1. (a) Let X be a discrete random variable. Let its support R_X be: $R_X = \{0, 1, 2, 3\}$. Let its probability mass function be:

$$p(x) = \begin{cases} 1/4 & \text{if } x \in R_X \\ 0 & \text{if } x \ni R_X. \end{cases}$$

Compute the mean and the variance of X !

(b) What is the chance of winning the lottery grand prize if you need to guess 6 numbers correctly out of 60? (Do not compute the numerical answer!)

2. Find the extremal value (and determine its type) of the function $f(x, y) = -x^2 + xy - 2y^2 - 2x + 4y + 2x$!

(a) Compute the partial derivatives of f up to second order!

(b) Find the location of the extremal value!

(c) Determine the type of the extremal value!

3. (a) Compute the $f'_x, f'_y, f''_{xx}, f''_{xy}, f''_{yx}, f''_{yy}$ partial derivatives of the following function!

$$f = y^2/x^3. \tag{1}$$

(b) Suppose that we toss a fair dice two times. The number of heads is counted by the random variable X. Compute the variance of X !

4. (a) Compute the $\int f(x)\, dx$ indefinite integrals of the following functions!

   i. $\sqrt[4]{4x^8} + \sqrt[5]{(3x)^7} + \frac{8}{x^6}$
   ii. $\frac{2}{1+4x^2}$
   iii. $e^{3x} + \sin(-3x)$

(b) Let X be a discrete random variable. Let its support R_X be: $R_X = \{1, 2, 3\}$. Let its probability mass function be:

$$p(x) = \begin{cases} x/6 & \text{if } x \in R_X \\ 0 & \text{if } x \notin R_X. \end{cases}$$

Compute the mean of X.