

1. Find the extremal value (and determine its type) of the function  $f(x, y) = 2x^2 - xy + 2y^2 - 2x + 4y + 2x$  !
  - (a) Compute the partial derivatives of  $f$  up to second order!
  - (b) Find the location of the extremal value!
  - (c) Determine the type of the extremal value!
2. (a) Compute the  $\int f(x) dx$  indefinite integrals of the following functions!
  - i.  $\sqrt[3]{4x^8} + \sqrt[4]{(3x)^7} + \frac{8}{x^8}$
  - ii.  $\frac{3}{1+16x^2}$
  - iii.  $\cos 3x + \sin(-3x)$(b) Compute  $\int_0^\pi \sin(-x) dx$  !
3. (a) There are 10 black and 5 white balls in a box. Suppose that we DO put back the balls after the drawings.
  - i. What is the chance of drawing firstly 3 white and then 2 black balls?
  - ii. What is the chance of drawing 3 white and then 2 black balls if the order is irrelevant?(b) Consider a sample space  $\Omega$  comprising four possible outcomes  $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ . Suppose that to the four possible outcomes the following probabilities are assigned:

$$P(\omega_1) = 2/10, \quad P(\omega_2) = 3/10, \quad P(\omega_3) = 1/10, \quad P(\omega_4) = 4/10.$$

Define two events:  $E = \{\omega_1, \omega_2\}$ ,  $F = \{\omega_2, \omega_3\}$  Compute  $P(E|F)$ , the conditional probability of  $E$  given  $F$  !

4. (a) Compute the following improper integral!  $\int_1^\infty 1/x^{77} dx$
- (b) There are two urns containing colored balls. The first urn contains 100 red balls and 0 blue balls. The second urn contains 10 red balls and 90 blue balls. One of the two urns is randomly chosen (both urns have probability 50% of being chosen) and then a ball is drawn at random from one of the two urns. If a red ball is drawn, what is the probability that it comes from the first urn?