Name:

1. A. Compute the derivatives of the following functions!

1.  $e^{2x} \cos(3x)$ 2.  $e^3 \sin(2x-1)$ 3.  $\frac{2x-1}{x-1}$ 

B. What is the prediction of the linear approximation of the function f(x) at  $x = x_0$  for the value of  $f(x_0 + \Delta x)$ ?  $f(x) = 1/x^2$ ,  $x_0 = 3$ ,  $\Delta x = 0.1$ .

- 2.
- A. Compute the limit of the following sequence!  $a_n = \left(1 \frac{4}{3n}\right)^{3n+7}$ .
- B. Let  $\phi(x) = 2x 1$ ,  $x_0 = 3$ ,  $x_{n+1} = \phi(x_n)$ . What are  $\phi^{-1}$  and  $\phi^n(3) = x_n$ ?
  - 1. Find the fixed point  $x_f$  of  $\phi$  !
  - 2. Introduce  $\Delta x = x x_f$  and  $\tilde{\phi}(\Delta x) = \phi(x_f + \Delta x) x_f$ . Calculate  $\tilde{\phi}$  and  $\tilde{\phi}^n$  !
  - 3. Compute  $x_n$  !

## 3.

Plot the solutions of the following differential equation! Find the equilibrium positions and determine their stability!

$$y' = y(y-2)$$

4.

A. Solve the following differential equation!  $y' = x^4$ , y(2) = 3!

- 1. Write down the general solution!
- 2. Write down the particular solution!

B. There are two urns containing colored balls. The first urn contains 30 red balls and 70 blue balls. The second urn contains 90 red balls and 10 blue balls. One of the two urns is randomly chosen (both urns have probability 50% of being chosen) and then a ball is drawn at random from one of the two urns. If a red ball is drawn, what is the probability that it comes from the first urn?