x < 0 \longrightarrow$ concave