A. Test 1. Econ.Anal 13.Oct.21. NEPTUN: Name:

1. A. Compute the derivatives of the following functions!

   1. $\ln x \sin(2x)$
   2. $\cos(4x) \ln(2x-1)$
   3. $\frac{\ln(2x)}{x^2-1}$

B. What is the prediction of the linear approximation of the function $f(x)$ at $x = x_0$ for the value of $f(x_0 + \Delta x)$ ?
$f(x) = 1/(x+7), \; x_0 = e, \; \Delta x = 0.01.$

2. A. Study the monotonicity, convexity and local extremal values of the following function!
$f(x) = e^{-5x}x.$
Draw its graph!

B. Study the monotonicity of the following sequence!
$\frac{3n}{5n+6}.$

3.A. Compute the limit of the following sequence! $a_n = \left(1 + \frac{1}{3n}\right)^{3n-7} \frac{2n}{3n+1}.$

B. Let $\phi(x) = 1.1x + 1.6$, $x_0 = 13$, $x_{n+1} = \phi(x_n)$. What are $\phi^{-1}$ and $\phi^n(1) = x_n$ ?

   1. Find the fixed point $x_f$ of $\phi$ !
   2. Introduce $\Delta x = x - x_f$ and $\tilde{\phi}(\Delta x) = \phi(x_f + \Delta x) - x_f$. Calculate $\tilde{\phi}$ and $\tilde{\phi}^n$ !
   3. Compute $x_n$ !

4. A. Let $\bar{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\bar{v}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 12 \\ 82 \end{pmatrix} = \alpha \bar{v}_1 + \beta \bar{v}_2$. Compute $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ !

B. Let $T$ be a $2 \times 2$ matrix formed by the transition probabilities of a two state (labeled by 1 and 2) stochastic system, where
$T(1 \leftarrow 1) = T_{11} = 0.6$, $T(2 \leftarrow 1) = T_{21} = 0.4$, $T(1 \leftarrow 2) = T_{12} = 0.5$, $T(2 \leftarrow 2) = T_{22} = 0.5$.

   1. Find an eigenvector $\bar{v}_1$ corresponding to the eigenvalue $\lambda_1 = 1$ ! (This is the equilibrium state.)
   2. Find the eigenvalue $\lambda_2$ of $T$ corresponding to the eigenvector $\bar{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ !
   3. Calculate $\alpha$ and $\beta$ in $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \alpha \bar{v}_1 + \beta \bar{v}_2$ !
   4. Calculate $T(\alpha \bar{v}_1 + \beta \bar{v}_2)$, $T^2(\alpha \bar{v}_1 + \beta \bar{v}_2)$, etc.