Vizsga. Econ.Anal. 14.jan.17. NEPTUN: Name:

1.

1. Compute the $f'_x, f'_y, f''_{xx}, f''_{xy}, f''_{yx}, f''_{yy}$ partial derivatives of the following function!

$$f = (x-1)^5 y^8.$$

2. Suppose that we toss a fair coin two times. The number of heads is counted by the random variable X. Write down and plot the cumulative distribution function
$F_X(x) = P(X \le x)$!
Calculate the variance of X !

2.

1. Compute the $\int f(x)\,dx$ indefinite integrals of the following functions!
   (a) $\frac{1}{4x^8} + \sqrt[3]{(2x)^4} + \frac{8}{\sqrt{x}}$
   (b) $\frac{3}{1+16x}$
   (c) $\cos 3x + \sqrt{-3x}$
2. Compute $\int_0^\pi \cos(2x)\,dx$!
3. Compute $\int_0^\infty e^{-x}\,dx$!

3.
A. Study the monotonicity, convexity and local extremal values of the following function!
$f(x) = x^2 - x^3$.
Draw its graph!
B. Study the boundedness and convergence of the following sequence!

$$a_n = \frac{3n}{2n+1}.$$

4.
A. Let $\bar{v}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\bar{v}_2 = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 0 \end{pmatrix} = \alpha\bar{v}_1 + \beta\bar{v}_2$. Compute $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$!

B. Let T be a 2×2 matrix formed by the transition probabilities of a two state (labeled by 1 and 2) stochastic system, where
$T(1 \leftarrow 1) = T_{11} = 0.6$, $T(2 \leftarrow 1) = T_{21} = 0.4$, $T(1 \leftarrow 2) = T_{12} = 0.5$, $T(2 \leftarrow 2) = T_{22} = 0.5$.

1. Find an eigenvector $\bar{v}_1$ corresponding to the eigenvalue $\lambda_1 = 1$! (This is the equilibrium state.)
2. Find the eigenvalue λ_2 of T corresponding to the eigenvector $\bar{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$!