C. Test 1. Econ.Anal 13.Oct.21. NEPTUN: Name:

1. A. Compute the derivatives of the following functions!

   1. $\sqrt[3]{1/x} + \frac{1}{(3x)^6} + \ln(3x)$
   2. $\ln(\sqrt[3]{x})$
   3. $x^{-7}\cos(2x-1)$

B. Let $f(x) = x^3$. Compute $\frac{f(5+\Delta x)-f(5)}{\Delta x}$! What is the limit of this fraction as $\Delta x \to 0$? What is $f'(5)$?

2. A. Study the monotonicity, convexity and local extremal values of the following function!

$f(x) = x\ln(x)$.

Draw its graph!

B. Study the monotonicity of the following sequence!

$(-1)^n \frac{3n+4}{5n+6}$.

3. A. Compute the limit of the following sequence! $a_n = \left(1+\frac{4}{3n}\right)^{7-3n}$.

B. Let $\phi(x) = -2x-12$, $x_0 = 13$, $x_{n+1} = \phi(x_n)$. What are ϕ^{-1} and $\phi^n(1) = x_n$?

   1. Find the fixed point x_f of ϕ !
   2. Introduce $\Delta x = x - x_f$ and $\tilde{\phi}(\Delta x) = \phi(x_f+\Delta x) - x_f$. Calculate $\tilde{\phi}$ and $\tilde{\phi}^n$!
   3. Compute x_n !

4. A. Let $\bar{n}_1 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$, $\bar{n}_2 = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$. Do $\bar{n}_1$ and $\bar{n}_2$ form and othonormed basis? Why? Let $\begin{pmatrix} 1 \\ 8 \end{pmatrix} = \alpha\bar{n}_1 + \beta\bar{n}_2$. Compute $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$!

B. Let T be a 2×2 matrix formed by the transition probabilities of a two state (labeled by 1 and 2) stochastic system, where
$T(1\leftarrow 1) = T_{11} = 0.8$, $T(2\leftarrow 1) = T_{21} = 0.2$, $T(1\leftarrow 2) = T_{12} = 0.5$, $T(2\leftarrow 2) = T_{22} = 0.5$.

   1. Find an eigenvector $\bar{v}_1$ corresponding to the eigenvalue $\lambda_1 = 1$! (This is the equilibrium state.)
   2. Find the eigenvalue λ_2 of T corresponding to the eigenvector $\bar{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$!
   3. Calculate α and β in $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \alpha\bar{v}_1 + \beta\bar{v}_2$!
   4. Calculate $T(\alpha\bar{v}_1+\beta\bar{v}_2)$, $T^2(\alpha\bar{v}_1+\beta\bar{v}_2)$, etc.